Premuneration Values, Investments, and Pricing in Matching Markets

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Introduction

- Many interesting economic problems involve matching of economic agents with complementary attributes.
- The output generated in a matched pair depends on the attributes of the members of the match.
- These attributes are affected by investments, which are typically made prior to matching.
- When ex ante contracting is impossible, inefficiencies may arise:
  - holdup problem, and
  - coordination failure.
Many interesting economic problems involve matching of economic agents with complementary attributes. The output generated in a matched pair depends on the attributes of the members of the match. These attributes are affected by investments, which are typically made prior to matching. When ex ante contracting is impossible, inefficiencies may arise:

- holdup problem, and
- coordination failure.

Does competition among potential partners ameliorate or exacerbate the inefficiencies?
Introduction
Part I: Personalized Prices and Efficiency

Competition for better matches can exactly offset the hold-up problem.
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Part I: Personalized Prices and Efficiency

• Competition for better matches can exactly offset the hold-up problem.

• Equilibria are constrained efficient equilibria if
  • the ex post matching market is competitive, and
  • prices are personalized—prices can be written as a function of the agents’ attributes.
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Equilibria may display
- underinvestment, or
- overinvestment.
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- prices are personalized—prices can be written as a function of the agents’ attributes.

Equilibria may display

- underinvestment, or
- overinvestment.

Moreover, the ex ante efficient outcome is an equilibrium.
What if prices cannot be personalized, but instead must be **uniform**, so that the price depends only on one side’s attribute? This might reflect

- unobservable attributes,
- prohibitive personalization costs, or
- legal or institutional prohibitions.
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**Premuneration values** now matter.

A matched agent’s premuneration value is her value from the match in the **absence of transfers** (i.e., pre-transfer).

Premuneration values are irrelevant if prices are personalized, but they are key to understanding the nature of the inefficiencies under uniform pricing.
The Papers

- **Part I:**
  - Cole, Mailath, Postlewaite (2001a, b): Infinite and finite population assignment games with prematch investments.

- **Part II:**
  - Mailath, Postlewaite, and Samuelson (2013a): Introduces premuneration values and a general one sided incomplete info model of prematch investments.

- **Related:**
Researchers $\rho \in [0, 1]$ and laboratories $\lambda \in [0, 1]$.

Researcher $\rho$ chooses attribute $r \in \mathbb{R}_+$, at cost $c(r, \rho)$, where $c$ satisfies the usual monotonicity and single crossing conditions.

Laboratory $\lambda$ chooses attribute $\ell \in \mathbb{R}_+$, at cost $\psi(\ell, \lambda)$.

Researchers and laboratories match, with a match between researcher $r$ and laboratory $\ell$ creating surplus $v(\ell, r)$, with $v$ supermodular.

Premuneration values:

$$v(\ell, r) = h_R(\ell, r) + h_L(\ell, r).$$
Suppose researcher and laboratory attribute choice functions \( r : [0, 1] \rightarrow \mathbb{R}_+ \) and \( l : [0, 1] \rightarrow \mathbb{R}_+ \) are strictly increasing when positive, and set

\[
\mathcal{R} := \text{cl}(r[0, 1]) \quad \text{and} \quad \mathcal{L} := \text{cl}(l[0, 1]).
\]

A feasible matching is \( \tilde{r} : \mathcal{L} \rightarrow \mathcal{R} \) and \( \tilde{\ell} : \mathcal{R} \rightarrow \mathcal{L} \) such that

\[
\tilde{r}(l(\lambda)) = r(\lambda) \quad \text{and} \quad \tilde{\ell}(r(\rho)) = l(\rho),
\]

so that

\[
\tilde{r}(\tilde{\ell}(r)) = r \quad \text{for all} \quad r \in \mathcal{R}.
\]

A feasible outcome is \(((r, l), (\tilde{r}, \tilde{\ell}))\).
Personalized Price Equilibrium

A personalized price is $p_P : \mathcal{R} \times \mathcal{L} \to \mathbb{R}_+$. 

**Definition**

Given $((r, l), (\tilde{r}, \tilde{l}), p_P)$, researcher $\rho$ is optimizing if

$$(r(\rho), \tilde{l}(r(\rho))) \in \arg \max_{(r, l) \in \mathcal{R} \times \mathcal{L}} h_R(r, l) - p_P(r, l) - c(r, \rho),$$

Laboratory $\lambda$ is optimizing if

$$(\tilde{r}(l(\lambda)), l(\lambda)) \in \arg \max_{(r, l) \in \mathcal{R} \times \mathcal{L}} h_L(r, l) + p_P(r, l) - \psi(l, \lambda).$$

A personalized price equilibrium is an outcome at which all agents are optimizing and no agent finds it profitable to deviate outside the set of chosen attributes.
A personalized price equilibrium is the equilibrium of a two stage game:

1. in the first stage, agents simultaneously choose attributes, and
2. in the second stage, as a function of the chosen attributes, agents are matched (and compensated) in a pairwise stable matching.

The second stage outcome is a stable outcome of the ex post assignment game with attributes $\mathcal{R}$ and $\mathcal{L}$. 
Ex Post Stability

Matching is stable in the ex post assignment game with attributes \( \mathcal{R} \) and \( \mathcal{L} \) if, for all pairs of attributes \( (r, \ell) \in \mathcal{R} \times \mathcal{L} \),

\[
\underbrace{h_R(r, \tilde{\ell}(r)) - p_P(r, \tilde{\ell}(r))}_{\text{researcher payoff}} + \underbrace{h_L(\tilde{r}(\ell), \ell) + p_P(\tilde{r}(\ell), \ell)}_{\text{laboratory payoff}} \geq v(r, \ell).
\]

Remark With personalized prices, the division of surplus into prenumeration values is irrelevant. Any reassignment of prenumeration values can be undone by prices.
Ex Post Stability

Matching is **stable** in the **ex post assignment game** with attributes $\mathcal{R}$ and $\mathcal{L}$ if, for all pairs of attributes $(r, \ell) \in \mathcal{R} \times \mathcal{L}$,

$$h_R(r, \tilde{\ell}(r)) - p_P(r, \tilde{\ell}(r)) + h_L(\tilde{r}(\ell), \ell) + p_P(\tilde{r}(\ell), \ell) \geq v(r, \ell).$$

**Remark**

With personalized prices, the division of surplus into premuneration values is irrelevant. Any reassignment of premuneration values can be undone by prices.
Ex Ante Stability

Given $((r, l), (\tilde{r}, \tilde{\ell}), p_P)$,

$$u_R(\rho) := h_R(r(\rho), \tilde{\ell}(r(\rho))) - p_P(r(\rho), \tilde{\ell}(r(\rho))) - c(r(\rho), \rho)$$

and

$$u_L(\lambda) := h_L(\tilde{r}(l(\lambda)), l(\lambda)) + p_P(\tilde{r}(l(\lambda)), l(\lambda)) - \psi(l(\lambda), \lambda).$$
Ex Ante Stability

Given \(( (r, l), (\tilde{r}, \tilde{\ell}), p_P) \),

\[ u_R(\rho) := h_R(r(\rho), \tilde{\ell}(r(\rho))) - p_P(r(\rho), \tilde{\ell}(r(\rho))) - c(r(\rho), \rho) \]

and

\[ u_L(\lambda) := h_L(\tilde{r}(l(\lambda)), l(\lambda)) + p_P(\tilde{r}(l(\lambda)), l(\lambda)) - \psi(l(\lambda), \lambda). \]

The matching is stable in the ex ante assignment game if, for all researcher-laboratory pairs \((\rho, \lambda) \in [0, 1]^2\),

\[ u_R(\rho) + u_L(\lambda) \geq \max_{r, \ell} v(r, \ell) - c(r, \rho) - \psi(\ell, \lambda). \]
Stability Comparison

A matching is **stable** in the ex post assignment game with attributes $\mathcal{R}$ and $\mathcal{L}$ if, for all pairs of attributes, $(r, \ell) \in \mathcal{R} \times \mathcal{L}$,

$$h_R(r, \tilde{\ell}(r)) - p_P(r, \tilde{\ell}(r))
\geq h_L(\tilde{r}(\ell), \ell) + p_P(\tilde{r}(\ell), \ell) + v(r, \ell).$$
Stability Comparison

A matching is stable in the ex post assignment game with attributes $\mathcal{R}$ and $\mathcal{L}$ if, for all pairs of attributes, $(r, \ell) \in \mathcal{R} \times \mathcal{L}$,

$$h_R(r, \tilde{\ell}(r)) - p_P(r, \tilde{\ell}(r))$$

$$+ h_L(\tilde{r}(\ell), \ell) + p_P(\tilde{r}(\ell), \ell)$$

$$\geq v(r, \ell).$$

A matching is stable in the ex ante assignment game if, for all researcher-laboratory pairs $(\rho, \lambda) \in [0, 1]^2$, with $r = \tau(\rho)$ and $\ell = l(\lambda),$

$$h_R(r, \tilde{\ell}(r)) - p_P(r, \tilde{\ell}(r)) - c(r, \rho)$$

$$+ h_L(\tilde{r}(\ell), \ell) + p_P(\tilde{r}(\ell), \ell) - \psi(\ell, \lambda)$$

$$\geq \max_{r', \ell'} v(r', \ell') - c(r', \rho) - \psi(\ell', \lambda).$$
Example

- $v(r, \ell) = \begin{cases} r\ell & \text{if } r\ell \leq \frac{1}{2}, \\ 2(r\ell)^2 & \text{if } r\ell > \frac{1}{2}. \end{cases}

- $c(r, \rho) = \frac{2r^5}{\rho + 2}$.

- $\psi(\ell, \lambda) = \frac{2\ell^5}{\lambda + 2}$.

- ex ante efficient attribute choices:

$$t^*(i) = l^*(i) = \begin{cases} \frac{3\sqrt{(i + 2)/10}}{10}, & \text{if } i \leq i^*, \\ \frac{(2i + 4)/5}{5}, & \text{if } i > i^*, \end{cases}$$

where $i^* \approx 0.1$. 

Example

net surplus
Example
attribute choices

\[ r^*, l^* \]
Theorem

Suppose \(((r, l), (\check{r}, \check{l}), p_P)\) is a personalized price equilibrium. Then, \(h_R(r, \check{l}(r)) - p_P(r, \check{l}(r))\) is differentiable where both \(\tau\) and \(l\) are continuous, and

\[
\frac{d}{dr} [h_R(r, \check{l}(r)) - p_P(r, \check{l}(r))] = \frac{\partial v(r, \check{l}(r))}{\partial r}.
\]

A similar statement holds for laboratories.
Efficiency

Theorem

Suppose \(((\tau, l), (\tilde{r}, \tilde{\ell}), p_P)\) is a personalized price equilibrium. Then, \(h_R(r, \tilde{\ell}(r)) - p_P(r, \tilde{\ell}(r))\) is differentiable where both \(\tau\) and \(l\) are continuous, and

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\]

A similar statement holds for laboratories.

Theorem

There exists a personalized price \(p_P\) such the ex ante efficient allocation is part of a personalized price equilibrium.
Constrained Efficiency

Definition

The allocation \(((r, l), (\tilde{r}, \tilde{l}), \rho, \tilde{\rho}), p_P\), is constrained efficient if

\[ u_R(\rho) + u_L(\lambda) \geq \sup_{r \in \mathbb{R}^+, \ell \in \mathcal{L}} v(r, \ell) - c(r, \rho) - \psi(\ell, \lambda) \]

and

\[ u_R(\tilde{\rho}) + u_L(\tilde{\lambda}) \geq \sup_{r \in \mathcal{R}, \ell \in \mathbb{R}^+} v(r, \ell) - c(r, \rho) - \psi(\ell, \lambda). \]
Constrained Efficiency

**Definition**

The allocation \(((r_l, l), (\tilde{r}, \tilde{l}), P_P)\), is constrained efficient if

\[
u_R(\rho) + u_L(\lambda) \geq \sup_{r \in \mathbb{R}^+, \ell \in \mathcal{L}} v(r, \ell) - c(r, \rho) - \psi(\ell, \lambda)
\]

and

\[
u_R(\rho) + u_L(\lambda) \geq \sup_{r \in \mathcal{R}, \ell \in \mathbb{R}^+} v(r, \ell) - c(r, \rho) - \psi(\ell, \lambda).
\]

**Theorem**

*Every personalized price equilibrium outcome is constrained efficient.*
Uniform Pricing

- Suppose researcher attributes are not public information (prices cannot condition on $r$).
- A laboratory deviating to an unpriced attribute now needs beliefs over the researcher attributes it attracts.
- Mailath, Postlewaite and Samuelson (2013a) introduces a general model of uniform pricing, establishing
  - sufficient conditions for existence of equilibrium, and
  - necessary and sufficient conditions for equilibria to be efficient.
- Mailath, Postlewaite and Samuelson (2013b) imposes more structure. In particular:
  - only one side makes investment decisions, and
  - the cost function and premuneration values have a simple functional form.
The Extended Example

- Researchers choose attributes $r$, at cost

\[ c(r, \rho) = \frac{r^{2+k}}{(2 + k)\rho^k}. \]

- Laboratories have attributes $\ell = \lambda$,

- Researchers and laboratories match, with a match between researcher $r$ and laboratory $\ell$ creating surplus $\ell r$.

- Premuneration values are given by
  - for the researcher
    \[ h_R(r, \ell) = \theta \ell r, \text{ and} \]
  - for the laboratory
    \[ h_L(r, \ell) = (1 - \theta)\ell r. \]
Equilibrium

Definition

A price function $p$ and researcher choices $(\ell_R, r_R)$ constitute a matching equilibrium if,

1. for every $\rho \in [0, 1]$, the choice $(\ell_R(\rho), r_R(\rho))$ solves the researcher optimization problem,

   $$\max_{\ell, r} \theta \ell r - p(\ell) - \frac{r^{2+k}}{(2 + k)\rho^k}.$$

2. every researcher and laboratory earns nonnegative payoffs, and

3. $\ell_R$ is market-clearing (i.e., $\ell_R$ is 1:1, onto and measure preserving).
Some Equilibrium Properties

Lemma

Every equilibrium price function \( p \) is strictly increasing and continuous.

Lemma

The equilibrium researcher attribute-choice function \( r_R \) is strictly increasing.

Lemma

The equilibrium researcher laboratory-choice function \( \ell_R \) is given by

\[
\ell_R(\rho) = \rho.
\]
The Efficient Outcome

Efficiency requires that each pair $\rho = \lambda$ match and maximize their surplus:

$$\max_r \rho r - \frac{r^{2+k}}{(2 + k)\rho^k}.$$ 

The first-order condition is

$$\rho = \frac{r^{1+k}}{\rho^k},$$

immediately implying

$$r = \rho.$$
The Equilibrium Outcome

Researcher $\rho$’s problem is to choose $\ell$ and $r$ to maximize

$$\theta \ell r - p(\ell) - c(r, \rho) = \theta \ell r - p(\ell) - \frac{r^{2+k}}{(2 + k)\rho^k}.$$

The first order conditions are

$$\theta \ell = \frac{r^{1+k}}{\rho^k}.$$

and

$$\theta r = p'(\ell).$$
Theorem

The unique matching equilibrium is given by

\[ r_R(\rho) = \rho \cdot \theta^{\frac{1}{1+k}}. \]

and

\[ p(\ell) = \frac{1}{2} \ell^2 \cdot \theta^{\frac{2+k}{1+k}}. \]

If \( \theta < 1 \), so that the laboratory premuneration value share is positive, then in equilibrium, researchers invest less than the efficient level.
Equilibrium Payoffs

\[ u_L(\theta, k, \lambda) = \frac{1}{2} \theta^{\frac{1}{1+k}} (2 - \theta) \lambda^2. \]
Suppose laboratories could, at fixed cost $\kappa$, acquire a technology allowing them to observe researchers’ types.

- Some laboratories would choose to do so, for sufficiently small $\kappa$.
- Some laboratories would not do so.
- We consider an equilibrium in which laboratories $\lambda > \tilde{\lambda}$ acquire information, laboratories $\lambda < \tilde{\lambda}$ do not.
Informed Laboratories \((\lambda \geq \tilde{\lambda})\)

Equilibrium prices

- Informed laboratories can set different prices for different researchers.
- For appropriate values of \(\phi\), the personalized price function

\[
\hat{p}(\ell, r) = \phi + \frac{\ell^2}{2} - (1 - \theta)\ell r
\]

implies efficient investments (because one-sided).

Researcher \(\rho\)'s payoff from \(\ell\) and \(r\) is

\[
\theta \ell r - \hat{p}(\ell, r) - \frac{r^{2+k}}{(2 + k)\rho^k} = \ell r - \phi - \frac{1}{2} \ell^2 - \frac{r^{2+k}}{(2 + k)\rho^k}.
\]

Maximizing the payoff yields \(\ell = \rho\) and \(r = \rho\).

- The value of \(\phi\) will be determined by the laboratory incentives to become informed.
Informed Laboratories

Exogenously fixed $\tilde{\lambda}$

- If $\tilde{\lambda} = 0$, then individual rationality implies $\phi = 0$, and the equilibrium is unique.
- If $\tilde{\lambda} > 0$, there is a one parameter family of equilibrium price functions, indexed by

\[
\phi \in [-\tilde{u}_L(\theta, k, \tilde{\lambda}), \tilde{u}_R(\theta, k, \tilde{\lambda})] = \left[ \frac{-\tilde{\lambda}^2}{2}, \frac{k\tilde{\lambda}^2}{2(2 + k)} \right];
\]

the bounds on $\phi$ come from the participation constraint.
- The total net surplus of the pair with index $\rho$ is

\[
\frac{k}{2(2 + k)}\rho^2 + \frac{1}{2}\rho^2 = \frac{(1 + k)}{2 + k}\rho^2,
\]

which is the maximum (i.e., efficient) value of the $\rho$-match.
Surpluses

$\theta = \frac{1}{4}$ and $k = 1$

$\rho_\tilde{\lambda} = 0.5$

maximum surplus $= \frac{2}{3} \rho^2$

$u_L + u_R = \frac{11}{24} \rho^2$

uninformed surplus

$\frac{16}{96} \approx 0.17$
$\frac{11}{96} \approx 0.11$
$\frac{2}{3} \approx 0.67$
$\frac{11}{24} \approx 0.46$
Endogenously Informed Laboratories

Researcher attribute choices

- \( \theta = \frac{1}{4}, k = 1 \) and \( \kappa = \frac{5}{96} \), so \( \tilde{\lambda} = \frac{1}{2} \).
- Laboratories with \( \lambda \geq \frac{1}{2} \) informed and \( \lambda < \frac{1}{2} \) uninformed.
Comparative Statics w.r.t. $\theta$

Black is $\theta = \frac{1}{4}$, red is $\theta = \frac{1}{9}$

$\lambda \hat{u}_L - \kappa + \phi$

$\lambda' \hat{u}'_L - \kappa + \phi'$

$\lambda = 0.5$

$\lambda' \approx 0.388$
Comparative Statics w.r.t. $\theta$

Black is $\theta = \frac{1}{4}$, red is $\theta = \frac{1}{9}$

$\tilde{\lambda}' \approx .388$

$\tilde{\lambda} = .5$

$\hat{u}'_R - \phi'$

$\hat{u}_R - \phi$

researcher payoffs

0 .001 .005 .023 .036

\rho
Laboratories Invest

- The Researchers’ Problem:
  - Researcher $\rho$ has exogenously determined attribute $r = \rho$.
  - Given the price function $p^*$, researcher $\rho$ chooses $\ell$ to maximize
    \[ \theta \ell \rho - p^*(\ell). \]

- The Laboratories’ Problem:
  - The cost of attribute $\ell \in \mathbb{R}_+$ to laboratory $\lambda$ is
    \[ \psi(\ell, \lambda) = \frac{\ell^{2+k}}{(2 + k)\lambda^k}, \quad k \in \mathbb{R}_+. \]
  - Laboratories choose attributes to maximize
    \[ (1 - \theta)\ell r^*_L(\ell) + p^*(\ell) - \psi(\ell, \lambda). \]
Equilibrium

Definition

A price function $p$, matching function $r^*_L$, and strictly increasing laboratory attribute choices $(\ell^*_L, \ell^*_R)$ constitute a matching equilibrium if

1. $\ell^*_R(\rho)$ is an optimal laboratory attribute for researcher $\rho$, for all $\rho \in [0, 1]$,
2. $\ell^*_L(\lambda)$ is an optimal laboratory attribute for laboratory $\lambda$, for all $\lambda \in [0, 1]$,
3. every researcher and laboratory earns nonnegative payoffs, and
4. markets clear: $r^*_L(\ell^*_R(\rho)) = \rho$ for all $\rho \in [0, 1]$ and $\ell^*_R(\lambda) = \ell^*_L(\lambda)$ for all $\lambda \in [0, 1]$. 
Theorem

A Matching Equilibrium is a vector \((p^*, r_L^*, \ell_R^*, \ell_L^*)\), where

\[
\begin{align*}
    p^*(\ell) &= \frac{\theta \ell^2}{2\alpha}, & \ell &\in \mathbb{R}_+, \\
    r_L^*(\ell) &= \frac{\ell}{\alpha}, & \ell &\in [0, \alpha], \\
    \ell_R^*(\lambda) &= \ell_L^*(\lambda) = \alpha \lambda, & \lambda &\in [0, 1],
\end{align*}
\]

for \(\alpha = (2 - \theta) \frac{1}{k+1}\). Laboratory payoffs are given by

\[
u_L^*(\theta, k, \lambda) = \frac{k}{2(k+2)}(2 - \theta)^{(k+2)/(k+1)} \lambda^2.
\]

Researcher payoffs are given by

\[
u_R^*(\theta, k, \rho) = \frac{1}{2} \theta(2 - \theta)^{1/(k+1)} \rho^2.
\]
Properties of Equilibrium

- If $\theta = 1$, the outcome is efficient.
- If $\theta < 1$, laboratories overinvest.
- Researchers’ payoffs increase in $\theta$; laboratories’ payoffs decrease in $\theta$. 
Premuneration values affect investment incentives.

When researchers’ attributes cannot be observed and \( \theta < 1 \), researchers underinvest.

Increasing \( \theta \) can mitigate this inefficiency: Labs’ eq payoffs increase in \( \theta \) if for small \( \theta \), and decrease in \( \theta \) otherwise. Researchers’ payoffs increase in \( \theta \).

When lab’s can become informed (at some cost), \( \theta \) determines which lab’s choose to become informed and the payoffs of all lab’s (both informed and uninformed) and researchers. Lab’s may gain by having \( \theta \) increase. The impact on researchers is even more ambiguous.

When laboratories invest, laboratories overinvest in order to more effectively compete for researchers.
Discussion

Competition

- $k = 0$ researchers are homogeneous, and so competitive.
- As $\rightarrow \infty$, researchers become more heterogenous.
- When researchers invest, as $k$ increases, there are enhanced investment incentives arising from reduced competition and reduced investment costs.
- When laboratories invest, reduced researcher competition leads to smaller laboratory investments.
- Lab payoffs under either scenario increase with $k$.
  - As $k \rightarrow \infty$, enhanced researcher investment incentives (when researchers make investments), and the reduced cost of investments (when laboratories make investments) dominate the impact of reduced competition for laboratories.
Discussion

Competition

maximum surplus

\[ u^* \left( \frac{1}{2}, k, 1 \right) + u^* \left( \frac{1}{2}, k, 1 \right) \]

\[ u_L \left( \frac{1}{2}, k, 1 \right) + u_R \left( \frac{1}{2}, k, 1 \right) \]

\[ u_L \left( \frac{1}{2}, k, 1 \right) \]

\[ u_R \left( \frac{1}{2}, k, 1 \right) \]
References