Published in final edited form as:
Behav Processes. 2013 February ; 93: 91–97. doi:10.1016/j.beproc.2012.11.001.

Nothing to it: Precursors to a Zero Concept in Preschoolers

Dustin J. Merritt*
Center for Cognitive Neuroscience, Duke University

Elizabeth M. Brannon
Department of Psychology and Neuroscience, Duke University

Abstract

Do young children understand the numerical value of empty sets prior to developing a concept of symbolic zero? Are empty sets represented as mental magnitudes? In order to investigate these questions, we tested 4-year old children and adults with a numerical ordering task in which the goal was to select two stimuli in ascending numerical order with occasional empty set stimuli. Both children and adults showed distance effects for empty sets.. Children who were unable to order the symbol zero (e.g., $0 < 1$), but who successfully ordered countable integers (e.g., $2 < 4$) nevertheless showed distance effects with empty sets. These results suggest that empty sets are represented on the same numerical continuum as non-empty sets and that children represent empty sets numerically prior to understanding symbolic zero.

Keywords

zero; empty-set; distance effect; numerical cognition; numerical continuum

Young children face a difficult cognitive challenge when learning the meaning of number words and the verbal count procedure (e.g., Fuson & Hall, 1983; Wynn, 1990, 1992; Carey, 2009). One of the many reasons why this is such a difficult process for children is that numbers are abstract symbols that are not bound to the physical and perceptual qualities of a stimulus set. For example, three trees and three cars differ in many important perceptual features, but they both share the common abstract feature of “threeness”. Other numbers that are not part of the count-list, such as zero, present an even greater challenge. Unlike the integers, zero does not represent the presence of a specific quantity; rather, it represents the absence of a quantity. As a result, the zero concept may present unique developmental and conceptual challenges for children.

The introduction of zero into modern symbolic notational systems occurred long after the incorporation of the count-list numbers. One of the earliest uses of zero was by the Babylonians (approximately 1500 BCE), who used zero as a placeholder to indicate the absence of a particular numerical value (e.g. zero in the number 101 represents the absence of a value in the “tens” column). Later, the Greeks used zero to indicate “absence”, but only

*Corresponding author Elizabeth M Brannon Duke University Center for Cognitive Neuroscience B203 LSRC, Box 90999 Durham, NC 27708 Brannon@duke.edu Phone: 919-668-6201 Fax: (919) 681-0815.

centuries later, in India, was zero introduced as a number that could be used in mathematical computations (Bialystok & Codd, 2000, Menninger, 1992).

The relatively delayed introduction of zero into modern notational systems is paralleled by the delayed developmental trajectory of mastering the meaning of the word "zero" in young children. Wellman and Miller (1986) demonstrated that children master the cardinal and ordinal properties of the count-list integers before they incorporate "zero". In one experiment, children were presented with an array of four cubes and asked to first count the cubes and then count backwards as one cube at a time was removed from the array. They found that children were much better at verbally identifying the number of cubes when there were more than zero cubes compared to when there were no cubes remaining. Similarly, when asked to make numerical magnitude comparisons, children were much more accurate comparing the numerals 1–5 with each other than comparing the count-list integers with zero.

Children's understanding of the symbol zero develops in a series of stages (Wellman and Miller, 1986). Children first learn to identify the symbol for zero without understanding what the symbol means. Later, children learn that zero represents "none" or "nothing", but, they still fail to recognize that zero is a numerical value that occupies a place on the numerical continuum. For example, when asked, "which is smaller, zero or one?" children will often insist that "one" is the smaller number (Wellman and Miller, 1986). Finally, children learn the relationship between zero and the other numbers on the continuum, and appreciate that zero is smaller than one. Interestingly, confusion surrounding the zero concept is not unique to children; even educated adults have difficulty grasping the status of zero as a number, and how zero operates in mathematical calculations (Wheeler & Feghali, 1983).

The delay in children's understanding of zero raises an important question. Do children incorporate zero into the same numerical continuum that is occupied by the count-list integers? Or do they initially develop a fundamentally different representation for zero, and only learn to evaluate it relative to other numbers through a system of rules?

Evidence that zero is represented differently than the count-list

Positive integers may be psychologically privileged because they can be represented as magnitudes by a numerical accumulator (Wynn, 1998; Meck & Church, 1983). Wynn argued that preverbal children and animals are unable to represent empty sets given their reliance on an accumulator to represent numerosities. There is no mental magnitude value for zero in such an accumulator.

Consistent with this line of reasoning, Wynn and Chiang (1998) found that 8-month old infants tested in a modified version of the well-known addition and subtraction paradigm developed by Wynn (1992) did not form expectations of empty set outcomes. In a magical disappearance condition, infants watched as an object was placed behind a screen. The experimenter then secretly removed the object, so that it appeared to have disappeared when the screen was lowered. In a magical appearance condition, a screen was raised in front of a single object while the infant watched. The experimenter then removed the object in view of the infant, but secretly replaced the object behind the screen. Thus, when the screen was lowered, the object seemed to have magically appeared. When compared to control conditions, infants looked longer at the magical disappearance, but did not look longer at the magical appearance. Wynn and Chiang argued that the infants did not notice the magical appearance because they were unable to represent the initial empty set, and therefore did not form the expectation that there should be no objects behind the occluder.

Experiments with adults also suggest that zero is represented differently than other natural numbers. Brysbaert (1995) examined numerical processing time for the Arabic numerals 0–99. Participant's eye movements were tracked as they sequentially viewed three numerals presented on a screen. After viewing all three numerals, participants answered whether the middle numeral in the sequence had a magnitude between the first and third value (e.g. 21, 23, 27), or, whether its magnitude was outside the first and third value (e.g. 27, 21, 23). Brysbaert found that the amount of time participants looked at each numeral could be predicted by the numeral's logarithmic value. However, the amount of time participants looked at zero was significantly longer than the amount of time they looked at the numeral “one”. Brysbaert concluded that a single system could account for scan patterns to numerals greater than zero, but that zero itself was treated qualitatively differently and was not part of the mental number line.

Other differences between zero and the counting numbers emerge when examining symbolic arithmetic (Butterworth, Zorzi, Girelli & Jonckheere, 2001). Wellman and Miller (1986) report that, unlike the counting integers, young children rely on simple algebraic rules when making calculations with zero such as $n + 0 = n$ and $n - 0 = n$. Wellman and Miller speculate that children's difficulty in understanding zero might contribute to their rule-based approach to mathematical operations. Similarly, other researchers have found that some patients with brain damage show deficits for fact-based arithmetic processing (e.g. $7 \times 3 = 21$) that differ from those of rule-based arithmetic processing involving zero (e.g. $N \times 0 = 0$) (McCloskey, Aliminosa, & Sokol, 1991; Semenza, Grana & Girelli, 2006). However, it is difficult to draw conclusions about how zero is represented from studying arithmetic operations given that rule-based calculations can be used even without a strong conceptual understanding of zero (Semenza, Grana & Girelli, 2006).

Evidence that zero is represented as part of the count-list

Although the previously mentioned studies suggest that empty sets and the symbol zero may be represented differently than the other numbers, some evidence suggests that zero is represented along a common continuum with positive integers. Bialystok and Codd (2000) placed 5 dolls representing “Sesame Street” characters on a table. In front of each doll were two boxes, one box for lunch, and the other for an afternoon snack. Children, aged 3–7 years old, were asked to distribute 2, 5, 0, ½, or ¼ cookies in each box. For example, the experimenter might say: “Can you give Big Bird 2 cookies for lunch?” Children were than asked to label the boxes with post-it notes so they could later identify the number of cookies in the boxes. Consistent with the findings of Wellman and Miller (1986), three-year-old children had more difficulty distributing zero cookies than whole number cookies. However, children also showed some remarkable similarities in their labeling methods for both zero and whole numbers. Three and four year olds were more likely to use iconic symbols than were older children. Specifically, they drew circles to represent cookies on a single post-it in one-to-one correspondence to the number of cookies that were placed into the box. For example, a child who was asked to place two cookies in a box labeled the quantity by drawing two circles on the post-it note. For boxes that contained zero cookies, young children left the post-it blank to represent the absence of cookies in the box.

Other evidence suggests that empty sets can be represented by nonhuman animals as analog magnitudes. Given that previous research has shown that humans and animals share an approximate number system (ANS) for processing numerical values, recent findings in animals may be particularly relevant for exploring nonverbal numerical processing in young children (Brannon & Terrace, 2000; Cantlon & Brannon, 2006; Feigenson, Dehaene & Spelke, 2004; Gelman & Gallistel, 2004; Neider & Miller, 2004). When comparing the relative magnitude of numerical arrays, humans and animals show similar distance and magnitude effects. Generally, as the disparity (distance) increases between two numerical sets, discrimination becomes easier. When the distance between two numerical sets is held constant, but the magnitude of those sets increases (e.g. 2 and 4 vs. 8 and 10), discrimination becomes more difficult. Therefore, if empty sets occupy a place on the numerical continuum, then when comparing empty sets with other values, we would expect lower accuracy when the distance from the empty set is small and greater accuracy when the distance from the empty set is large.

Biro and Matsuzawa (2001) trained a chimpanzee (named Ai) to match arrays of dots to corresponding Arabic numerals. The numeral zero was matched to a blank square that did not contain any dots. Ai was then tested with an ordinal task in which she was required to select the Arabic numbers in order from smallest to largest. Ai was unable to spontaneously transfer the symbol zero from the matching task to the ordinal task initially; although not surprisingly, she eventually learned to correctly order the symbol zero relative to the other symbols. In another set of studies, Pepperberg and colleagues found that an African Gray Parrot (named Alex) spontaneously used the word “none” to identify absence in some numerical contexts, but not in others (Pepperberg & Gordon, 2005; Pepperberg, 2006).

Although results with Ai the chimpanzee and Alex the parrot suggest that animals may possess some important features of a zero-like concept, both animals showed limitations in their ability to transfer a zero symbol to a novel context. Would animals fare better if the symbolic requirement were removed? To address this question and to assess whether animals have precursors to a zero concept when not required to learn arbitrary stimuli. Merritt, Rugani, and Brannon (2009) tested rhesus monkeys, who were already proficient at numerical matching and ordering tasks, on their ability to respond to empty sets. The matching task required monkeys to select a target that numerically matched a previously shown sample array. The numerical ordering task required monkeys to respond to two numerical arrays in ascending (red background) or descending (blue background) order. In both tasks, standard trials contained numerical arrays with values between 1 and 12. Correct answers produced positive (juice) feedback and incorrect answers produced negative (timeout) feedback. On probe trials, monkeys were tested on their ability to match or order empty set stimuli that consisted of a blank square that varied in color and size but contained no elements. In order to prevent the monkeys from learning the correct choice during probe testing, the probe-trial choices were not differentially reinforced. Results indicated that the monkeys were able to spontaneously match and order empty sets at accuracy levels comparable to those of the other numerosities. Further, in both tasks the monkeys showed distance effects, with accuracy increasing as the distance between the empty set and the other numerosity increased. This pattern of results suggests that the monkeys were not treating the empty set as a qualitatively different non-numerical stimulus, but rather, they viewed the empty set a numerical value that could be directly compared to other numerical values.

Here, we tested four-year old children and adults using procedures similar to those of Merritt et al (2009). Four-year old children were chosen because this is the age where children start to resolve their confusion about zero, and begin to understand both its cardinal and ordinal properties (Wellman & Miller, 1986). Our study was designed to answer two main questions. First, are mental magnitudes generated for empty sets? If so, then children and adults should show distance effects indicating representational continuity with the other numerosities. Alternatively children may view empty sets as a qualitatively different non-numerical stimulus. Second, our study investigated the relationship between a child's ability to order empty sets and their developing understanding of the meaning of the symbol zero. If the ability to order and/or match empty sets serves as a foundation for learning the meaning of the symbol zero, we may see this capacity emerging before reliable usage of the symbol zero in young children.

Experiment 1

Methods

Subjects—Participants were 21 four-year old children (mean age = 4.5, *SD* = .32). One additional child was excluded from the analyses because he/she avoided selecting the empty set on 96% of all probe trials.

Apparatus—Participants were tested in a small room while seated in front of a 17-inch computer screen affixed with a *MagicTouch* touch sensitive screen. A custom-built program written in RealBasic presented the stimuli and registered the responses.

Procedure—The task required children to select the numerically smaller of two numerosities[1] (see Figure 1). At the beginning of each trial, a start stimulus (a picture of a white rabbit) appeared in the lower right corner of the screen. Pressing the start stimulus resulted in the presentation of two numerical stimuli in random locations on the screen. If the child correctly selected the smaller numerosity, a black border appeared around the stimulus, and the stimulus remained on the screen until the second stimulus was selected. Once the child selected the second stimulus correctly, she received computer generated visual and auditory feedback (a 1-second audio clip and a picture of a sun). If the larger numerosity was selected first, the child received negative visual and auditory feedback (a 1-second "Try Again" audio clip and a 3-second black screen) and the trial ended. Stimuli were yellow squares within which circular elements were randomly placed. Elements varied in size, shape, and color so that none of these dimensions could be used as ordering cues.

Training and testing: The stimuli were trial-unique exemplars of 0, 1, 2, 4, and 8. For any trial that contained an empty set, selecting the stimuli in either ascending or descending order produced positive visual and auditory feedback. This was done so that no information about the empty set was conveyed during the experiment. All possible pairs were presented in a pseudo-random order, with the constraint that each pair was presented an equal number of times throughout the session. The children were given a total of 60 trials.

Controls: In order to eliminate the possibility that children were using background surface area as a cue on empty set trials density was controlled by varying the background size of the two stimuli. The larger numerosity had the larger background size (12.73 cm × 10.6 cm) on half of the trials and the smaller background size (6.36 × 5.32 cm) on the other half of the trials.

Instructions and Demonstration: Prior to each session, children watched as the experimenter demonstrated the task. The children were instructed to touch the picture with the smaller number of objects first, and to touch the picture with the larger number of objects second. Children were further instructed that they could take as much time as they needed to press the start stimulus, but afterward, they should respond as quickly as possible without counting. On two occasions a child attempted to count aloud, she/he was interrupted and reminded not to count. There were a total of 8 practice trials. Following the demonstration by the experimenter, the child was allowed to complete the remainder of the practice trials. On three occasions when the child failed to get the last two trials correct, the

[1]In research with adults and monkeys performance has been shown to be equivalent for choosing the larger or smaller array (e.g., Cantlon and Brannon, 2005).

child was given four additional practice trials. The practice trials did not contain the empty set.

Symbolic Number Assays—Children were tested with four post-test numerical assays to assess their knowledge of symbolic numbers.

Give-a-number and How Many tasks: The "Give a number" and "How many?" tasks were based on Wynn (1992). During the "how many" task, the experimenter placed a set of 1–6 plastic dolphins in a line in front of the child. The child was presented with a different set size on each trial until all 6 sets had been presented. During each trial, the child was asked "How many dolphins are there? Can you count them out loud?" In the "give a number" task, the standard titration procedure was used. Twelve plastic dolphins were placed in a pile, and children were first asked "Can you give me one dolphin?" The number of dolphins requested was increased by one each time the child gave the requested number correctly and was decreased by one each time the child erred. The task was completed when the child successfully gave N dolphins twice and was unsuccessful on two requests for N + 1. For both tasks, the experimenter encouraged the children to count and to check their answers by asking "Can you count them for me to make sure?"

Smallest Number query: Children were asked, "What is the smallest number in the world?" If the child identified a positive integer, the child was asked "Is there a number smaller than that?" This process was repeated until the child failed to provide a number that was smaller than their previous answer.

Symbolic Ordering Task: Children were shown two 3 by 3 inch index cards with Arabic numbers written on them. The child was asked "which one is the smaller number?" The values shown were 1 vs. 6, 2 vs. 4, 0 vs. 6, and 0 vs. 1. The left-right positions of the cards were randomized, and pair comparisons were chosen randomly by shuffling the pairs before each session.

Scoring the symbolic number assays

How Many: Scoring for the "how many" and "give a number tasks" followed the procedure used by Cantlon et al (2007). In the "how many" task, children received 3 points if they were able to correctly count all 6 dolphins and correctly indicate the cardinal number. They received 2 points if they counted a set of six incorrectly, but were able to correct the mistake. They also received 2 points if they counted correctly, but failed to accurately report the total number of dolphins when asked "how many?" They received 1 point if they were able to count at least two dolphins. Zero points were given if the child was unable to count and label correctly for sets of 2 dolphins.

Give a number: For the "give a number task", children received 3 points if they were able to give the experimenter exactly 6 dolphins. They received 2 points if they gave the experimenter an incorrect number of dolphins when asked for 6, but were able to correct their mistake. Children were given 1 point if they were able to accurately hand the experimenter at least two dolphins, and zero points if they were unable to produce at least two dolphins correctly.

Smallest number and Symbolic ordering: The "smallest number" and "symbolic ordering" tasks were scored in a binary fashion, as either pass or fail. In order to pass the smallest number task, the child had to answer that zero was the smallest possible number. If the child answered with a number greater than zero, then the child was scored as having failed the task. In the symbolic ordering tasks, comparisons involving zero were scored separately from nonzero comparisons. For zero comparisons, children who identified zero as the smallest number for *both* comparisons were scored as passing (0 vs. 1, and 0 vs. 6). In contrast, children who chose the larger number in either comparison were scored as failing. Similarly, children who identified the smallest number for both non-zero comparisons (1 vs. 6, and 2 vs. 4) were scored as passing, whereas children who did not answer both correctly were scored as failing.

Results

Symbolic Performance

On the "how many" task, 75% of all children scored the maximum score of 3, 25% scored a 2, and no children scored a 0 or 1. On the "give a number" task, 75% of children scored the maximum score of 3, 15% scored a 2, and 10% scored a 1. Further, 75% of children accurately ordered both pairs of positive numbers (binomial, $p < .05$), and 45% accurately ordered both pairs containing zero (binomial, $p < .05$). A Wilcoxon signed ranks test revealed that children had more difficulty ordering comparisons involving zero than comparisons of positive integers ($Z = -2.12$, $p < .05$). When tested on the "smallest number" task, 57% of children identified zero as the smallest number; the remaining children identified numbers larger than zero[1].

Non-Symbolic Performance

Overall, children ordered (non-empty) numerical sets at above chance accuracy [$M = 73.9\%$; $t(19) = 5.9$, $p < .05$]. However, children were considerably more accurate ordering non-empty numerical sets compared to ordering pairs that contained an empty set (Figure 2; paired samples t-test, [$t(19) = 3.12$, $p < .05$]. Empty set accuracy did not exceed chance for pairs 0,1, [$t(19) = 0$, $p = 1.0$] and 0,2 [$t(19) = 1.25$, $p = 0.12$] but did exceed chance for pairs 0,4 [$t(19) = 1.74$, $p < .05$] and 0,8 [$t(19) = 2.89$, $p < .05$]. Due to large variability and inconsistencies in RT performance, we did not analyze RT for distance effects (e.g., M = 2.09 s, SD = 1.65 s).

Because empty sets do not lend themselves to ratio comparisons, we compared distance effects for empty set probe trials with standard trials that contained the numerosity one (see also Merritt, Rugani, & Brannon, 2009). If children treat empty sets as numerical values that can be compared with other numerical values, then we would expect that distance effects with empty sets should be similar to those observed with sets of one. As demonstrated in Figure 3, children showed distance effects for both empty sets $F(3,57) = 3.88$, $p < .05$, and for numerical comparisons involving one item, $F(2,38) = 7.22$, $p < .05$. The empty set distance effects shown here are qualitatively similar to those found by Merritt et al., (2009) in rhesus monkeys.

If empty sets are represented as mental magnitudes, then children who are more proficient at forming representations of positive numerosities should also be more proficient at forming representations of empty sets. In order to assess whether proficiency ordering sets of one translated into proficiency ordering empty sets, we divided children based on a median split according to their accuracy on standard trials that included the numerosity one. We then examined accuracy for empty sets as a function of distance in each of these groups.

Children were divided into two groups based on their performance on numerosity comparisons that did not include empty sets. The children who were in the bottom half of this group were above chance on numerosity comparisons with sets of one, $t(9) = 2.51$, $p <$.

[1]One child failed to provide an answer on the "smallest number" task.

05, but were at chance on comparisons with empty sets, $t(9) = -0.04$, $p = 0.49$. Further these children did not show distance effects with empty sets ($F(3,27) = 1.05$, $p = 0.39$) or for sets of one. While a repeated-measures ANOVA on distance for comparisons with the numerosity one was significant, $F(2,18) = 8.67$, $p < .05$; trend tests revealed a quadratic function, $F(1,9) = 23.65$, $p < .05$, rather than a linear function $F(1,9) = 0.34$, $p = 0.58$ reflecting that accuracy was highest for the middle distance rather than the largest distance.

For top half performers, accuracy was above chance for both empty sets $t(9) = 2.17$, $p < .05$, and sets of one, $t(9) = 13.28$, $p < .05$, and, they showed reliable distance effects for both empty sets, $F(3,27) = 3.59$, $p < .05$, and sets of one $F(2,18) = 6.77$, $p < .05$.

Relationship between performance on empty sets and symbolic number knowledge

Fifteen of the twenty children scored at ceiling on the “how many” and “give a number” tasks (not the exact same 15 children). For each task, accuracy on empty sets (e.g., choosing empty sets before a comparison array with 1–9 elements) was better for the 15 children who scored at ceiling compared to those children who scored lower (60% vs 52% and 61% vs 54% for the “give a number” and “how many” tasks respectively). Given the small and uneven sample sizes, neither of these comparisons were significantly different.

The final assay required knowledge of Arabic numerals rather than number words. Children who failed to order both pairs with the symbol zero were less accurate ordering empty sets ($M = 50\%$) than children who successfully ordered both pairs with the symbol zero ($M = 71\%$; $t(18) = -1.91$, $p < .05$). Similarly, children who identified zero as the smallest number were also significantly more accurate ordering empty sets ($M = 74\%$) compared to children who did not identify zero as the smallest number ($M = 43\%$, $t(17) = 2.9$, $p < .05$). Thus, performance on symbolic zero tasks was generally predictive of performance on ordering empty sets.

We examined performance on empty sets for the children who correctly ordered both pairs of positive numbers, but failed to order the two pairs with symbolic zero. Overall performance for these children did not exceed chance levels on empty sets [$M = 57\%$; $t(6) = 0.75$, p = 0.24]. Curiously however, as shown in Figure 4, these children did show a distance effect, with accuracy increasing as distance between the empty set and the other numerosity increased, $F(3,18) = 3.77$, $p < .05$. This suggests that the children may actually possess a rudimentary understanding of how empty sets relate to other numerosities. To explore this possibility we compared empty set performance for children who were unable to successfully order the symbol zero, but were able to order the positive numbers (n = 7), with children who failed both (n = 5). Unlike children who were able to order positive numbers, children who failed both tasks did not show a distance effect for empty sets [$F(3,9) = 0.16$, $p = 0.92$]. Further, their performance was significantly lower than children who correctly ordered the positive numbers [$t(10) = -1.9$, $p <.05$], and as shown in Figure 4, their overall accuracy was significantly below chance, thereby demonstrating a bias against selecting empty sets [$M = 36\%$, $t(4) = -2.5$, $p < .05$].

Overall, these results suggest that children may have a burgeoning understanding of the ordinal relationship between empty sets and other numerosities before they understand how the symbol zero relates to other symbolic numbers. They also suggest that comprehension symbolic zero's numerical meaning is unlikely to be critical in and of itself for children to appreciate empty sets as magnitudes on a mental number line. We return to the issue of the relationship between symbolic number knowledge and empty set performance in the discussion.

Experiment 2

When human adults are tested in numerical tasks that avoid verbal counting, their accuracy and RT is ratio dependent and appears to tap an approximate number system (ANS) shared with a variety of nonhuman animals (Cantlon & Brannon, 2006; see also Cordes et al, 2001; Platt & Johnson, 1971; Whalen, Gallistel, & Gelman, 1999). Given that zero is not a countable number, it may be that it is appreciated as categorically different from the integers. Certainly, as explained earlier, some evidence suggests that zero is not represented by the same way as other numbers (Brysbaert, 1995). Thus Experiment 2 investigates whether human adults represent empty sets as analogue numerical magnitude values, and thus show distance effects or alternatively whether they treat empty sets as categorically different from other numerosities.

Subjects

The participants were 10 undergraduate students from Duke University who participated in exchange for payment.

Procedure and apparatus

The procedure and apparatus were identical to that of the ordinal task in Experiment 1 with a few exceptions. First, rather than using a touch screen, participants responded by clicking each stimulus with a mouse. Second, participants were given 540 trials with all numerosities 0–9 within a single session. Third, correct choices were rewarded with a white screen that displayed the word “Correct!” in black letters. Incorrect choices produced a black screen with the word “Incorrect!” in white letters. Like the children, adults were rewarded for both correct and incorrect responses made during the empty set trials. No symbolic numerical assays were given to the adult participants. There was no time limit, but participants were asked to select the numerically smaller array as quickly and accurately as they could without counting.

Results

As with the previous experiments, we compared distance effects for empty set probe trials with standard trials that included the numerosity one. Overall accuracy was extremely high for empty set probe trials ($M = 98.5\%$) as well as trials containing the numerosity one ($M = 98.8\%$). There were no difference in accuracy between empty set probe trials and standard trials that included the numerosity one [$t(9) = -0.54$, $p = 0.60$]. No further analyses were done on accuracy given that it was at ceiling levels.

Figure 5 shows that adults exhibited a distance effect in RT for both empty sets and standard trials with the numerosity one. A repeated measures one-way ANOVA revealed that RT on correct trials decreased as distance increased for comparisons involving the empty set [$F(8,72) = 2.78$, $p < .05$] and those involving a set of one [$F(7,63) = 3.93$, $p < .05$]. Further, a two-way ANOVA with factors of distance (1–8) and comparison type (empty set vs. one) showed that there was no difference in the slope between the two comparison types [$F(7,63) = 0.92$, $p = 0.50$]

General Discussion

Our results suggest that children as young as four years of age represent empty sets along the same numerical continuum as other numerosities and thus represent them as analog magnitudes. At a group level children showed distance effects for comparisons with empty sets that were similar to reaction-time patterns in human adults (Experiment 2) and accuracy in monkeys (Merritt et al., 2009). However, there was variability in children's ability suggesting that the representation of empty sets may be in flux at 4 years of age.

When children were divided into two groups based on their performance on numerosity comparisons that did not include the empty sets, the higher performing group showed high accuracy levels and distance effects with empty sets. In contrast, the lower performing group was at chance on empty set comparisons and did not show a distance effect. Taken together, these findings suggest that children are incorporating empty sets into their mental number line in this developmental window.

A second question we investigated was whether young children understand the ordinal relationship between empty sets and the other numerosities before they understand the ordinal properties of the symbol for zero. We found that children who successfully ordered all four symbol pairs (two with zero and two without), were above chance on their ability to order empty sets and showed a distance effect for empty sets. Children who successfully ordered both of the positive integers but failed to order both of the pairs with the symbol zero were at chance on empty sets but nevertheless showed a distance effect for empty sets. Children, with the most limited knowledge of numerical symbols, who failed to correctly order one or more within each of the two trial types performed at chance on empty sets and did not show a distance effect for empty sets.

One possible explanation of these results is that two different factors are contributing to children's developing empty set performance. The first is an initial bias to avoid empty sets and the second is increasing understanding that empty sets represent a value less than one on the mental number line. Children may have an initial bias against selecting the empty set. This bias may emerge from children's experience with a verbal count list, which starts with “one.” This may create top-down interference that disrupts their ability to order empty sets. Parents rarely draw a child's attention to the absence of countable objects nor do they frequently start counting with the word “zero”. This enculturation may lead children to actively avoid empty sets. This avoidance may decrease as empty sets become fully incorporated into the child's mental number line, which may be occurring gradually as a separate process.

The protracted emergence of an appreciation of empty sets as numerical entities is consistent with Wynn and Chiang's (1998) finding that infants do not represent empty sets. However, the fact that empty sets are indeed treated as analog magnitudes by 4 years of age and into adulthood (in the current context) suggests that the nonverbal system for representing number as analog magnitudes is in fact capable of a zero setting. Acquisition of the symbol zero appears to occur in concert with, but is slightly delayed relative to their appreciation of the numerical value of empty sets. This raises the interesting question of how symbolic knowledge contributes to the non-symbolic appreciation of empty sets. What kinds of experience are most critical? Our study cannot speak to the question of whether or how developing ordinal symbolic knowledge informs a child's understanding of nonsymbolic empty sets. But one possibility the research suggests is that it may be necessary for children who do not yet understand the ordinal properties of symbolic zero to learn the ordinal relationship between the positive (non-zero) numbers prior to incorporating empty sets into the mental number line.

Conclusions

Prior work from our research group has demonstrated that monkeys represent empty sets as a value along the numerical continuum (Merritt, Rugani, & Brannon, 2009). This finding inspired us to ask whether adult humans and young children also represent empty sets in an analog fashion. We found unequivocal evidence that adults represent empty sets as mental magnitudes. However, our findings with 4-year-old children were more variable. In both tasks, only children who successfully ordered countable numerosities showed distance effects for empty sets. Thus consistent with the claim that infants may be unable to represent empty sets numerically we find that the capacity for appreciating empty sets as values along the numerical continuum has a protracted development (Wynn and Chiang, 1998). Further research, with younger children will be necessary to pinpoint when representing empty sets on the numerical continuum emerges over development and whether mapping of numerical symbols to numerosities plays an important role in empty set representations.

Acknowledgments

This paper is contributed as a tribute to Dr. Tony Wright and his research program. While neither of the authors had the opportunity to work with Tony as a student, our research programs have been greatly influenced by his contributions to the field of comparative cognition. This work was supported by an NSF Career award Grant No. 0448250, a James McDonnell scholar award to EMB, and by the National Institutes of Health under Ruth L. Kirschstein National Research Service Award No. F32MH082623 to Dustin J. Merritt. We also thank Maggie Vogel for assisting with this research and the members of the Brannon lab for their helpful discussion of these data.

References

Bialystok E, Codd J. Representing quantity beyond whole numbers: some, none, and part. Canadian Journal of Experimental Psychology. 2000; 54:117–128. [PubMed: 10881395]

Biro D, Matsuzawa T. Use of numerical symbols by the chimpanzee (pan troglodytes): Cardinals, ordinals, and the introduction of zero. Animal Cognition. 2001; 4:193–199.

Brannon EM, Terrace HS. Representation of the numerosities 1–9 by rhesus macaques (macaca mulatta). Journal of Experimental Psychology: Animal Behavior Processes. 2000; 26(1):31–49. [PubMed: 10650542]

Brysbaert M. Arabic number reading - on the nature of the numerical scale and the origin of phonological recoding. Journal of Experimental Psychology-General. 1995; 124(4):434–452.

Butterworth B, Zorzi M, Girelli L, Jonckheere AR. Storage and retrieval of addition facts: The role of number comparison. Quarterly Journal of Experimental Psychology. A, Human Experimental Psychology. 2001; 54(4):1005–1029.

Cantlon J, Brannon EM. Semantic congruity facilitates number judgments in monkeys. Proceedings of the National Academy of Sciences. 2005; 102(45):16507–16511.

Cantlon J, Fink R, Safford K, Brannon EM. Heterogeneity impairs numerical matching but not numerical ordering in preschool children. Dev Sci. 2007; 10(4):431–440. [PubMed: 17552933]

Cantlon JF, Brannon EM. Shared system for ordering small and large numbers in monkeys and humans. Psychological Science. 2006; 17(5):402–407.

Carey, S. The origin of concepts. Oxford University Press; New York: 2009.

Cordes S, Gelman R, Gallistel C, Whalen J. Variability signatures distinguish verbal from nonverbal counting for both large and small numbers. Psychonomic Bulletin and Review. 2001; 8(4):698–707. [PubMed: 11848588]

Feigenson L, Dehaene S, Spelke E. Core systems of number. Trends in Cognitive Sciences. 2004; 8(7):307–314. [PubMed: 15242690]

Fuson, KC.; Hall, JW. The acquisition of early number word meanings: A conceptual analysis and review. In: Ginsburg, HP., editor. The development of mathematical thinking. Academic Press; New York: 1983.

Gelman R, Gallistel CR. Language and the origin of numerical concepts. Science. 2004; 306(5695):441–443. [PubMed: 15486289]

Mccloskey M, Aliminosa D, Sokol SM. Facts, rules, and procedures in normal calculation - evidence from multiple single-patient studies of impaired arithmetic fact retrieval. Brain and Cognition. 1991; 17(2):154–203. [PubMed: 1799451]

NIH-PA Author Manuscript

NIH-PA Author Manuscript

Meck WH, Church RM. A mode control model of counting and timing processes. Journal of Experimental Psychology-Animal Behavior Processes. 1983; 9(3):320–334. [PubMed: 6886634]

Menninger, K. Number words and number systems. Ziffer, Z., editor. Dover; Mineola: 1992.

Merritt DJ, Rugani R, Brannon EM. Empty sets as part of the numerical continuum: Conceptual precursors to the zero concept in rhesus monkeys. Journal of Experimental Psychology: General. 2009; 138(2):258–269. [PubMed: 19397383]

Nieder A, Miller EK. Analog numerical representations in rhesus monkeys: Evidence for parallel processing. Journal of Cognitive Neuroscience. 2004; 16(5):889–901. [PubMed: 15200715]

Pepperberg IM. Grey parrot (psittacus erithacus) numerical abilities: Addition and further experiments on a zero-like concept. Journal of Comparative Psychology. 2006; 120(1):1–11. [PubMed: 16551159]

Pepperberg IM, Gordon JD. Number comprehension by a grey parrot (psittacus erithacus), including a zero-like concept. Journal of Comparative Psychology. 2005; 119(2):197–209. [PubMed: 15982163]

Platt JR, Johnson DM. Localization of position within a homogeneous behavior chain: Effects of error contingencies. Learning and Motivation. 1971; 2:386–414.

Semenza C, Grana A, Girelli L. On knowing about nothing: The processing of zero in single- and multi-digit multiplication. Aphasiology. 2006; 20(9):1105–1111.

Wellman HM, Miller KF. Thinking about nothing: Developmental concepts of zero. British Journal of Developmental Psychology. 1986; 4:31–42.

Whalen J, Gallistel CR, Gelman R. Nonverbal counting in humans: The psychophysics of number representation. Psychological Science. 1999; 10(2):130–137.

Wheeler M, Feghali I. Much ado about nothing: Preservice elementary school teachers' concept of zero. Journal of Research in Mathematics Education. 1983; 14(3):147–155.

Wynn K. Childrens understanding of counting. Cognition. 1990; 36(2):155–193. [PubMed: 2225756]

Wynn K. Childrens acquisition of the number words and the counting system. Cognitive Psychology. 1992; 24(2):220–251.

Wynn K. Psychological foundations of number: Numerical competence in human infants. Trends in Cognitive Sciences. 1998; 2(8):296–303. [PubMed: 21227212]

Wynn K, Chiang WC. Limits to infants' knowledge of objects: The case of magical appearance. Psychological Science. 1998; 9(6):448–455.

NIH-PA Author Manuscript

NIH-PA Author Manuscript

Highlights

Both children and adults show distance effects for empty sets.

Empty sets are represented on same numerical continuum as numerical sets.

Children represent empty sets numerically before they understand symbolic zero.

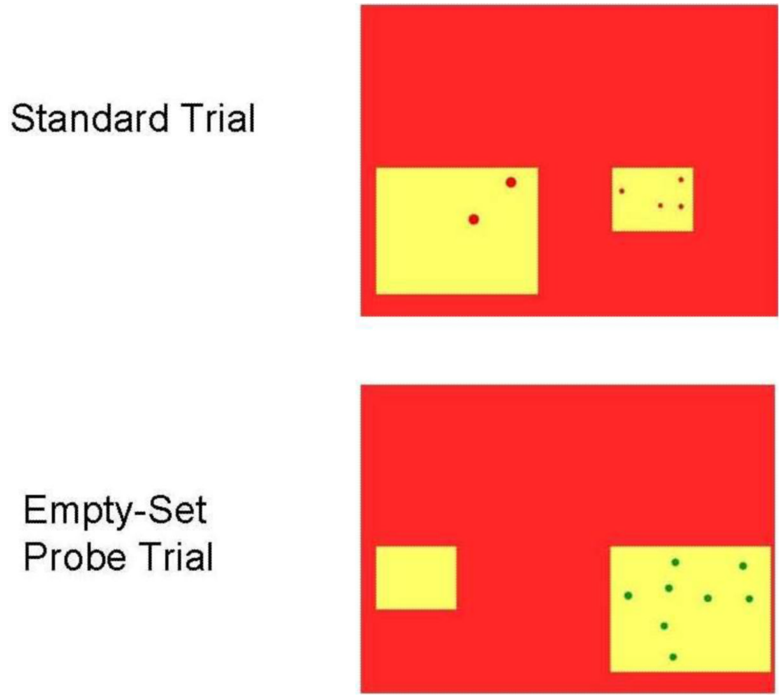

Figure 1.
Example screen shots for training and probe trials. The task was to select the smaller numerosity first and the larger numerosity second.

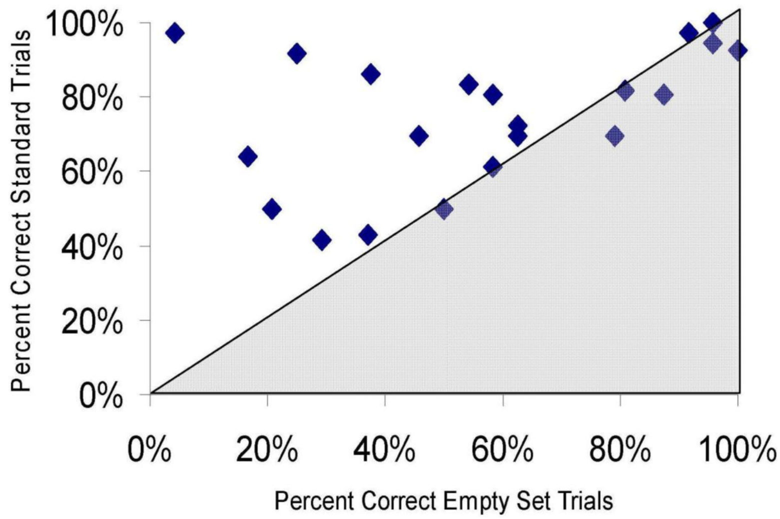

Figure 2.

A comparison of ordering performance on empty set and non-empty set trials. Points in the shaded area indicate greater accuracy on empty sets than on non-empty sets. Points in the unshaded area indicate greater accuracy on non-empty set than on empty set trials.

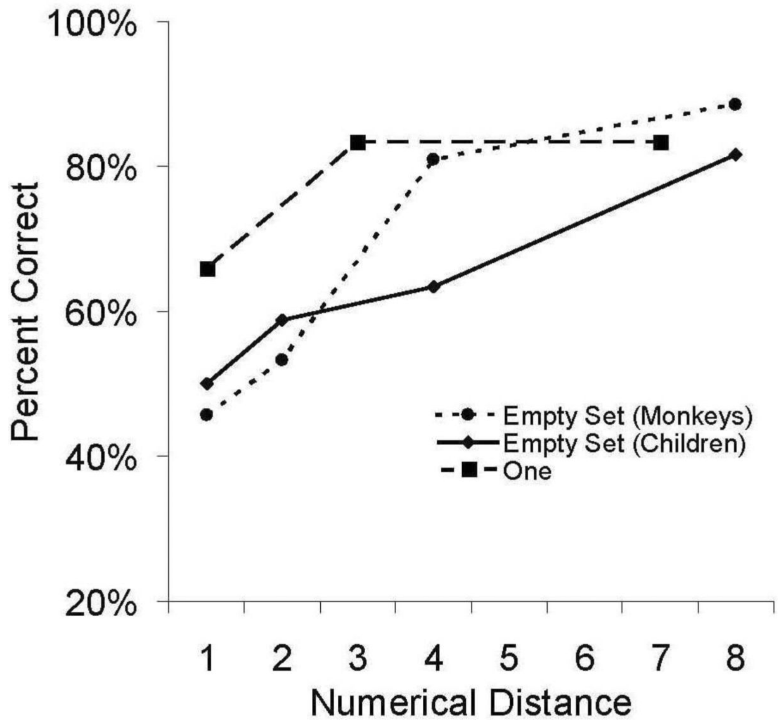

Figure 3.
Accuracy for empty sets and sets containing one item as a function of distance. Also included for comparison purposes, is the empty set accuracy across distances of 1, 2, 4, and 8 previously obtained from rhesus monkeys. The monkey data is from Merritt et al. (2009).

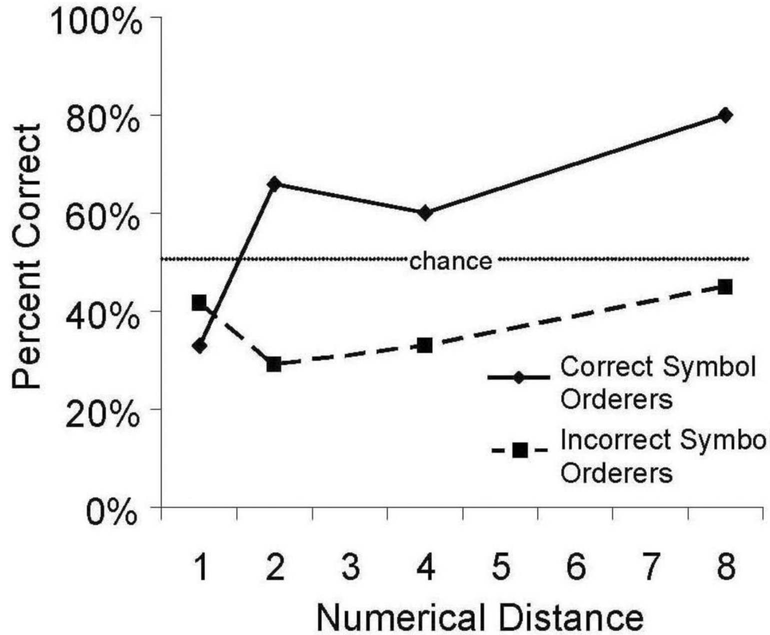

Figure 4.
Empty set accuracy for children who failed to correctly order the zero symbolic number pairs as a function of whether they correctly or incorrectly ordered symbolic number pairs containing positive symbolic (non-zero) numbers.

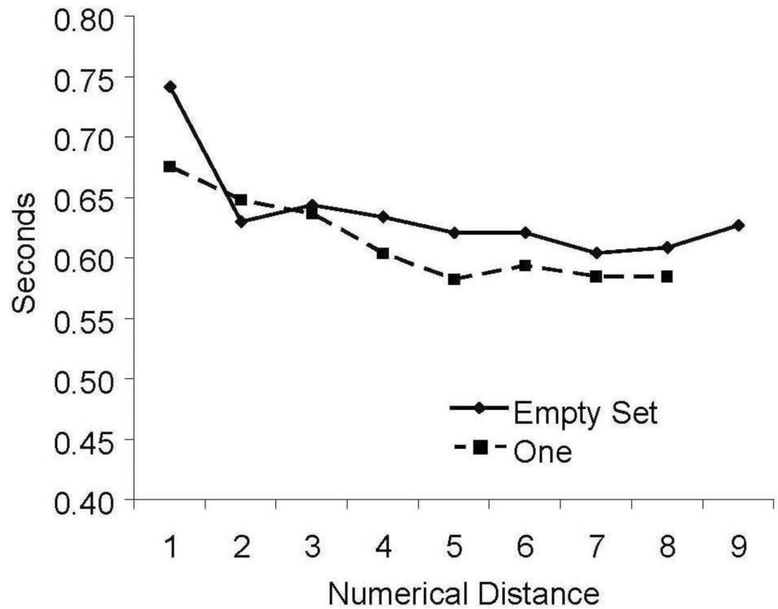

Figure 5.
Reaction time as a function of distance for adult participants.

NIH-PA Author Manuscript
NIH-PA Author Manuscript