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The contributions of numerical acuity and non-numerical stimulus features to the development of the number sense and symbolic math achievement

Ariel Starr[a,*], Nicholas K. DeWind[b], Elizabeth M. Brannon[b]

[a] Department of Psychology, University of California, Berkeley, United States
[b] Department of Psychology, University of Pennsylvania, United States

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ABSTRACT

Numerical acuity, frequently measured by a Weber fraction derived from nonsymbolic numerical comparison judgments, has been shown to be predictive of mathematical ability. However, recent findings suggest that stimulus controls in these tasks are often insufficiently implemented, and the proposal has been made that alternative visual features or inhibitory control capacities may actually explain this relation. Here, we use a novel mathematical algorithm to parse the relative influence of numerosity from other visual features in nonsymbolic numerical discrimination and to examine the strength of the relations between each of these variables, including inhibitory control, and mathematical ability. We examined these questions developmentally by testing 4-year-old children, 6-year-old children, and adults with a nonsymbolic numerical comparison task, a symbolic math assessment, and a test of inhibitory control. We found that the influence of non-numerical features decreased significantly over development but that numerosity was a primary determinate of decision making at all ages. In addition, numerical acuity was a stronger predictor of math achievement than either non-numerical bias or inhibitory control in children. These results suggest that the ability to selectively attend to number contributes to the maturation of the number sense and that numerical acuity, independent of inhibitory control, contributes to math achievement in early childhood.

1. Introduction

Beginning with Piaget, there has been great controversy regarding how children make decisions about number. In his classic number conservation studies, Piaget observed that although children correctly viewed two lines composed of the same number of objects as equally numerous when the spacing of the objects was equal, they erroneously judged one line as more numerous when the objects were spaced further apart (Piaget, 1952). As a result, Piaget concluded that children are unable to disentangle representations of number and space until they reach the concrete operations stage around seven years of age. More recent work, however, suggests that even infants can selectively attend to both number and size, particularly for large set sizes (see Cantrell & Smith, 2013; Mou & vanMarle, 2013 for reviews). At the same time, it is clear that numerical representations, even in adulthood, are influenced by non-numerical properties such as element size, field area (also referred to as convex hull), and density (Allik & Tuulmets, 1991; Dakin, Tibber, Greenwood, Kingdom, & Morgan, 2011; Defever, Reynvoet, & Gebuis, 2013; Gebuis & Gevers, 2011; Gebuis, Herfs, Kenemans, & van der Smagt, 2009; Gebuis & Reynvoet, 2011b, 2012a, 2012b; Ginsburg & Nicholls, 1988; Hurewitz, Gelman, & Schnitzer, 2006; Rousselle & Noël, 2008; Rousselle, Palmers, & Noël, 2004; Soltész, Szűcs, & Szűcs, 2010; Sophian & Chu, 2008; Szűcs, Nobes, Devine, Gabriel, & Gebuis, 2013; Tibber, Greenwood, & Dakin, 2012; Tokita & Ishiguchi, 2010, 2013). In general, numerical decisions are more accurate when these non-numerical features co-vary reliably with number compared to when they do not.

In most assessments of nonsymbolic numerical discrimination, the stimuli are constructed such that in half of the trials numerosity is congruent with area (i.e., the more numerous array contains larger individual elements and has a larger total surface area than the less numerous array). In the other half of trials, numerosity and area are incongruent such that the more numerous array has smaller elements and a smaller total surface area (e.g., DeWind &

* Corresponding author at: Department of Psychology, UC Berkeley, 134 Barker Hall, Berkeley, CA 94720, United States.
E-mail address: arielstarr@berkeley.edu (A. Starr).

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Brannon, 2012; Halberda & Feigenson, 2008; Inglis & Gilmore, 2013; Piazza et al., 2010). With this design, performance is typically better for congruent trials compared to incongruent trials. Furthermore, the effects of congruency are strongest for young children and attenuate with age, which suggests that younger children may be more biased by non-numerical cues than older children when making numerical comparison judgments (Defever, Sasanguie, Gebuis, & Reynvoet, 2011; Fuhs & Mcneil, 2013; Gilmore, Attridge, Clayton, et al., 2013; Rousselle & Noël, 2008; Rousselle et al., 2004; Soltész et al., 2010). For example, Soltész et al. (2010) tested 4- to 7-year-old children with a nonsymbolic magnitude comparison task in which the overall surface area or overall perimeter of the dot arrays was either congruent or incongruent with number. All children were more accurate when surface area and perimeter were congruent with number, but the difference in performance between congruent and incongruent trials decreased between 4 and 7 years of age. In addition, the precision with which numerical comparison judgments are made regardless of congruency condition follows a similar developmental trajectory, undergoing rapid development in infancy and early childhood and continuing to improve into adulthood (Halberda & Feigenson, 2008; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Piazza et al., 2010).

Despite the wealth of research demonstrating that congruency effects decrease with age while numerical acuity improves, it is unclear whether these changes arise from a common mechanism or whether they represent differentiable developmental effects. Does overall improvement in numerical acuity over development reflect an increase in the precision of the internal numerical representations themselves or, alternatively, an increase in the ability to selectively attend to number and inhibit attention towards other stimulus features? A second open question concerns the relative influence of different types of non-numerical features on numerical discrimination. Does the influence of all non-numerical cues decrease in parallel, or do the trajectories vary by feature?

The development of selective attention to number may also have implications for the acquisition of symbolic math skills. Many studies have documented a relation between performance on numerical comparison tasks and symbolic math performance (DeWind & Brannon, 2012; Fazio, Bailey, Thompson, & Siegler, 2014; Gilmore, McCarthy, & Spelke, 2010; Halberda, Mazzocco, & Feigenson, 2008; Halberda et al., 2012; Libertus, Feigenson, & Halberda, 2011; Libertus, Odic, & Halberda, 2012; Mazzocco, Feigenson, & Halberda, 2011; Mussolin, Nys, Leybaert, & Content, 2012; Starr, Libertus, & Brannon, 2013). However, a growing number of studies have observed no relation (Holloway & Ansari, 2009; Nosworthy, Bugden, Archibald, Evans, & Ansari, 2013; Sasanguie, Defever, Maertens, & Reynvoet, 2013) or a relation that exists only in young children or children with low math ability (Bonny & Lourenco, 2012; Inglis, Attridge, Batchelor, & Gilmore, 2011; Sasanguie, De Smedt, Defever, & Reynvoet, 2011). Three recent meta-analyses on this topic concluded that there is a significant yet small relation between symbolic math and non-symbolic numerical acuity that is strongest in young children (Chen & Li, 2013; Fazio et al., 2014; Schneider et al., 2016), lending credence to the hypothesis that approximate number representations may be foundational for the acquisition of symbolic math (Dehaene, 1997; Gallistel & Gelman, 1992). According to this view, having a more precise internal representation of number is advantageous for learning numerical symbols and symbolic arithmetic operations.

An alternative theory, however, is that the apparent relation between numerical acuity and math ability actually reflects the well-documented relation between inhibitory control and math ability (e.g., Bull & Lee, 2014; Bull & Scerif, 2001; St Clair-Thompson & Gathercole, 2006). One proposal is that the need to attend to number and ignore irrelevant non-numerical features in nonsymbolic numerical comparison tasks taps inhibitory control as much as, or to an ever greater extent, than numerical acuity (Fuhs & Mcneil, 2013; Gilmore, Attridge, Clayton, et al., 2013). According to this view, children (and adults) with stronger inhibitory control perform more accurately on nonsymbolic numerical comparison tasks because they are better able to inhibit attention towards non-numerical stimulus features and focus their attention on number. Likewise, this stronger inhibitory control also contributes to higher math performance. Therefore, if better performance on nonsymbolic numerical comparison tasks is actually reflecting inhibitory control rather than the precision of internal numerical representations, numerical acuity in and of itself may not directly contribute to math achievement.

The present study was designed to investigate how children's attention to numerical and non-numerical stimulus features shifts over development and the degree to which attention to these features is related to symbolic math performance. We tested 4- and 6-year-old children, as well as adults, on a nonsymbolic numerical comparison task, a symbolic math assessment, and an inhibitory control task. The nonsymbolic numerical comparison task and analysis method were adapted from DeWind, Adams, Platt, and Brannon (2015). This method is unique in its treatment of non-numerical stimulus features (e.g. field area, total surface area, and individual element size). Instead of defining stimulus subsets that control for a particular feature by holding it constant (Halberda & Feigenson, 2008) or trying to minimize the correlation between number and all non-numeric features overall (Gebuis & Reynvoet, 2011a), non-numerical stimulus features are intentionally varied, such that their effect on numerical discrimination can be explicitly modeled. Critically, this enables us to go beyond the dichotomy of congruent and incongruent trials to quantitatively estimate how different non-numerical features influence numerical decision-making. The model, fit to each participant's data, returns coefficients that represent the effect of number and non-numerical size and spacing features on ostensibly numerical discriminations. By comparing the coefficients across different age groups, we can examine how the influence of different non-numerical features changes over development. This framework also allows an explicit test of the hypothesis that the ability to inhibit attention towards non-numerical features drives the relation between performance on nonsymbolic numerical comparison tasks and symbolic math skill through an investigation of the relation between the non-numerical feature coefficients, symbolic math performance, and inhibitory control.

2. Method

2.1. Participants

Thirty-nine 4-year-old children (mean age 4.64 years, range: 4.45–4.96 years, 17 females), 45 6-year-old children (mean age 6.63 years, range: 6.29–6.96 years, 19 females), and 30 adults (mean age 21.9 years, range 18.5–54.4 years, 20 females) participated in the study. An additional 11 4-year-olds were excluded due to failure to complete all tasks (n = 10), or inability to speak English (n = 1), and one 6-year-old was excluded due to failure to complete all tasks. All participants or their parents consented to a protocol approved by the local IRB. Children were given a small gift and parents were compensated monetarily at each visit. Adults were compensated either monetarily or with course credit.

2.2. Measures

2.2.1. Nonsymbolic numerical comparison task

This task and stimulus construction was adapted from DeWind et al. (2015). Construction of the dot arrays was based on the principle that although there are many non-numerical features that covary with number within dot arrays, only three degrees of freedom are available for experimental manipulation: the number of dots, one parameter that describe the size of the dots (i.e., individual element size, total surface area, and total perimeter), and one parameter that describes the spacing of the dots (i.e., field area, which is the area of the circle in which the dots are drawn and is very closely related to convex hull or envelope size, and sparsity, which is the average field area per item or the inverse of density). In other words, if the number of items in an array is fixed and the size of the dots is changed, the item area, item perimeter, total area, and total perimeter are necessarily changed as well. All four of these features share a single degree of freedom and including more than one in a regression model will over-specify it; there will be no single solution to the linear equation and the model will fail to converge. Similarly, if the number of items in the array is fixed and the spacing of the dots is altered, the field area, sparsity, and density are necessarily changed as well. Only three terms are needed to fully specify all these features of any given array; we call these three terms number, size and spacing. These three parameters can be thought of as a three-dimensional stimulus space in which any given array of dots can be defined by a single point that reflects the number, size, and spacing of that array (Fig. 1).

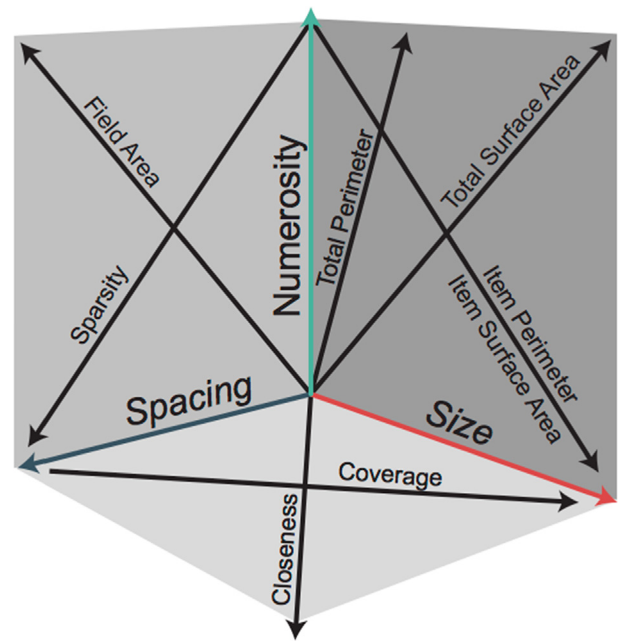

Fig. 1. Illustration of the 3D stimulus space demonstrating the relations between the number, size, and spacing dimensions. adapted from DeWind et al., 2015

Within this three-dimensional space, we constructed a set of stimuli that divided numerosity, size, and spacing into 13 levels that were evenly spaced on a logarithmic scale in which the largest values for each dimensions was four times greater than the smallest value. The logarithmic scale affords the advantages that the distance between stimulus points in the three-dimension space is proportional to their ratio and produces linear equations relating numerosity, size, and spacing to the other non-numerical features. Within this framework, numerosity can be defined as

$$\log_2(n) = \log_2\left(\frac{TSA}{ISA}\right) = \log_2\left(\frac{FA}{Spar}\right)$$

in which n is the number of items, TSA is the total surface area, ISA is the individual item surface area, FA is the field area and $Spar$ is the sparsity (the inverse of density). Size and spacing can then be defined independent of numerosity as

$$\log_2(Size) = \log_2(TSA) + \log_2(ISA)$$

$$\log_2(Spacing) = \log_2(FA) + \log_2(Spar)$$

Example stimuli that demonstrate how varying each of these parameters affects the appearance of the dot arrays are illustrated in Fig. 2.

A critical point is that the stimulus set was not constructed to explicitly eliminate any correlations between numerosity and non-numerical features but rather to evenly sample the stimulus space. Indeed, within the stimulus set, number, size, and spacing are uncorrelated, but correlations do exist between number and other non-numerical features (see Supplementary Table 1). The advantage of the present approach is that our analysis enables us to deconvolve the effects of number, size, and spacing on numerical decision-making because we can model their independent effects on choice behavior. As described in DeWind et al. (2015), the goal of the stimulus design was therefore to orthogonalize and create variance in the regression predictor variables so as to minimize error in coefficient estimates.

Dot arrays were generated in real-time before each trial. The experimental program randomly chose one of four different numerical ratios (closest whole numbers to $1{:}2^{1/6}$, $1{:}2^{1/3}$, $1{:}2^{1/2}$, or 1:4 ratios), one of the 13 size ratios, and one of the 13 spacing ratios. Note that although four numerical ratios were sampled whereas 13 size and spacing ratios were sampled, all ratios fell within the same range (between $1{:}2^{1/6}$ and 1:4). A greater range of ratio was tested for size and spacing because there exists an extensive literature characterizing numerical acuity in children and adults, whereas acuity for size and spacing is not as clearly defined. Therefore, we tested a larger subset of ratios for size and spacing in order to best model choice performance. To sample the stimulus space evenly along a logarithmic scale, all values were rounded to the nearest whole number. The number of dots in each array ranged from 8 to 32, and dot size was homogenous within each array. All dots in a given trial were homogenous in color, but the color of the dots varied across trials.

On each trial, participants were presented with two arrays of dots and asked to indicate which side had the greater number of dots. Each trial began with a white fixation cross in the center of the screen. After 500 ms, the dot arrays were displayed on either side of the fixation cross. The dots were visible for a set presentation time that varied as a function of age (1000 ms for 4-year-olds, 750 ms for 6-year-olds, and 250 ms for adults), after which participants were prompted to provide a response using the keyboard. These variations in presentation time were implemented such that we could use the same stimuli and ratios for all age groups while keeping accuracy within a sensitive range for modeling choice behavior. Pilot testing was used to determine the appropriate presentation times for 4- and 6-year-olds. No feedback was given during test trials. To ensure that all participants understood the task, 4- and 6-year-olds began with four practice trials and adults began with eight practice trials. Practice trials included feedback, had a slower presentation time, and always presented a 1:4 numerical ratio. If children did not answer at least three out of four practice trials correctly, the practice was repeated. Four-year-old children completed 60 test trials per session at each of two sessions (120 trials total), 6-year-old children completed 150 trials per session

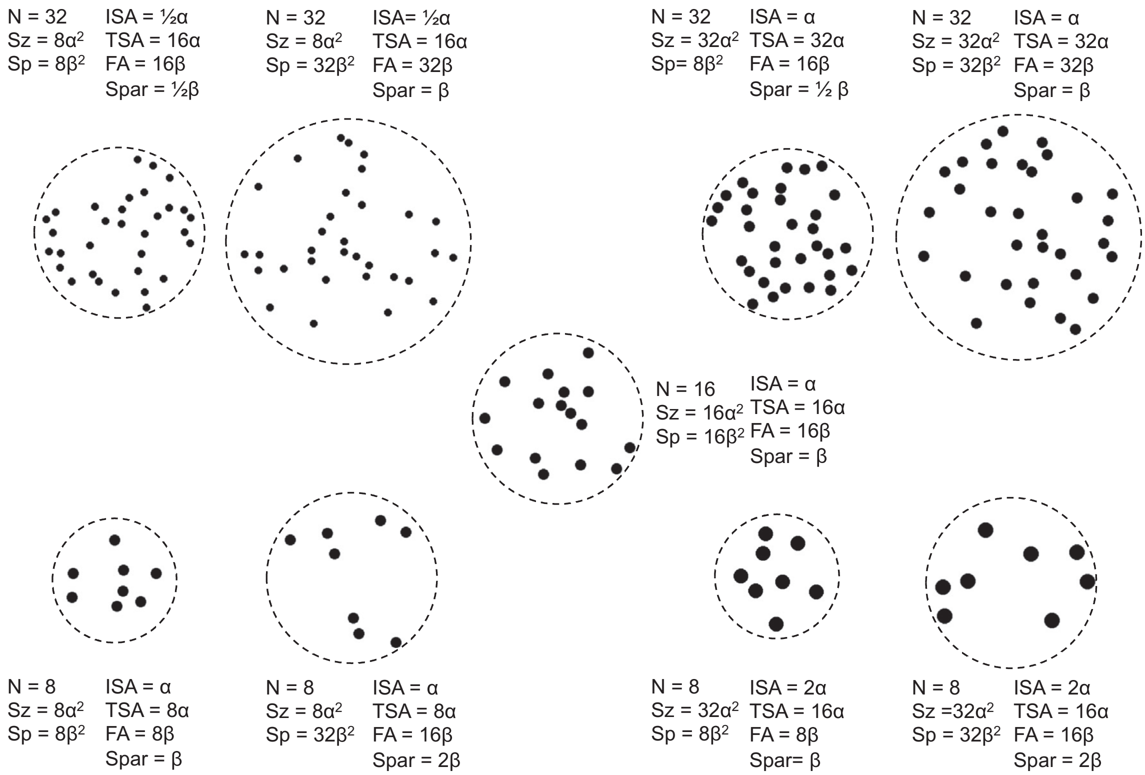

Fig. 2. Example stimuli illustrating how changes to number and non-numerical stimulus features influence the number, size, and spacing of the arrays. Note that these arrays are not sample pairs of test arrays; rather the pairs demonstrate the effects of changing specific parameters. Using the center array as a reference, the arrays in the top and bottom rows illustrate how halving or doubling the various parameters influences the appearance of the arrays. All arrays in the top row contain 32 dots; all arrays in the bottom row contain 8 dots. Dots in the top left arrays are half as large as those in the reference array; dots in the bottom right arrays are twice as large as those in the reference array. Within each pair of arrays, the number and size of the dots is held constant and the spacing variables change by a factor or two. *N* is the number of items, *Sz* is the size parameter, *Sp* is the spacing parameter, *TSA* is the total surface area, *ISA* is the individual item surface area, *FA* is the field area and *Spar* is the sparsity (the inverse of density). α and β represent arbitrary baseline values for item area and sparsity, respectively.

at each of two sessions (300 trials total), and adults performed 500 trials in a single session. Previous work using these stimulus parameters and model has found Weber fractions to be reliable with this number of trials in adults (DeWind & Brannon, 2016). To determine if differences in presentation time and number of trials across age groups influenced the reliability of the task, we calculated Cronbach's alpha separately for each group using the psych package in R (Revelle, 2014). Reliability was high for all age groups (0.79 for 4-year-olds, 0.89 for 6-year-olds, and 0.96 for adults).

2.2.2. Number Sense Screener

The Number Sense Screener (NSS; Glutting & Jordan, 2012) was used to assess math skill in 4-year-old children. The NSS is orally administered and assesses counting skill, number recognition, symbolic numerical magnitude comparison, and arithmetic. Standardized scores in the NSS are based on level of schooling. Scores were standardized based on the norms for the first semester of kindergarten, which is the lowest level of schooling for which standardized scores are available for this measure.

2.2.3. Wide Range Achievement Test-4th Edition, calculation subtest

The Wide Range Achievement Test-4th Edition (WRAT-4) Calculation test (Wilkinson & Robertson, 2006) was used to assess math skill in 6-year-old children and adults.[1] The WRAT-4 consists of orally-administered number knowledge and arithmetic questions (6-year-old children only) and a written portion in which participants have 15 min to complete arithmetic problems that sequentially increase in difficulty. The orally-administered problems are similar in format to many of the items on the NSS. Scores were standardized based on participants' age.

2.2.4. Day/Night task

The Day/Night task (Gerstadt, Hong, & Diamond, 1994) was used to assess inhibitory control in 4- and 6-year-old children. In the warm-up version, children were shown a card containing 16 sun and moon pictures in a pseudo-random order and instructed to say "day" for the sun pictures and "night" for the moon pictures as quickly as possible. Next, children were told they were going to play a silly version of the game that required saying the opposite picture names ("day" for the moon picture and "night" for the sun picture). They were then shown a new card with 16 sun and moon pictures and instructed to say the opposite picture names as quickly as possible. The total time and number of errors were combined into a single efficiency score (number of correct responses divided by total time).

2.2.5. Flanker task

The Flanker task was used to assess inhibitory control in adults. On each trial, a row of five fish was presented centrally on a computer screen, and participants were instructed to push a button indicating whether the center fish was facing to the left or to the right. The center fish was either facing the same direction as the surrounding fish (congruent) or the opposite direction (incongruent). Participants were instructed to respond as quickly

[1] There is no standardized test that would be appropriate for both 4-year-olds and adults, thus it was necessary to use two distinct assessments. We could have used either the NSS or the WRAT-4 with 6-year-old children and chose to use the WRAT-4 to better enable a comparison between children and adults.

as possible. The fish stimuli appeared on the screen for 300 ms. After the fish disappeared, a green fixation cross remained on the screen until a response was made. After the response, the fixation cross turned white and there was a two second interval before the start of the next trial. Participants performed four practice trials with feedback followed by 144 test trials without feedback. A version of this task was also administered to child participants but was excluded from further analyses because as a group children did not exhibit the classic Flanker effect. Six-year-old children tended to perform very slowly and deliberately, such that there was no performance difference between congruent and incongruent trials, whereas 4-year-old children struggled to perform at chance level on incongruent trials.

2.3. Procedure

Child participants completed two laboratory visits lasting approximately 30 min each. At the first visit, children performed the nonsymbolic numerical comparison task, Day/Night, and the age-appropriate symbolic math test. At the second visit, children performed the nonsymbolic numerical comparison task and the Flanker task. Children were tested in two separate sessions in order to maximize task engagement. Adults were tested in a single visit lasting approximately 1 h in which they completed the nonsymbolic numerical comparison task, the Flanker task, and the WRAT-4 Calculation test.

2.4. Modeling choice behavior in the nonsymbolic number comparison task

For each subject, we fit a generalized linear model (GLM) with a binomial error distribution to their choice data with regressors for the log of the ratio of numerosity, size, and spacing for the dot arrays appearing on the right and left sides of the screen (DeWind et al., 2015). The model equation is as follows:

$$p(ChooseRight) = (1-\gamma)\left(\frac{1}{2}\left[1+\mathrm{erf}\left(\frac{\beta_{side}+\beta_{num}\log_2(r_{num})+\beta_{Size}\log_2(r_{Size})+\beta_{Spacing}\log_2(r_{Spacing})}{\sqrt{2}}\right)\right]-\frac{1}{2}\right)+\frac{1}{2}$$

We included a guessing term in our model, γ, to account for trials in which participants guessed randomly. This allows the choice curves to asymptote below 100% (Halberda & Feigenson, 2008; Pica, 2004). The guess parameter was set as each participant's mean accuracy on the easiest numerical ratio (1:4) subtracted from one. If γ is set to 0 the model simplifies to a GLM with a probit link function and a binomial error distribution. Model fitting was implemented by the generalized linear model fitting algorithms in Matlab (Mathworks) using the glmfit and fitglm functions. Code for fitting the model is available in Appendix B of DeWind et al. (2015).

When we fit the model to an individual participant's data, four coefficients are returned: β_{number}, β_{size}, $\beta_{spacing}$, and β_{side}. β_{side} indicates a bias towards the stimulus presented on the right versus on the left irrespective of what stimulus it is (i.e., side bias), and is unrelated to our hypotheses. The three other coefficients comprise a vector that provides detailed information regarding how these features influence participants' perception of numerical quantity. If participants are able to ignore all non-numerical influences, then β_{number} is positive and β_{size} and $\beta_{spacing}$ are zero. The value of β_{number} indicates the acuity in making numerical discriminations: a large β_{number} indicates participants can discriminate difficult ratios closer to 1:1; a small β_{number} indicates participants can only differentiate easier ratios.

However, if participants are influenced by non-numerical features related to the size of the dots (total area or individual item area), or the spacing of dots (density or field area), then the β_{size} and $\beta_{spacing}$ terms enable us to quantify these influences. A positive β_{size} indicates that, at a given numerical magnitude, larger dots are perceived as more numerous; a negative β_{size} indicates that smaller dots are perceived as more numerous. Similarly, a positive $\beta_{spacing}$ indicates that more spaced out dots are perceived as more numerous, whereas a negative $\beta_{spacing}$ indicates that more densely packed dots are perceived as more numerous. Examining age-related changes in β_{number}, β_{size}, and $\beta_{spacing}$ therefore provides insight into how bias towards non-numerical cues in the nonsymbolic number comparison task changes over development.

β_{size}, $\beta_{spacing}$, and β_{number} can also be imagined as the X, Y, and Z coordinates of a point in space, which allows us to create useful summary statistics based on the geometry of that point. The distance of the point from the origin (X = 0, Y = 0, Z = 0) indicates acuity, the precision with which participants discriminated small changes in the stimuli. The direction of a line drawn through the origin to the point indicates the features that contributed to perceived numerosity. Returning to the case of an unbiased participant, if β_{number} is positive and β_{size} and $\beta_{spacing}$ are zero, then the line connecting the coefficient-point with the origin will be vertical (assuming β_{number} is plotted as "up"). A vertical line is an indication of numerical discrimination unaffected by the size or spacing of the items. If, however, a participant's coefficients include positive or negative β_{size} or $\beta_{spacing}$, the line will deviate from vertical. This deviation, quantified as the angle at the origin formed between the participant's coefficient-point line and the vertical line of a hypothetical unbiased participant, can be taken as an index of non-numerical feature bias. The greater this angle, the further the participant is from unbiased number discrimination. In other words, the geometry of the coefficient-point line provides two key metrics for quantifying participants' numerical decision-making behavior: the length of the line reflects *acuity* and the angle of the line reflects *non-numerical bias*. The coefficient-point line length and angle can therefore be used in conjunction with the measures of inhibitory control to determine how each of these abilities relates to symbolic math achievement.

Within the three-dimensional stimulus space, we can also relate specific non-numerical features (e.g., total surface area and field area) to the primary number, size, and spacing axes. The linear equations that relate these features to number, size, and spacing can be found in Supplementary Table 2. These feature lines are represented in Fig. 1, and their two-dimensional projections are shown in Figs. 3 and 4. In Figs. 3 and 4, if a coefficient-point is closer to one of these labeled features lines than to the number line (vertical axis), then it is more parsimonious to describe the discrimination behavior as discriminating that feature rather than discriminating number itself.

3. Results

3.1. Developmental change in the nonsymbolic number comparison task

All age groups performed the task better than would be expected by chance and at approximately the same level (4-year-olds: 72.6%; 6-year-olds: 77.5%; adults: 78.5%), indicating that our attempt to equalize performance by manipulating stimulus presentation time was relatively successful. Adults responded more rapidly than either 6-year-olds or 4-year-olds, and 6-year-olds responded more rapidly than 4-year-olds (adults: mean RT = 0.39 s, SEM = 0.038; 6-year-olds: mean RT = 1.25 s, SEM = 0.073; 4-year-olds: mean RT = 1.56 s, SEM = 0.084; all *ts* > 2.78, all *ps* < 0.007).

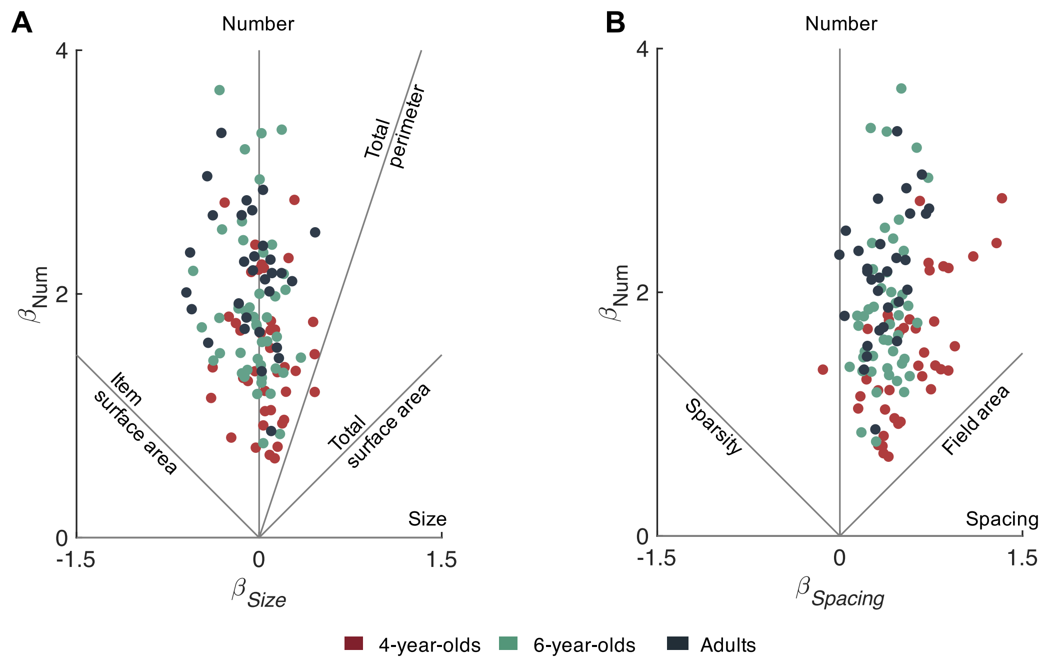

Fig. 3. Scatterplots of the coefficient weights after fitting the choice model to the nonsymbolic numerical comparison data. (A) Number and size dimensions. (B) Number and spacing dimensions. Each dot represents a single participant.

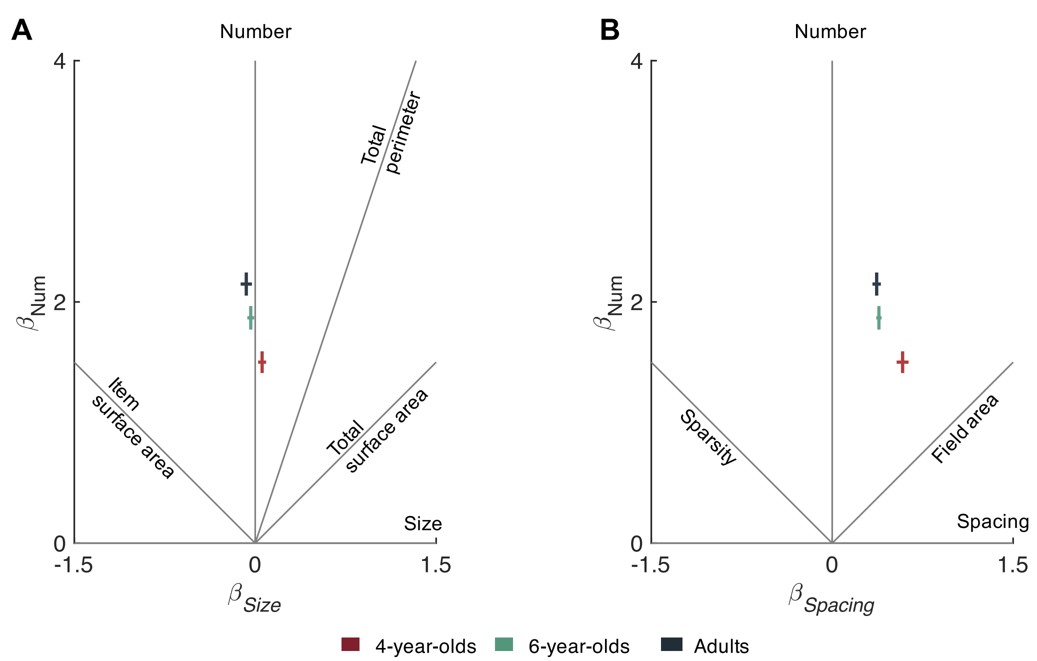

Fig. 4. Group means for the coefficient weights after fitting the choice model to the nonsymbolic numerical comparison data. (A) Number and size dimensions. (B) Number and spacing dimensions. The length of the vertical and horizontal crosshatches indicates the standard error of each coefficient.

In the first set of analyses we investigated how bias towards non-numerical cues in the nonsymbolic number comparison task changes over development by investigating changes in β_{number}, β_{size}, and $\beta_{spacing}$ across the three age groups. We fit each participant's choice data using our generalized linear model (Fig. 3). Individual model fits for all participants were significant (4-year-olds: mean adjusted $R^2 = 0.434$, mean chi-squared value = 62.45; 6-year-olds: mean adjusted $R^2 = 0.451$, mean chi-squared value = 157.42; adults: mean adjusted $R^2 = 0.495$, mean chi-squared value = 299.93; all $ps < 0.001$). Individual coefficient fits analyzed at the group level revealed that all age groups exhibited β_{number} terms significantly greater than zero (all $ts > 16$, $ps < 0.001$),[2] as

[2] Given that recent studies have demonstrated that variations in stimulus parameters can lead to vastly different estimations of *w* (e.g., Defever et al., 2013; Szűcs et al., 2013), comparing Weber fractions across studies may not be meaningful. However, the *w* values obtained here using the standard Halberda model are well within the range of those reported for these age groups in the literature.

well as $\beta_{spacing}$ terms significantly greater than zero (all *ts* > 10, *ps* < 0.001) (Fig. 4). However, none of the age groups exhibited β_{size} terms significantly different from zero (all *ts* > 1.3, *ps* > 0.09). We next compared the values of the individual coefficient weights across the three age groups (Fig. 4). For all features there was a main effect of age (number: $F(2,111) = 10.44$, $p < 0.001$, $\eta^2 p = 0.158$; size: $F(2,111) = 3.59$; $p < 0.05$, $\eta^2 p = 0.06$; spacing: $F(2,111) = 10.53$, $p < 0.001$, $\eta^2 p = 0.42$). Follow-up comparisons revealed that 4-year-olds had significantly lower β_{number} terms compared to 6-year-olds and adults (*ps* ≤ 0.005), and 6-year-olds had significantly lower β_{number} terms than adults ($p < 0.05$). Four-year-olds had also significantly higher β_{size} and $\beta_{spacing}$ terms compared to 6-year-olds and adults (*ps* < 0.05), whereas 6-year-olds and adults did not differ (*ps* > 0.4). These results suggest that the biasing effects of size and spacing variables become attenuated to adult-like levels between 4 and 6 years of age, whereas numerical acuity continues to improve between 6 years and adulthood (Fig. 5).

To further explore how numerical decision-making changes over development, we next investigated age-related differences in coefficient-point line length and angle, which represent participants' overall discrimination performance and bias towards non-numerical features, respectively. Unsurprisingly, coefficient-point line length was significantly greater than zero in all age groups (all *ts* > 17, *ps* < 0.001), indicating that all groups made discriminations based on *some* visual feature. Coefficient-point angles were also significantly greater than zero in all age groups (all *ts* > 13, *ps* < 0.001), indicating that all groups were significantly biased away from a pure numerical discrimination. In addition, there was a main effect of age on both angle ($F(2,111) = 39.49$, $p < 0.001$, $\eta^2 p = 0.42$) and line length ($F(2,111) = 7.42$, $p = 0.001$, $\eta^2 p = 0.118$). Follow-up comparisons revealed that 4-year-olds had significantly larger angles and smaller line lengths in comparison to 6-year-olds and adults (*ps* < 0.05). Six-year-olds and adults exhibited no significant difference in angle ($p = 0.14$), but adults exhibited longer line lengths ($p = 0.05$) (Fig. 6). These analyses support the previous analyses in suggesting that the influence of non-numerical cues decreases between the ages of 4 and 6 and then remains relatively stable, whereas discrimination acuity has a more protracted developmental trajectory.

The next series of analyses compared β_{number} to the full list of stimulus features accounted for by the model to determine which specific non-numerical features exert the greatest influence on numerical decision-making. Although these other stimulus features (individual element area/perimeter, total surface area, total perimeter, field area, and sparsity) are not explicitly included in the model, their linear relations to number, size, and spacing enables us to calculate coefficient weights that reflect their influence on numerical comparison performance (DeWind et al., 2015). We conducted paired *t*-tests comparing β_{number} and the coefficient weights for each of the other stimulus features in each of the age groups. In 6-year-olds and adults, β_{number} was greater than the coefficient weights for all of the other non-numerical stimulus features (all *ts* > 8.1, all *ps* < 0.001). In 4-year-olds, however, β_{number} was greater than the coefficient weight for all of the non-numerical features with the exception of field area, which had a coefficient weight that was not statistically different from β_{number} ($t(38) = 0.99$, $p = 0.33$). Therefore, whereas 6-year-olds and adults successfully focused their attention on number, 4-year-olds were strongly influenced by field area and used number and field area information equally when ostensibly making decisions about number. This effect can be visualized in Fig. 4B: 4-year-olds fall between the field area and number feature lines, which indicates that as a group they equally weighted these two features. Six-year-olds and adults are biased by field area, but are closer to the number feature line, indicating that they primarily relied on number.

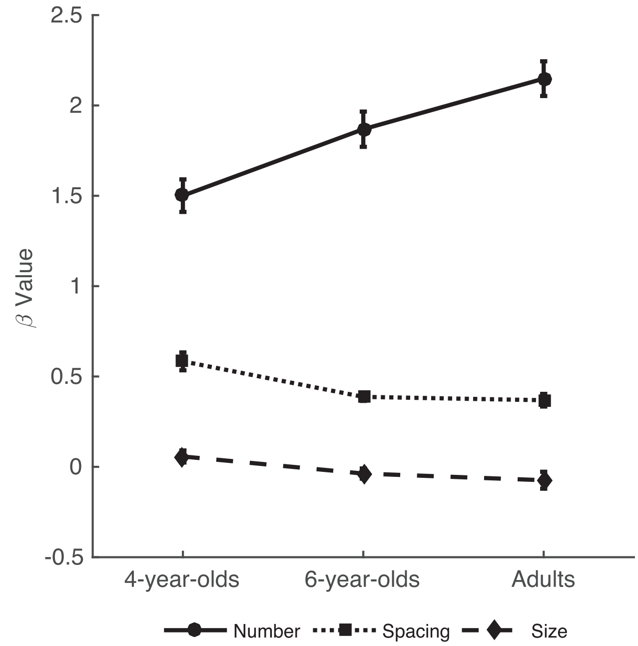

Fig. 5. Mean coefficient weights for number, size, and spacing in each age group. The coefficient weight for number increases at each time point, whereas the coefficient weights for size and spacing decrease between 4 and 6 years of age and then remain stable between 6 years of age and adulthood. Error bars indicate standard error.

3.2. Assessing the relation between symbolic math skill, non-numerical stimulus bias, and inhibitory control

The aim of the final set of analyses was to investigate the potential role of inhibitory control in the relation between numerical acuity and math achievement. First, we leveraged our independent measure of non-numerical feature bias (coefficient-point line angle) to assess the hypothesis that ignoring non-numerical stimulus features relies on the same inhibitory control mechanisms as more traditional measures of executive control, including the Day/Night task (Fuhs & Mcneil, 2013; Gilmore, Attridge, Clayton, et al., 2013). We found that coefficient-point line angle was not significantly correlated with Day/Night performance (4-year-olds: $r(37) = 0.02$, $p = 0.92$; 6-year-olds: $r(43) = -0.14$, $p = 0.37$), which suggests that inhibitory control and non-numerical feature bias are separate constructs.

Even if non-numerical bias and inhibitory control are independent, it remains possible that both, either, or neither could influence symbolic math performance. To test this possibility, we conducted linear multiple regressions including coefficient-point line length, coefficient-point line angle, and Day/Night performance as possible predictors of math performance in each age group. In both 4- and 6-year-old children, coefficient-point line length was a significant predictor of math achievement and there was a trend towards inhibitory control contributing unique variance as well (*ps* for coefficient-point line length ≤0.05, *ps* for inhibitory control <0.1; see Table 1). However coefficient-point line angle did not contribute significant variance (*ps* > 0.2). These results suggest that numerical acuity and inhibitory control each contribute significant and unique variance to math achievement in young children. In adults, however, the model was unable to significantly predict math performance. This suggests that none of the predictors exerted significant influence on symbolic math performance in adults. Descriptive statistics and zero-order correlations for all variables of interest in all age groups can be found in Supplementary Tables 3–6, and the zero-order correlations between

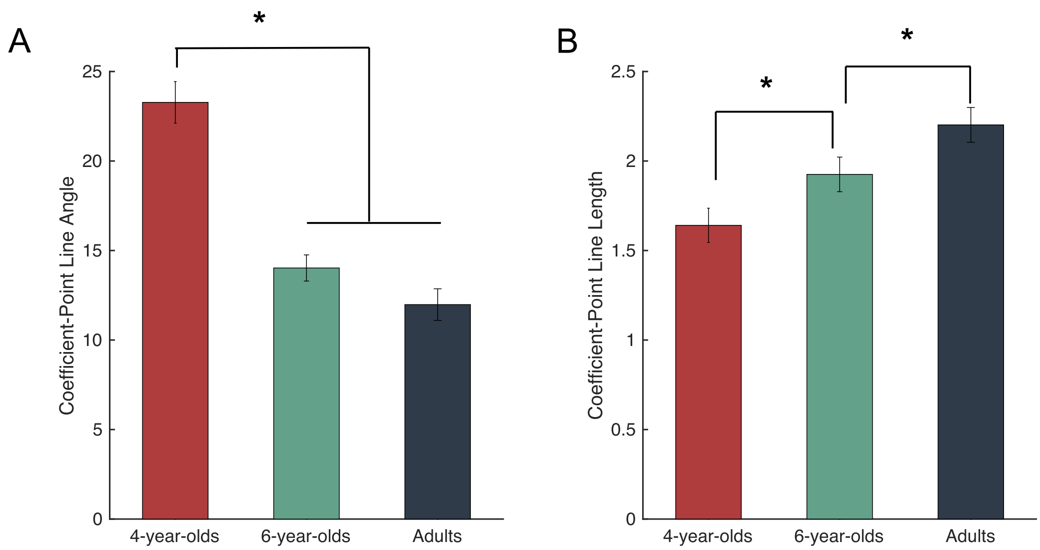

Fig. 6. Group averages for the coefficient-point line angle (A) and coefficient-point line length (B). Four-year-olds have higher coefficient-point line angles in comparison to 6-year-olds and adults. Coefficient-point line length increases between 4 and 6 years of age and between 6 years of age and adulthood. Error bars indicate standard error.

Table 1
Linear regression analyses predicting symbolic math performance from coefficient-point line length, coefficient-point line angle, and inhibitory control.

	4-year-olds		6-year-olds		Adults	
R^2	0.207		0.193		0.08	
F-stat	3.041		3.264		0.635	
p-stat	0.042		0.031		0.60	
	β	*p*	β	*p*	β	*p*
Coefficient-point line length	0.309	**0.053**	0.379	**0.039**	0.226	0.293
Coefficient-point line angle	0.177	0.253	−0.047	0.787	0.154	0.464
Inhibitory control	0.266	0.090	0.264	0.089	−0.169	0.425

coefficient-point line length and math performance are illustrated in Supplementary Fig. 1.

As noted above, 4-year-olds as a group relied on both number and field area equally in the nonsymbolic numerical discrimination task, as evidenced by statistically equivalent β_{number} and field area model coefficients. Individually, however, some 4-year-olds appeared to be basing their decisions entirely on field area, whereas others were successfully using number. For 4-year-olds who are basing their decisions entirely on field area, acuity as measured by coefficient-point line length would actually reflect the precision of their field area discrimination rather than their numerical discrimination. Because coefficient-point line length was a significant predictor of math achievement in 4-year-olds, we wondered if in children with strong non-numerical biases, field area precision would be related to math achievement. To test this, we performed a median split based on coefficient-point line angle (i.e., high or low non-numerical bias) and then assessed the relation between coefficient-point line length and math achievement. In 4-year-olds with smaller coefficient-point line angles (less non-numerical bias), there was a significant correlation between coefficient-point line length and math achievement ($r(19) = 0.48$, $p < 0.05$). However, in 4-year-olds with larger coefficient-point line angles (greater non-numerical bias), there was no significant relation between coefficient-point line length and math achievement ($r(18) = 0.27$, $p > 0.27$). Therefore, in children who were relying on field area rather than number when performing the nonsymbolic numerical comparison task, field area acuity was not predictive of math achievement. Only in children who successfully based their choice decisions on number did we find a relation between discrimination acuity and math achievement. This suggests that the relation between discrimination acuity and math achievement is specific to numerical discrimination acuity.

4. Discussion

The aims of this study were threefold. First, we characterized the changing influence of non-numerical stimulus features on nonsymbolic numerical discrimination over development. Second, we investigated whether the biasing effects of these non-numerical features contributes to the relation between numerical acuity and symbolic math skill. Third, we tested the hypothesis that inhibitory control may drive the link between nonsymbolic numerosity discrimination and symbolic math skill. To address these questions, we used the algorithm described in DeWind et al. (2015) to determine the relative weight that participants placed on numerical and non-numerical stimulus features when making decisions about number. A critical aspect of this method is that it aims to quantify the effects of non-numerical stimulus features using regression models rather than designing the stimuli to control for them. As a result, this method enabled us to objectively quantify the biasing effects of non-numerical stimulus features and to dissociate numerical acuity from attention towards non-numerical features. The present work therefore both extends the use of the model and analysis framework developed by DeWind et al. (2015) to developmental populations and applies it to clarifying the relation between nonsymbolic numerical representations, symbolic math, and inhibitory control.

4.1. Developmental changes in the influence of non-numerical stimulus features

Consistent with previous results (Allik & Tuulmets, 1991; Dakin et al., 2011; Defever et al., 2013; DeWind et al., 2015; Gebuis & Gevers, 2011; Gebuis & Reynvoet, 2011b, 2012a, 2012b; Ginsburg & Nicholls, 1988; Hurewitz et al., 2006; Rousselle & Noël, 2008; Rousselle et al., 2004; Soltész et al., 2010; Sophian & Chu, 2008; Szűcs et al., 2013; Tibber et al., 2012; Tokita & Ishiguchi, 2010, 2013), we found that numerical decision-making in all age groups was influenced by non-numerical features. When participants were attempting to make decisions based on the numerosity of the arrays, even adults were unable to ignore the spacing of items within the arrays. Interestingly, though the overall influence of size parameters was not significant in any age group, large individual differences were found: some participants tended to view larger dots as more numerous while others viewed smaller dots as more numerous.

The biasing effects of non-numerical features decreased significantly with age. Four-year-old children's numerical judgments were most influenced by non-numerical features. Specifically, four-year-olds' numerical judgments were strongly influenced by the spacing of the dots: they weighted numerosity and field area equally when deciding which of two arrays contained more dots. Six-year-old children, on the other hand, exhibited strikingly adult-like performance. The similarities in the weighting of non-numerical cues by 6-year-olds and adults, in contrast to the larger biases observed in 4-year-olds, suggests that numerical decision-making undergoes substantial change between 4 and 6 years of age. The 4-year-olds in the present study behaved much like the children in Piaget's classic number conservation task: as the dots became more spread out over a larger field area, the arrays were viewed as being more numerous (Piaget, 1952). The biasing effects of field area have been noted elsewhere (Allik & Tuulmets, 1991; Dakin et al., 2011; Kramer, Di Bono, & Zorzi, 2011; Tokita & Ishiguchi, 2013), and suggest that field area is a dimension that requires greater consideration in future studies. However, the influence of field area and other non-numerical features was markedly decreased in 6-year-olds. This overall decrease in influence from non-numerical features between 4 and 6 years of age suggests that the ability to selectively attend to number contributes to the development of the number sense.

The time frame between 4 and 6 years of age is associated with significant improvements in executive functions (e.g., Gerstadt et al., 1994; Wright, Waterman, Prescott, & Murdoch Eaton, 2003), which is consistent with the hypothesis that domain-general improvements in inhibitory control may be contributing to the decrease in non-numerical bias. However, in the present study we did not find a relation between non-numerical bias and inhibitory control. The time frame between 4 and 6 years of age is also when children typically enter the formal educational system. Although none of the 4-year-olds in the present study had begun kindergarten, the 6-year-old participants had all entered first grade. Therefore, the 6-year-olds but not the 4-year-olds had begun formal mathematics education. Previous research in adults with varying levels of exposure to formal schooling has demonstrated that experience with numerical symbols and calculation appears to sharpen the precision of internal number representations (Nys et al., 2013; Piazza, Pica, Izard, Spelke, & Dehaene, 2013). These findings, in combination with the present results, suggest that exposure to mathematics education may increase children's attention to number, perhaps by making number a more salient dimension and thereby increasing the facility with which children can inhibit attention towards non-numerical features (see also Gebuis et al., 2009). At the same time, the fact that we observed continued improvements in numerical acuity between 6 years of age and adulthood, a time period in which bias towards non-numerical features remains relatively stable, suggests that selective attention to number may be just one of multiple mechanisms that promotes refinement of the number sense.

Note, however, that under some circumstances numerical decision-making does not appear to be influenced by non-numerical cues. For example, when numerical arrays are relatively sparse, numerosity may be perceived directly without an effect of density. It appears that it is only once arrays reach a critical density that density exerts an influence on numerical decision-making (Anobile, Cicchini, & Burr, 2014). Complimentary lines of research from computational modeling and electrophysiology also suggest that there may be situations in which numerosity perception is independent of non-numerical features. Recent modeling work suggests that visual numerosity is a statistical property that emerges independently from other visual features during unsupervised learning in hierarchical generative models of sensory input (Stoianov & Zorzi, 2012). Consistent with this view, data from ERPs suggest that the brain exhibits sensitivity to changes in visual number independent from changes in other visual cues beginning extremely early in the visual processing pathway (Park, DeWind, Woldorff, & Brannon, 2015). Furthermore, behavioral data from non-human primates, children, and adults from both high- and low-numeracy cultures demonstrate that in some discrimination and categorization tasks, participants spontaneously attend to number rather than non-numerical cues such as size or density (Cicchini, Anobile, & Burr, 2016; Ferrigno, Jara-Ettinger, Piantadosi, & Cantlon, 2017). Indeed, in some situations in appears that participants are more sensitive to changes in number than they are to other non-numerical features (e.g., Cicchini et al., 2016; Libertus, Starr, & Brannon, 2014; Odic, 2017; Starr & Brannon, 2015). Although non-numerical features did bias numerical decision-making in the present experiment, the situations in which non-numerical visual cues do and do not influence numerical decision-making require additional research. In particular, there may be interaction effects between numerical and non-numerical cues not captured by our model, such that non-numerical cues may be more or less influential depending on both the numerical and non-numerical feature ratios at play. Complicating matters further, numerosity perception can be biased by configural and topographic effects (Cicchini, Anobile, & Burr, 2014; Franconeri, Bemis, & Alvarez, 2009), thus raising the question of how these effects may interact with the non-numerical features discussed here.

An important caveat for interpreting our results is that stimulus presentation time varied across the age groups, such that presentation time decreased with increasing age. Although varying the presentation time between children and adults is common practice in nonsymbolic numerical comparison studies (e.g., Gilmore, Attridge, De Smedt, & Inglis, 2013; Halberda & Feigenson, 2008; Inglis et al., 2011), it is possible that differences in stimuli exposure influence numerical decision-making strategies. Note that this concern also applies to studies in which the stimuli remained on the screen until a response is made, as response times decrease with age (e.g., Defever et al., 2013; Szűcs et al., 2013). In the present study, we chose to vary presentation time as a function of age so that we could use the same stimuli and ratios across all age groups while equating performance to best model numerical decision-making strategies. Children were given longer stimulus presentation times than adults not because we assumed that children require more processing time to extract numerical information from the visual arrays, but rather due to differences in attention span. With short presentation durations, children are more likely to look away from the screen and miss the stimulus presentation all together. Extending the presentation duration for children therefore increases the number of the stimuli that they attend to, and consequently minimizes random guessing. Thus, from our data we can conclude that at a given level of performance, 4-year-olds exhibit stronger non-numerical biases than 6-year-olds, and 6-year-olds exhibit non-numerical biases that are equivalent to those exhibited by adults.

However, even if presentation time influences bias and acuity differentially, previous studies suggest that it would be in the direction of attenuating our effects rather than generating spurious ones. Prior work has demonstrated that as presentation times increases, numerical representations become more precise (Inglis & Gilmore, 2013). If children were able to form more precise numerical representations due to longer stimulus presentation, they should have exhibited smaller non-numerical biases and greater numerical acuity. Likewise, if numerical representations require the integration of multiple non-numerical visual cues, shorter presentation times should then increase non-numerical bias and decrease numerical precision. However, we found the opposite pattern of results. In the present experiment, numerical acuity increased with age whereas non-numerical bias decreased, despite older participants experiencing shorter presentation times than younger participants. Therefore, it seems unlikely that the age-related differences in numerical decision making observed here can be attributed solely to differences in stimulus presentation time. Nevertheless, the exact effect of presentation time on the biasing effects of non-numerical stimulus features and how that effect interacts with development will be important avenue for future research.

4.2. Relations between numerical acuity, non-numerical bias, and math ability

The second focus of the study concerns the role that attention towards non-numerical stimulus features may play in the relation between numerical acuity and symbolic math skill. In two recent studies, inhibitory control was found to mediate the relation between children's numerical acuity and math skill (Fuhs & Mcneil, 2013; Gilmore, Attridge, Clayton, et al., 2013). In these studies, the relation between math skill and numerical acuity was significant only for the incongruent trials in a nonsymbolic numerical comparison task, in which the more numerous dot arrays had smaller dots than the less numerous arrays. In these trials, participants needed to disregard dot size, which may be a salient feature, and instead focus on the potentially less-salient feature of numerosity. Furthermore, these studies found that the relation between math skill and numerical acuity did not hold once a domain-general measure of inhibitory control was accounted for. These studies suggest that it may be the ability to selectively attend to number, rather than numerical acuity itself, that relates to math achievement.

An advantage of the present study, however, is that the methodology allowed for precise and dissociated measures of acuity and non-numerical bias rather than simply assessing the difference in performance between congruent and incongruent trials. This enabled us to directly test the relations between children's numerical discrimination acuity, non-numerical bias, symbolic math performance, and inhibitory control. If demands on inhibitory control on incongruent trials in the nonsymbolic numerical comparison task are driving the relation between numerical acuity and math achievement, we hypothesized that non-numerical bias should correlate with inhibitory control. However, we did not find a significant relation between children's non-numerical bias and their inhibitory control, which suggests that the ability to selectively attend to number in the presence of non-numerical features may draw on a separate cognitive resource from domain-general inhibitory control.

We also tested the unique contributions of numerical acuity, non-numerical bias, and inhibitory control towards children's math achievement and found that acuity was the strongest predictor of variance in math achievement. Non-numerical bias was not a significant predictor of math achievement. However, there was a trend towards a significant contribution of inhibitory control, which is in line with previous studies demonstrating a link between inhibitory control and math performance (Bull & Lee, 2014; Bull & Scerif, 2001; St Clair-Thompson & Gathercole, 2006). Note that while the inhibitory control task used here was similar to those used in previous studies that have found that inhibitory control mediates the relation between numerical acuity and symbolic math (Fuhs & Mcneil, 2013; Gilmore, Attridge, Clayton, et al., 2013), this task is only a single measure of inhibitory control and it is possible that with a different measure of inhibitory control there may be a stronger relation to non-numerical bias or math achievement. However, the present findings and those of a recent study that also failed to find evidence that inhibitory control mediates the relation between numerical acuity and symbolic math (Keller & Libertus, 2015) suggest that domain-general inhibitory control is driving neither developmental improvements in numerical acuity nor the relation between numerical acuity and symbolic math achievement.

Critically, we found that the relation between math achievement and acuity was specific to children who were attending to number in the nonsymbolic numerical comparison task. For children who were basing their decisions on non-numerical features, discrimination acuity was unrelated to math achievement. This suggests that it is specifically numerical acuity, rather than overall task performance, that is related to math achievement. Taken together, these findings do not support the hypothesis that inhibitory control drives the relation between nonsymbolic numerical discrimination performance and math achievement. Instead, the present results suggest that numerical acuity and inhibitory control each make independent contributions to children's math achievement.

We did not find any evidence for a relation between numerical acuity and math achievement in adults. Although many previous studies have found such a relation (e.g., Agrillo, Piffer, & Adriano, 2013; DeWind & Brannon, 2012; Halberda et al., 2012; Libertus et al., 2012), two recent meta-analyses suggest that the relation between nonsymbolic numerical acuity and symbolic math achievement may be strongest in young children (Chen & Li, 2013; Fazio et al., 2014). The present results are consistent with the idea that approximate number representations may be most critical when children are first acquiring symbolic number knowledge and arithmetic, and the link between nonsymbolic number representations and formal math may therefore decrease with increasing age and math experience (Fazio et al., 2014; Mussolin et al., 2012). In particular, the precision of nonsymbolic number representations may influence the acquisition of numerical symbols, and indirectly influence math performance through facility with these symbols (Starr et al., 2013; vanMarle, Chu, Li, & Geary, 2014). Therefore, children who have greater precision in their internal number representations, regardless of their bias towards non-numerical features, may be more adept at learning number words and digits in comparison to their peers who exhibit poorer nonsymbolic numerical precision, and this may provide an advantage for performing basic arithmetic operations. In older children and adults, approximate number representations may serve as an online error-monitoring system, such that gross errors in arithmetic can be detected and discarded (Feigenson, Libertus, & Halberda, 2013; Lourenco, Bonny, Fernandez, & Rao, 2012).

Several recent studies also suggest that the strength of the relation between numerical acuity and symbolic math varies depending on which aspect of symbolic math is assessed (Anobile, Stievano, & Burr, 2013; Lourenco et al., 2012; Lyons & Beilock, 2011; Piazza et al., 2010). These studies suggest that the relation is strongest between numerical acuity and symbolic math tasks that tap numerical magnitude (e.g., ordering symbolic digits, performing arithmetic operations in which the answer is not retrieved from memory). Although 6-year-olds and adults both performed the same assessment of symbolic math, it is possible that adults were able to retrieve a larger portion of the responses from memory, and therefore did not need to process the magnitude of the digits to the same degree as the 6-year-olds. This could explain why we found a relation between symbolic math achievement and numerical acuity in 6-year-olds but not in adults. However, it is impossible to draw strong conclusions based on a null correlation, and further work is required in order to clearly delineate the relation between nonsymbolic number representations and symbolic math throughout development.

4.3. Conclusions

The present study demonstrates that the ability to selectively attend to number over non-numerical stimulus features is a unique component of the development of the number sense. The ability to selectively attend to number and ignore non-numerical features undergoes rapid maturation in early childhood, and reaches adult-like levels by the age of six. Numerical acuity, by contrast, continues to improve into adulthood, which suggests that acuity and non-numerical bias mature through distinct mechanisms. Importantly, this study also reveals that numerical acuity is a unique predictor of symbolic math achievement in 4- and 6-year-old children, above and beyond the contributions of inhibitory control, whereas non-numerical bias is unrelated to math performance. Taken together, these findings demonstrate that decreasing non-numerical bias and increasing acuity are separable mechanisms of number sense development, and that the relation between the number sense and symbolic math in young children is grounded in the acuity of the number sense.

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Appendix A. Supplementary material

The complete dataset can be accessed at https://osf.io/y56u2/. Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.cognition.2017.07.004.

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