Quantifying Conceptual Flexibility in a Compositional Network Model

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Abstract
A single concept can manifest in many varied forms, depending on the context in which it is activated. That is, concepts appear to be flexible rather than static. Here we implement a compositional model of conceptual knowledge in which basic-level concepts are represented as graph theoretical networks, with the specific goal of quantifying conceptual flexibility. We collect within-concept statistics using online participants, construct network models, and validate these models in a classification analysis. We then extract network measures and find that network diversity and core-periphery structure correspond to conceptual flexibility and stability, respectively. These results suggest that a compositional network model can be used to extract formal measures that are interpretable and useful in the study of conceptual knowledge.

Keywords: conceptual knowledge, network science, flexibility, semantics

Introduction
The APPLE information evoked by “apple pie” is considerably different from that evoked by “apple picking”: the former representation is soft, warm, and wedge-shaped, whereas the latter is firm, cool, and spherical. Though APPLE is considered to be one basic-level concept, its information content can be flexibly adjusted to reflect contextual demands. This conceptual flexibility enables concepts to be represented in varied and fluid ways, a central characteristic of the semantic system that has not yet been captured in a formal model of conceptual knowledge.

Perhaps the most basic form of conceptual flexibility is that a single concept has many distinct subordinates that differ from each other. The concept APPLE can be instantiated as a Granny Smith or as a Macintosh, and either one can easily be brought to mind. But even a representation of a single token of APPLE can be flexibly adjusted: activated properties might be RED and ROUND while shopping, whereas they might be SWEET and CRISPY while eating. A concept can also be represented in varied states, each with its own distinct features: the representation of an APPLE is FIRM versus SOFT before and after baking, and SOLID versus LIQUID before and after juicing. Conceptual flexibility is further evidenced in the frequent non-literal use of concepts: one should stay away from “bad apples” and should not “compare apples with oranges;” and, one can use concepts fluidly in novel analogies and metaphors. Though conceptual flexibility is a pervasive phenomenon, it poses a formidable challenge: what kind of conceptual structure permits this flexibility to occur?

In vector-based approaches, concepts are represented as vectors of features. These features can span a range of information-types (e.g., visual, functional, encyclopedic), consistent with a distributed account of conceptual knowledge (e.g., McRae et al., 1997; Tyler & Moss, 2001). Models that represent basic-level concepts in terms of their constituent features are valuable because they can be implemented in computational architectures such as parallel distributed processing models and attractor networks (e.g. Cree et al., 1999). More generally, they provide an account of concepts’ internal structure, which is arguably essential for a theory of conceptual flexibility. However, the feature-vectors that represent individual concepts are static and unchanging — a clear limitation if one aims to incorporate flexibility into conceptual structure.

In current network-based approaches, individual concepts are characterized in terms of their relation to other concepts by virtue of their word-association strengths or text-based co-occurrence statistics. In this framework, concepts are represented as nodes in a network, and their relations are encoded as the links, or edges, between them (e.g., Steyvers & Tenenbaum, 2005; De Deyne et al, 2016). These models are valuable because semantic structure can be analyzed using a rich set of network science tools. However, current network-based implementations do not provide the internal conceptual structure that is necessary — we argue — to model conceptual flexibility. In other words, it is hard to provide a model of conceptual flexibility (in the sense described above) when the features that are being flexibly adjusted are not explicitly represented.

Here we introduce a new model in which concepts are represented as their own feature-based networks. We believe that a feature-based conceptual framework paired with network science techniques provides a platform on
which to model conceptual flexibility. In our concept-specific networks, nodes represent individual features and edges (i.e., the links between the nodes) represent the statistical relationship between features within that concept. That is, edges capture the extent to which certain properties tend to covary with each other within a concept. The creation of such networks thus requires the calculation of within-concept statistics. These statistics provide the scaffolding to build our networks, and also reveal how a concept’s information may be appropriately adjusted to form valid, yet varied, instances of that concept. Our specific goals here are (1) to show that creation of such networks is possible, (2) that these networks contain concept-specific information, and (3) that they permit the extraction of formal measures of conceptual flexibility.

Another phenomenon relating to conceptual flexibility is the distinction between context-independent and context-dependent conceptual properties (here, we use this synonymously with “features”; Barsalou, 1982). Context-independent properties are those that are automatically activated for a concept in all contexts, and are sometimes referred to as “core” properties. Context-dependent properties are those that are only activated when the context renders them relevant. In the apple example, sweet and has skin may be context-independent and –dependent properties, respectively. Concepts are composed of both kinds of properties, such that some properties are stable and occur across all instances, and some are more variable and only occur some of the time. Furthermore, some concepts may have a stronger “core” than others, and this might relate to the flexibility of those concepts. One of our additional goals was thus to extract network-based measures that characterize this element of conceptual structure in a formal way.

Many networks by their very nature permit flexibility, because a single network can support different states, each characterized by different patterns of activation across nodes. This kind of network flexibility is determined by the connections between nodes and how those connections give rise to a larger network structure. Most natural systems exhibit “small-world” network structure (Bassett & Bullmore, 2006), which means that there are clusters of nodes in a network with strong connections between them. These are called “modules”, and nodes can interact with these modules in different ways. Some nodes may have links that are highly distributed across the modules in a network, whereas other nodes may have links only in one module. This tendency is captured in the diversity coefficient, a version of the participation coefficient calculated using normalized Shannon entropy. We interpreted network diversity as a likely candidate for a formal flexibility measure, and predicted that it would correlate with a measure of “semantic diversity” calculated separately using word co-occurrence statistics (SemD; Hoffman et al., 2013).

Network science also provides techniques for assessing core-periphery structure (Borgatti & Everett, 2000). In network terms, a core is a set of nodes that are densely interconnected and therefore often co-activated, whereas the periphery consists of nodes with sparser connections. A measure can be extracted that represents the extent to which a given network has a core-periphery structure; some networks might have more prominent cores than others. We hypothesized that this construct of core-periphery structure could provide a way to formally capture the notion of context-dependent and context-independent conceptual properties. More specifically, concepts characterized by large sets of context-independent properties might correspond with networks characterized by a strong core-periphery structure. It also seems reasonable to suggest that concepts with a stronger core might be less flexible in the ways described above. If we interpret a core as a set of properties whose activation patterns are stable across contexts, then there is less room for variability in the expression of these properties, and therefore less flexibility overall. We therefore predicted a negative relationship between our network measures of flexibility and core-periphery structure.

**Methods**

**General Methods**

**Network Construction** In order to create our networks we first had to define our nodes. Since our nodes represent individual conceptual properties, we compiled a list of properties that applied to all of our target concepts. Participants were recruited from Amazon Mechanical Turk and were asked to list all of the properties that must be true or can be true for each concept. It was emphasized that the properties do not have to be true of all types of the concept. Participants were required to report at least 10 properties per concept, but there was no limit on the number of responses they could provide. Once these data were collected, we organized the data as follows. For each concept, we collapsed across different forms of the same property (e.g., “sugar”, “sugary”, “tastes sugary”), and removed responses that were too general (e.g., “taste”, “color”). For each concept, we only included properties that were given by more than one participant. We then combined properties across all concepts to create our final list of \( N \) properties that will be represented as nodes in our concept networks.

The same participants also provided “sub-concepts”: these included subordinate concepts and possible concept states (e.g., chocolate chips, wine bottle). For each concept, participants were asked to think about that object and all the different kinds, forms, types, or states in which that object can be found. For each concept, we removed responses that we considered properties rather than types (e.g., “sweet chocolate”), and responses that were non-generic trademarks (e.g., “Chiquita banana”). We only included responses that were given by more than one participant, resulting in a set of \( K \) sub-concepts for each concept. It should be noted that the classification of “concepts” and “sub-concepts” is arbitrary: networks could theoretically be
constructed at any level of the conceptual hierarchy (e.g., FOOD, CHOCOLATE, DARK CHOCOLATE). We chose to model basic-level concepts in the present work.

A separate set of participants was presented with one sub-concept of each of the target concepts in random order (e.g., “chocolate chips”, “frozen banana”), and were asked to select the properties that are true of that specific sub-concept. The full list of \( N \) properties was displayed in a multiple-choice format. For each sub-concept, responses were combined across participants and represented in a binary fashion. To reduce noise, a property was only considered “true” for a sub-concept if more than one participant made that response. At this point, each concept’s data include a set of \( K \) sub-concepts, each of which corresponds to a \( N \)-length vector that indicates the presence or absence of each property. Each sub-concept is weighted equally. We can also view these data as a set of \( N \) conceptual properties, each of which corresponds to a \( K \)-length vector that indicates its presence or absence in each of the sub-concepts.

For each concept, we excluded properties that were not present in any of the sub-concepts, resulting in a smaller set of \( M \) properties. We created a network by correlating the \( M \) binary property-vectors with each other to create a \( M \times M \) symmetrical, weighted correlation matrix. These networks were filtered using the triangulation filtering method in order to remove spurious correlations (e.g., Massara et al., 2016). This filtering approach generates a simpler subgraph that maximizes information content while reducing the influence of noise, and is appropriate for graphs where edges are defined as correlations between nodes, as is the case here. No parameter fitting is required to apply the filter. These final, filtered concept networks were then analyzed using standard network science methods.

We created two sets of randomly-selected object concepts such that our results would not be specific to particular node definitions or network sizes. For all analyses, we used rank-based (spearman) correlations and an alpha criterion of 0.05.

**Classification Analysis** Our primary goal is to extract measures from concept networks that relate to individual concept’s flexibility; this will only work if our networks differ across concepts. In order to establish that this is the case, we ran a classification analysis to confirm that our networks could discriminate between new concept exemplars. Exemplar data were generated from sets of photographs for each concept; there was at least one image for each sub-concept, though co-existing states (e.g. dark chocolate, chocolate chips) precluded a one-to-one mapping. AMT participants were shown one image per concept, were presented with the full list of \( N \) properties in multiple-choice format, and were asked to select the properties that they believed applied to the object in the image. Individual participants’ responses to each sub-concept were represented as \( N \)-length property vectors and were used as test data in the classification analysis.

By performing eigendecomposition on each concept network (i.e., adjacency matrix) we can assess the extent to which a property vector is expected given an underlying network structure (e.g., Medaglia et al., 2017). For each adjacency matrix \( A, V \) is the set of \( N_c \) eigenvectors, ordered by eigenvalue. \( M \) is the number of ordered eigenvectors to include in analysis, and designates a subset of \( V \). For each eigenvector \( v \), we find the dot product with signal vector \( x \), which gives us the projection of \( x \) on that dimension in the eigenspace of \( A \). That is, it gives us an “alignment” value for that particular signal and that particular eigenvector. We can include all eigenvectors in \( M \) by taking the sum of squares of the dot products for each eigenvector. The alignment value for each signal is defined as

\[
\mathcal{X} = \sum_{i=1}^{M} (v_i \cdot x)^2, \tag{1}
\]

where \( x \) is a property vector, \( M \) is the number of eigenvectors to include in alignment (sorted by eigenvalue), \( v_i \) is one of \( M \) eigenvectors of the adjacency matrix, and \( \mathcal{X} \) is the scalar alignment value for signal \( x \) with adjacency matrix \( A \), given the eigenvectors \( 1:M \). In our case, signal \( x \) is a property vector corresponding to a particular exemplar image, which we align with each of the concept networks. Each exemplar was restricted to the properties included in each concept model before transformation; that is, exemplar data \( (x) \) were reduced to \( N_c \)-length vectors. The concept network that resulted in the highest alignment value \( (\mathcal{X}) \) was taken as the “guess” of the classifier; each exemplar was either classified correctly (1), or incorrectly (0). We averaged these data across all exemplars to calculate the average classifier accuracy. To calculate a baseline measure of classification accuracy, we created traditional vector models for each concept. For each concept, we averaged the \( K \) sub-concept vectors resulting in an \( N_c \)-length vector containing mean property strength values. Each concept’s traditional vector model and network model contained the same conceptual properties. We ran a separate classification analysis using these traditional models and a correlational classifier. Each exemplar property-vector was correlated with each of the traditional concept vector models; the concept model that resulted in the highest correlation value was taken as the guess of the classifier. We calculated average measures of classifier performance using the same methods described above, and also calculated classification accuracy within each concept.

**Network Analysis** We extracted network metrics from our concept networks using the Brain Connectivity Toolbox (Rubinov & Sporns, 2010). The set of nodes in each network is designated as \( N \), and \( n \) is the number of nodes. The set of links is \( L \), and \( l \) is the number of links. The existence of a link between nodes \((i,j)\) is captured in \( a_{ij} \): \( a_{ij} = 1 \) if a link is present and \( a_{ij} = 0 \) if a link is absent. The weight of a link is represented as \( w_{ij} \), and is normalized such that \( 0 \leq w_{ij} \leq 1 \). \( l^w \) is the sum of all weights in the
network. The network metrics we extracted included node strength, node degree, modularity (Q), core-periphery structure, and diversity coefficients.

Nodes within a network differ in the number and strength of their connections to other nodes. Node degree (k) is the number of connections that each node has with other nodes in the network (Eq. 2; Rubinov & Sporns, 2010). In weighted (i.e., non-binary) networks, node strength (kwi) is calculated by summing the weights of the connections with other nodes (Eq. 3; Rubinov & Sporns, 2010). We separately averaged node strength and node degree within each network to obtain mean strength and degree measures for each concept network.

\[ k_i = \sum_{j \in N} a_{ij} \]  
\[ k_i^w = \sum_{j \in N} w_{ij} \]

Modularity (Q) is a metric that describes a network’s community structure. We can attempt to partition a weighted network into sets of non-overlapping nodes (i.e., modules) such that within-module connections are maximized and between-module connections are minimized. Some networks exhibit more of a modular structure than others; Qwi is a quantitative measure of modularity for each weighted network (Eq. 4; Rubinov & Sporns, 2010).

\[ Q^w = \frac{1}{N} \sum_{i,j \in N} \left[ w_{ij} - \frac{k_i^w k_j^w}{\langle w \rangle} \right] \delta_{m_i m_j} \]  

where \( \delta_{m_i m_j} = 1 \) if nodes \( i,j \) are in the same module \( m \), \( w_{ij} \) is the specific strength between nodes \( i,j \), and \( \frac{k_i^w k_j^w}{\langle w \rangle} \) scales \( w_{ij} \) by the total strengths of nodes \( i,j \) across the network. Given a network’s community structure, we can observe how individual nodes participate with each of the modules in the set of modules (M): Nodes may have connections to many different modules, or have very few such connections. The diversity coefficient \( (h_i^\pm) \) is a measure ascribed to individual nodes that reflects the diversity of connections that each node has to modules in the network. This is a version of the participation coefficient, and is calculated using normalized Shannon entropy; we have previously used entropy to model property flexibility, and so predicted that diversity would be a good candidate for a network-based measure of conceptual flexibility. The diversity coefficient (Eq. 5; Rubinov & Sporns, 2011) for each node is defined as

\[ h_i^\pm = -\frac{1}{\log m} \sum_{u \in M} p_i^+(u) \log p_i^+(u) \]

where \( p_i^+(u) = \frac{s_i^+(u)}{s_i^+} \), \( s_i^+(u) \) is the strength of node \( i \) within module \( u \), and \( m \) is the number of modules in modularity partition \( M \). We averaged diversity coefficients across nodes in a network to obtain a mean measure of diversity for each concept network.

Core-periphery structure is another way to describe the structure of a network. Here, we attempt to partition a network into two non-overlapping sets of nodes such that connections within one set are maximized (i.e., the “core”) and connections in the other are minimized (i.e., the “periphery”). Core-periphery fit (\( Q_c \)) is a quantitative measure of how well each network can be partitioned in this way (Eq. 5), and can be defined as

\[ Q_c = \frac{1}{v_c} \left( \Sigma_{i,j \in C_c} (w_{ij} - y_c \bar{w}) - \Sigma_{i,j \in C_p} (w_{ij} - y_c \bar{w}) \right) \]  

where \( C_c \) is the set of all nodes in the core, \( C_p \) is the set of nodes in the periphery, \( \bar{w} \) is the average edge weight, \( y_c \) is a parameter controlling the size of the core, and \( v_c \) is a normalization constant (Rubinov et al., 2015).

**Methods: Set 1**

The 5 concepts used in Set 1 were CHOCOLATE, BANANA, BOTTLE, TABLE, and PAPER. Participants on Amazon Mechanical Turk (N=66) provided general properties for each concept along with sub-concepts. An additional group of participants (N=198) made property judgments on specific sub-concepts, and an additional group of participants (N=60) generated test data for the classification analysis by making property judgments on individual images.

The final property list included 129 properties. The number of sub-concepts for each concept were as follows: chocolate=14, banana=15, bottle=11, table=14, paper=20.

In the classification analysis, test data comprised a total of 300 property-vectors, with 60 exemplars/concept.

**Methods: Set 2**

The 10 concepts used in Set 2 were KEY, PUMPKIN, GRASS, COOKIE, PICKLE, KNIFE, PILLOW, WOOD, PHONE, and CAR.

Participants on Amazon Mechanical Turk (N=60) provided general properties for each concept along with sub-concepts. An additional group of participants (N=108) made property judgments on specific sub-concepts, and an additional group of participants (N=30) generated test data for the classification analysis by making property judgments on individual images.

The final property list included 276 properties. The number of sub-concepts for each concept were as follows: key=19, pumpkin=18, grass=16, cookie=22, pickle=17, knife=15, pillow=16, wood=22, phone=16, car=20.

In the classification analysis, test data comprised 300 property-vectors, with 30 exemplars/concept.

**Results**

**Classification Results**

In order to determine whether our concept networks contained concept-specific information, we ran a
classification analysis using eigendecomposition for both Set 1 and Set 2. We ran multiple analyses using different ranges of eigenvectors, which were sorted by eigenvalue (positive to negative). We started by only using the first eigenvector in each of the concept networks and determined whether this dimension alone could be used to classify the property vector. One dimension was enough to classify exemplars in Set 2 (Mean Accuracy=0.27; SE=0.03; Chance=0.10) but not Set 1 (M=0.11; SE=0.02; Chance=0.20). Increasing the number of dimensions improved classification performance for both sets (Fig. 1): for example, classification performance is significantly above chance when only 10 dimensions are used in Set 1 (M=0.38; SE=0.03; Chance=0.10) and Set 2 (M=0.38; SE=0.03; Chance=0.20). As more dimensions were included in the analysis, classification performance approaches that of the vector-based classifier. The increased success of the vector-based model (Set 1: M=0.85, SE=0.03, Chance=.20; Set 2: M=0.84, SE=0.03, Chance=0.10) suggests that the presence or absence of individual features is highly informative for discriminating between concepts. However, the success of the network-based model suggests that our concept networks do contain concept-specific information, motivating us to look within a concept for structural elements that relate to conceptual flexibility. It is this main goal that we pursue in the subsequent analyses.

Network Measures of Conceptual Structure
Networks across the two sets differed in node assignments, since they were constructed using different properties. However, once classification and network measures were extracted, we could pool the concepts together (N=15) and examine relationships between these network-related measures and other variables of interest.

We extracted network measures from the concept networks and explored how they relate to cognitive measures of conceptual flexibility and stability. Hoffman et al. (2013) use word co-occurrence statistics to quantify the context-dependent variations in word meanings found in language. The authors provide a measure of semantic diversity (SemD) that captures this variability, and we extracted SemD values for our 15 concepts. We also extracted their reported mean cosine similarity of a word’s contexts and used this as a measure of semantic stability (which we refer to as SemS). As expected, SemD negatively correlated with SemS across our 15 concepts (r(15)=−0.96, p<0.0001).

One of our primary goals was to extract a network measure that reflects conceptual flexibility. We used SemD (Hoffman et al., 2013) as a benchmark for conceptual flexibility and determined whether our hypothesized network measures of flexibility correlated with SemD across our 15 concepts. A priori, we hypothesized that the mean diversity (i.e., the average of a concept network’s diversity coefficients across nodes) could reflect conceptual flexibility. This network measure captures the extent to which properties within a concept associate with different modules, or property clusters. Another possible candidate measure was network modularity, which reflects the extent to which a concept’s network can be partitioned into separate property clusters. Network modularity (M=0.72, SD=0.04) was not significantly associated with either SemD (r(15)=0.22, p>0.4) or SemS (r(15)=−0.19, p=0.5). On the other hand, mean diversity was positively associated with SemD (r(15)=0.56, p=0.03; Fig. 2) and negatively associated with SemS (r(15)=−0.60, p=0.02). Mean diversity (M=0.07, SD=0.02) was not significantly associated with either mean node strength (r(15)=−0.08, p>0.7) or mean node degree (r(15)=0.42, p=0.12). These results suggest that the network measure of mean diversity is a strong candidate for a quantitative measure of conceptual flexibility.

We also assessed the core-periphery structure for each concept network, which determines how well a network can be divided into a highly-connected core and a sparsely-connected periphery. If the core of a concept network corresponds to the notion of a context-independent conceptual “core”, we predicted that more stable (i.e., less flexible) concepts would have networks with a stronger core-periphery structure. Consistent with this prediction, core-periphery structure (M=0.56, SD=0.08) was positively associated with SemS (r(15)=0.54, p=0.038), though the
relationship with SemD was only marginally significant ($r(15)=-0.50, p=0.059$). Furthermore, mean diversity and core-periphery structure were negatively correlated ($r(15)=-0.61, p=0.02$), suggesting these measures may be used to capture conceptual flexibility and stability, respectively. We also found that core-periphery structure was positively correlated with classification accuracy using the standard vector model ($r(15)=0.56, p=0.03$). This suggests that standard cognitive models perform better on more stable concepts, highlighting the need for a model that can adequately capture conceptual flexibility.

**Discussion**

Here our goal was to model basic-level concepts using graph-theoretical networks. We argue that the within-concept statistics encoded in these models capture useful, concept-specific information. Using standard network science tools, we further reveal the usefulness of these models by extracting formal metrics that relate to cognitive notions of conceptual flexibility and stability.

A model structured using within-concept statistics provides a framework in which varied yet appropriate instantiations of a concept may be flexibly activated. An APPLE network may contain a strong connection between CRUNCHY + FRESH and between SOFT + BAKED, enabling the conceptual system to know what sets of properties should be activated in a particular APPLE instance — for example, in the representations evoked by “apple picking” versus “apple pie.” The property-covariation statistics for a given concept will determine which sets of properties tend to be co-activated, and how individual properties relate to those sets and to each other. We thus sought to use our compositional concept network models, which contain within-concept statistics, to extract quantitative measures of these phenomena. We found that mean-diversity and core-periphery structure can be interpreted as measures of conceptual flexibility and stability, respectively: a concept network-model’s mean-diversity positively predicts semantic diversity (SemD; Hoffman et al., 2013), a network-model’s core-periphery fit positively predicts semantic stability (mean cosine similarity; Hoffman et al., 2013), and these two network measures are negatively related to each other across our concepts. We also found that traditional property-vector models were better at capturing the representation of stable versus flexible concepts, suggesting that a different kind of conceptual model may be necessary to capture the intrinsic flexibility of the conceptual system. We argue that a network-based model of basic-level concepts is one such option.

Here we have constructed concept network models, confirmed their ability to capture concept-specific information, and extracted network measures that relate to cognitive measures of conceptual flexibility and stability. We believe the application of network science to conceptual knowledge will provide a set of tools that will enable the intrinsic flexibility of the conceptual system to be explored and quantified.

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**References**


