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A common representational system governed by Weber's law: Nonverbal numerical similarity judgments in 6-year-olds and rhesus macaques

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Abstract

This study compared nonverbal numerical processing in 6-year-olds with that in nonhuman animals using a numerical bisection task. In the study, 16 children were trained on a delayed match-to-sample paradigm to match exemplars of two anchor numerosities. Children were then required to indicate whether a sample intermediate to the anchor values was closer to the small anchor value or the large anchor value. For two sets of anchor values with the same ratio, the probability of choosing the larger anchor value increased systematically with sample number, and the psychometric functions superimposed when plotted on a logarithmic scale. The psychometric functions produced by the children also superimposed with the psychometric functions produced by rhesus monkeys in an analogous previous experiment. These examples of superimposition demonstrate that nonverbal number representations, even in children who have acquired the verbal counting system, are modulated by Weber's law.

Keywords: Numerical competence; Cognitive development; Bisection; Weber's law; Comparative psychology; Nonverbal cognition

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Introduction

Adult humans are thought to represent number using at least two distinct systems: one system that depends on language and one or more systems that are independent of language. Culturally specific, linguistically mediated counting systems allow adults to verbally represent number linearly with precision. For example, using language, adults can easily differentiate 1001 from 1002 and appreciate that the difference between these two values is equivalent to the difference between 8 and 9. Another way in which number can be represented, however, is by mental magnitudes that are proportional to number. An analog magnitude system for representing number has been proposed to mediate many of the nonverbal numerical abilities of adult humans (Barth et al., 2006; Boisvert, Abroms, & Roberts, 2003; Cantlon & Brannon, 2005; Moyer & Landauer, 1967; Pica, Lemer, Izard, & Dehaene, 2004; Whalen, Gallistel, & Gelman, 1999). This system of representing number obeys Weber's law, which states that the size of a just noticeable difference in stimulus intensity is a constant proportion of the original stimulus magnitude. For example, if a person required an increase or a decrease of 4 pounds to detect a change in a 20-pound sack of flour, Weber's law would predict that this person would require an 8-pound increment or decrement in a 40-pound sack of flour to detect a change. Thus, under an analog magnitude system for representing number, discrimination between two quantities depends on their ratio rather than on their linear distance.

A great deal of evidence suggests that the analog magnitude system for representing number is shared by adult humans, developing humans, and nonhuman animals (Beran, 2004; Beran & Beran, 2004; Beran & Rumbaugh, 2001; Brannon & Roitman, 2003; Brannon & Terrace, 1998; Cantlon, Safford, & Brannon, 2006b; Feigenson, Dehaene, & Spelke, 2004; Gallistel & Gelman, 1992; Hauser, Tsao, Garcia, & Spelke, 2003; Jordan & Brannon, 2006; Lipton & Spelke, 2004; Nieder, Freedman, & Miller, 2002; Xu, Spelke, & Goddard, 2005). A particularly striking example of the developmental and evolutionary continuity found with respect to this analog magnitude representational system is the body of data showing that the accuracy and/or speed with which human adults, children, and nonhuman primates compare numerical magnitudes are modulated by ratio (Beran, 2001, 2004; Beran, Beran, Harris, & Washburn, 2005; Brannon & Terrace, 1998, 2000; Dehaene, Dupoux, & Mehler, 1990; Judge, Evans, & Vyas, 2005; Moyer & Landauer, 1967; Nieder & Miller, 2004; Rumbaugh, Savage-Rumbaugh, & Hegel, 1987; Sekuler & Mierkiewicz, 1977; Smith, Piel, & Candland, 2003; Temple & Posner, 1998; Washburn & Rumbaugh, 1991). For example, even during the first year of life, infants possess a ratio-dependent nonverbal system for representing number; by 6 months of age, infants discriminate large arrays with a 1:2 ratio but fail to discriminate sets with a 2:3 ratio (Lipton & Spelke, 2003; Xu & Spelke, 2000). In a separate study, Huntley-Fenner and Cannon (2000) found that 3- to 5-year-olds could also more easily compare the numerosity of arrays occurring in a 1:2 ratio than in a 2:3 ratio. Furthermore, Barth, La Mont, Lipton, and Spelke (2005) found that for arrays of large numerosities, both within and across sensory modalities, preschool children's nonverbal numerical comparisons—including the addition of two arrays to compare with a third array—were modulated by ratio. Similarly, Temple and Posner (1998) tested 5-year-olds and adults on a numerical comparison task in which participants rapidly judged whether the numerosities 1, 4, 6, and 9—presented as Arabic numerals on some trials and presented as arrays of dots on other trials—were greater than or less than 5. Results indicated that the distance between 5 and the target number modulated accuracy and reaction time in both age groups and for both types of stimulus presentation format. A further illustration is a study by Huntley-Fenner (2001) in which 5- to 7-year-olds were asked to rapidly estimate the number of items in an array by pointing to the corresponding Arabic numeral on a number line. In the nonverbal number representations produced by these children, researchers found direct evidence of scalar variability, whereby variability in responses increased proportionally with magnitude.

Few studies have directly compared the performance of children with that of nonhuman animals. However, Cantlon and colleagues (Cantlon & Brannon, 2006a; Cantlon, Fink, & Brannon, 2006a) demonstrated that the ordinal numerical judgments made by rhesus monkeys and preschool children were similarly affected by the heterogeneity (in color, shape, and size) of items within to-be-compared sets. Specifically, performance on ordering pairs of numerosities between 1 and 9 was not affected by degree of stimulus heterogeneity in either species. In a second pair of studies, Cantlon et al. (2006b) and Cantlon and Brannon (2006b) demonstrated that children and monkeys spontaneously match arrays based on number rather than on surface area.

Here we used a numerical bisection task to test the nature of nonverbal number representations in 6-year-olds and to compare their numerical representations with those of rhesus monkeys. In the bisection paradigm, participants are trained to discriminate between a small anchor value and a large anchor value along some dimension (e.g., time, number) and are then tested with intermediate values. The intermediate value probe trials require participants to judge whether the probe is relatively more similar to the small anchor value or the large anchor value. If participants have an underlying systematic representation of the dimension tested, the probability that they choose the large anchor value (the "choose large" response) typically is found to increase with the probe value. Bisection experiments have demonstrated that Weber's law governs numerical and temporal similarity judgments in a wide range of species. For example, in rats, pigeons, and adult humans, the psychometric functions relating the probability of a "choose large" response to the probe value have been found to superimpose when plotted on a logarithmic scale for different sets of anchor values with the same ratio (e.g., Allan & Gibbon, 1991; Fetterman, 1993; Roberts, 2005).

A second aspect of results from bisection tasks is that the test value for which participants are equally likely to choose the small or large anchor value, the point of subjective equality (PSE), often is found to be at the geometric mean[1] of the anchor values (Allan & Gibbon, 1991; Church & Deluty, 1977; Fetterman, 1993; Jordan & Brannon, 2006; Meck & Church, 1983; Meck, Church, & Gibbon, 1985; Platt & Davis, 1983; Roberts, 2005; Roitman, Brannon, Andrews, & Platt, 2006; Stubbs, 1976). A bisection point at the geometric mean is consistent with two distinct hypotheses concerning the format of nonverbal numerical representations. Under one hypothesis, number (and time) is represented linearly with scalar variance (Brannon, Wusthoff, Gallistel, & Gibbon, 2001; Clement & Droit-Volet, 2006; Droit-Volet, 2002, 2003; Droit-Volet, Clement, & Wearden, 2001; Gibbon, 1977, 1981, 1986; Meck & Church, 1983; Wearden, 1994). Under a second hypothesis, number is scaled logarithmically; for example, the psychological distance between 4 and 6 is smaller than the psychological distance between 2 and 4 (Dehaene & Changeux, 1993; Dehaene, 1997; Nieder & Miller, 2003). In contrast, a PSE at the arithmetic mean implies a

[1] The geometric mean of two numbers is defined as the square root of the product of those two numbers.

linear representation of number or time (for examples of PSEs at the arithmetic mean during temporal bisection in humans, see Droit-Volet & Wearden, 2001; Rattat & Droit-Volet, 2001; Wearden, 1991; Wearden, Rogers, & Thomas, 1997).

Thus, if nonverbal number representations are governed by Weber's law, the PSE should be at the geometric mean in bisection tasks. The only previous study of numerical bisection in children, however, found that the PSE was closer to the arithmetic mean (Droit-Volet, Clement, & Fayol, 2003). This numerical bisection task required 5- and 8-year-olds and adults to process the number of stimuli in a sequence while ignoring the duration of the sequence. The experiment included both counting and noncounting conditions. Predictably, in the counting condition, the PSE was found to be closer to the arithmetic mean than to the geometric mean of the anchor numerosities 2 and 8, implicating a traditional linear verbal representation of number. However, in the noncounting condition, the PSE was also found to be closer to the arithmetic mean. This finding does not conform to the predictions of Weber's law and is surprising given previous studies suggesting that Weber's law holds over nonverbal representations of number in children, adults, and nonhuman animals (Barth, Kanwisher, & Spelke, 2003; Barth et al., 2006; Beran & Beran, 2004; Beran & Rumbaugh, 2001; Boisvert et al., 2003; Brannon & Roitman, 2003; Cantlon & Brannon, 2005; Feigenson et al., 2004; Gallistel & Gelman, 1992; Nieder et al., 2002; Pica et al., 2004; Whalen et al., 1999).

In light of the divergent results obtained by Droit-Volet and colleagues (2003), further investigation is needed to resolve the issue of whether nonverbal numerical matching in children conforms to the predictions of Weber's law. In the experiment reported here, we used a delayed match-to-sample paradigm with a bisection task and show that Weber's law governs nonverbal numerical similarity judgments in 6-year-olds. We first trained children to match two anchor numerosities on a touch-sensitive computer monitor. We then used the bisection procedure to examine whether the probability that children would make a "choose large" response increased with the numerosity of probe sample values and to determine whether children's PSE was closer to the geometric mean or the arithmetic mean of the anchor values. We further tested the predictions of Weber's law by asking whether the psychometric functions from the bisection task superimpose on a logarithmic scale when the anchor numerosities are varied in absolute value but maintain a constant ratio. The final question our research addressed was whether the nonverbal numerical representations of children are similar to those of rhesus monkeys. We tested this by comparing children's performance in the bisection task with monkeys' performance in the same task (Jordan & Brannon, 2006). Superimposition of the psychometric functions for children and monkeys in this bisection task would provide evidence of a shared system for representing number nonverbally in human children and nonhuman animals.

Method

In the experiment, 6-year-olds were first trained to match novel exemplars of two sets of anchor numerosities (2 vs. 8 and 3 vs. 12) in a DMTS task on a touch-sensitive monitor. Children were encouraged to match as quickly as they could while still maintaining accuracy. After children reached a performance criterion when matching the anchor numerosities, they were tested in the bisection task with intermediate sample values and a forced choice between the two trained anchor values.

Participants

Participants were 16 children (6 girls and 10 boys, mean age = 5.62 years, range = 5.00–6.98). No children were excluded from the final sample. Informed consent from a parent of each participant was obtained before starting the experiment. The order of the two conditions (2 vs. 8 and 3 vs. 12) was counterbalanced across participants.

Apparatus

Each child was seated at a desk in front of a touch-sensitive monitor. A Dell OptiPlex GX400 computer executing a software program written in-house in Java was used to create, run, and control the sessions and to register responses. The experimenter was seated next to the child during the experiment. The parent remained in the room but was asked not to talk to the child.

Stimuli

Stimuli were yellow 10 × 10-cm rectangles that contained a variable number of elements. The elements within each stimulus were circles and varied in diameter (0.25–2.50 cm) and color (green, red, blue, orange, purple, or black) between stimuli. Elements within a stimulus were uniform in size and color. For each numerosity, stimuli were drawn from a pool of eight different exemplars. Sample stimuli always differed in color, element size, cumulative surface area, perimeter, and orientation of elements from the two choice stimuli; within trials, the two choice stimuli always were uniform in element size and color. On approximately 50% of trials, the cumulative surface area and perimeter of elements in the sample were closer to the cumulative surface area and perimeter of elements in the larger anchor numerosity choice stimulus than to the cumulative surface area and perimeter of elements in the smaller anchor numerosity choice stimulus; on the other 50% of trials, the cumulative surface area and perimeter of elements in the sample were closer to the cumulative surface area and perimeter of elements in the smaller anchor numerosity choice stimulus. On approximately 50% of trials, the size of individual elements in the sample was larger than that used in the choice values; on the other 50% of trials, the size of individual elements in the sample was smaller than that used in the choice values. Examples of stimuli in the training and bisection conditions are shown in Fig. 1.

Procedure

Task

To reduce variability in the reaction times to the sample, each child was required to initiate each trial by pressing a picture of a giraffe in the lower center portion of the screen. Following trial initiation, a sample appeared. A touch to the sample resulted in its disappearance and the presentation of two choice stimuli: one stimulus containing the same number of elements as the sample (match) and one stimulus containing a different number of elements (nonmatch). The choice stimuli appeared randomly in two of eight possible screen locations. After each correct response, the child was given a sticker and both computer-generated visual feedback (700-ms green border around the correct choice) and

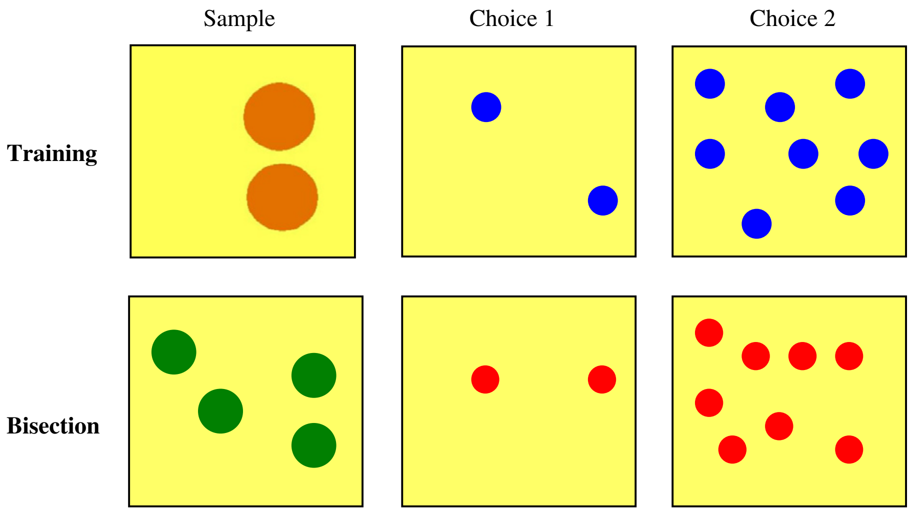

Fig. 1. Examples of stimuli used in the training and bisection conditions of the delayed match-to-sample tasks. The sample is depicted in the first column, and the two choice stimuli are depicted in the second and third columns.

auditory feedback (brief pair of ascending tones). After an incorrect response, no sticker was provided, the computer screen turned black (2000 ms), and negative auditory feedback occurred (brief pair of descending tones).

Experimenter presentation and pretraining

The experimenter first announced that she had two number-matching computer games for the child to play and that she would like to show the child how the games work. The experimenter explained, “You have to choose the picture that has the same number of things in it as the picture that just disappeared.” The experimenter then demonstrated two correct trials and one incorrect trial. The participant was then encouraged to try two trials on his or her own.

Training

Training continued until each child reached a performance criterion of 80% correct and a minimum of eight training trials had been completed. The experimenter reminded the child at least once during training to make choices rapidly to prevent him or her from verbally counting.

Bisection test

After children had learned to numerically match the training values, they were tested in a bisection procedure in which the sample was any value from 2 to 8 or from 3 to 12 but the choice stimuli were always the two training values (2 and 8 or 3 and 12). Here 70% of trials were reinforced (positive or negative) and involved only the sample numbers 2 and 8 or 3 and 12. The remaining 30% of trials (probe trials) involved only sample values intermediate to these trained anchor values: 3 to 7 or 4 to 11. Children were tested with three probe trials at each of these intermediate numerosities; therefore, they completed 50 trials in the 2-to-8 condition and 72 trials in the 3-to-12 condition. Children were told to "play the game the same way you played before." They received a sticker for each probe trial completed, regardless of their response, so that these trials were nondifferentially reinforced. If children commented that there was no exact numerical match available, the experimenter answered, "Just make your best guess! I'm sure you'll do a good job, you've been doing so well!"

Posttests

After each participant completed the games with the two sets of anchor values, the experimenter asked him or her the following debriefing questions to determine the child's proficiency with the verbal counting system:

1. *Verbal Count:* "Can you show me how you count? How many stickers are here?" The experimenter showed the participant six stickers.
2. *Give a Number:* "Here's a cup full of stickers. Can you give me six stickers?" If the participant gave the experimenter the wrong number of stickers, the experimenter asked, "Let's check that. Can you count them?" If the participant counted to a number other than six, the experimenter asked, "Can you fix it?"

Results

On average, children required 8.5 trials (range = 8–13) to reach the 80% accuracy criterion on matching the anchor values 2 and 8, and they required 8.4 trials (range = 8–10) to reach the 80% accuracy criterion on matching the anchor values 3 and 12. Fig. 2 shows that when novel intermediate numerical values were presented as samples, the probability of

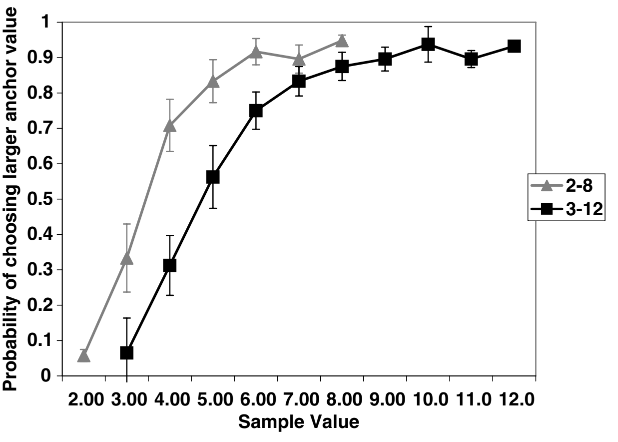

Fig. 2. Probability of choosing the larger anchor value as a function of the numerical value of the sample. Data reflect averages over 16 children in two bisection conditions with different trained anchor values. As sample number increased, children chose the larger anchor value as the match. The probability of choosing the larger anchor value by chance was 50%.

choosing 8 or 12 increased with the numerosity of the sample: second-order polynomial regression, 2-to-8 condition, $R^2 = .98$, $p < .00031$; 3-to-12 condition, $R^2 = .98$, $p < .000001$. Each data set was fit with a cumulative normal distribution to determine the PSE, which was determined by the mean of the fit. A fit with a PSE at 3.53 ± 0.15 accounted for 96.4% of the variance in the 2-to-8 condition, and a fit with a PSE at 4.96 ± 0.20 accounted for 94.5% of the variance in the 3-to-12 condition. The Weber fraction was calculated by finding the ratio of the difference limen (half of the difference between the numerosity corresponding to 75% choices of the larger value and the numerosity corresponding to 25% choices of the larger value) to the PSE. The Weber fraction was .24 for the 2-to-8 condition and .26 for the 3-to-12 condition.

Fig. 3 shows the probability that the children chose the larger of the two anchor values as a function of the sample number, represented as a proportion of the PSE. This is analogous to the method commonly used to determine superimposition on a logarithmic scale of psychophysical functions in temporal bisection tasks, in which the probability of a "choose large" response is plotted against sample duration as a proportion of the PSE (e.g., Allan & Gibbon, 1991; Droit-Volet & Wearden, 2001). The psychophysical curves for the two conditions do not differ significantly: comparing individuals' PSE/geometric mean, $t(15) = -0.420$, $p = .680$; comparing individuals' Weber fractions, $t(15) = -0.164$, $p = .872$. The superimposition of the functions for two sets of absolute values with the same ratio confirms the prediction made by Weber's law.

With regard to the Verbal Count task, 15 of the 16 children correctly counted six objects on their first try, and the 1 child who failed to do so corrected herself spontaneously on a recount. For the Give a Number task, in which the experimenter asked children to give her six stickers, 15 children counted and gave six stickers on their first try, and 1 child (different from the child who at first counted incorrectly in the Verbal Count task) initially gave the experimenter seven stickers but corrected herself when the experimenter asked her to check her count. Thus, the posttest data suggest that all children had mastered the basics of the verbal counting system.

Children were, however, encouraged to answer rapidly and to avoid verbal counting during the bisection task. If children were using a verbal counting strategy during the

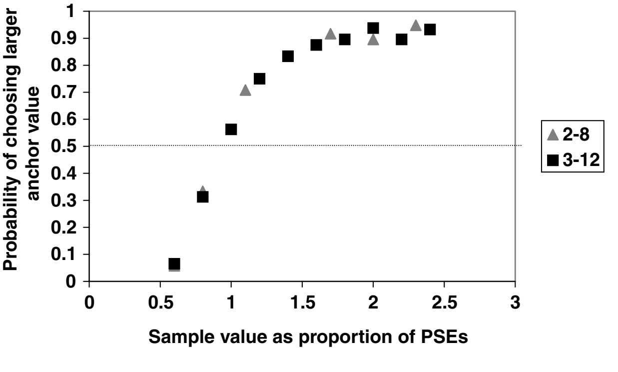

Fig. 3. Probability of choosing the larger anchor value as a function of the sample number (normalized as a proportion of PSEs). The plot reflects superimposition of data from the 2-to-8 and 3-to-12 scales. Specifically, gray triangles represent data from the 2-to-8 scale, and black squares represent data from the 3-to-12 scale.

bisection task, enumeration time should have increased with number. Fig. 4 demonstrates that children's reaction times to touch the sample did not vary as a function of sample number: linear regression, 2-to-8 condition, $R^2 = .0723$, $p = .560$; 3-to-12 condition, $R^2 = .00460$, $p = .852$. These data suggest that children were using a nonverbal enumeration strategy.

A final analysis investigated whether children were relying on a system for representing number shared with nonhuman animals. Fig. 5 shows data from the current experiment alongside data from three rhesus monkeys tested in the same two experimental conditions (Jordan & Brannon, 2006). All three monkeys were first trained to reach a performance criterion of 80% correct over two consecutive sessions. Sessions were approximately 2.5 h, and monkeys completed anywhere from 30 to 1500 trials per session. After reaching the performance criterion, monkeys were tested in the bisection experiment with the 2-to-8 and 3-to-12 conditions. The monkeys were experimentally naive with respect to numerical tasks before beginning the bisection training.

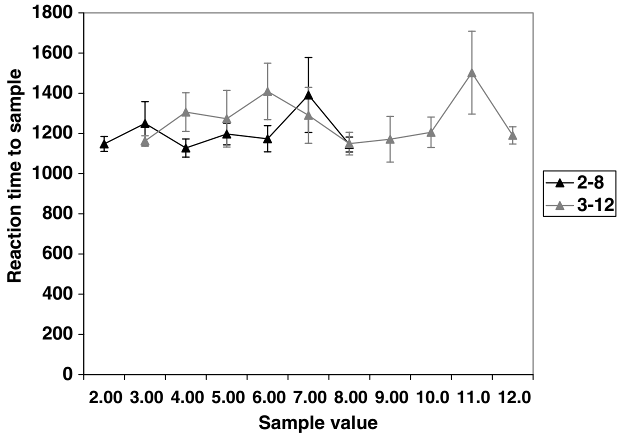

Fig. 4. Latency to touch the sample numerosity in the bisection tasks.

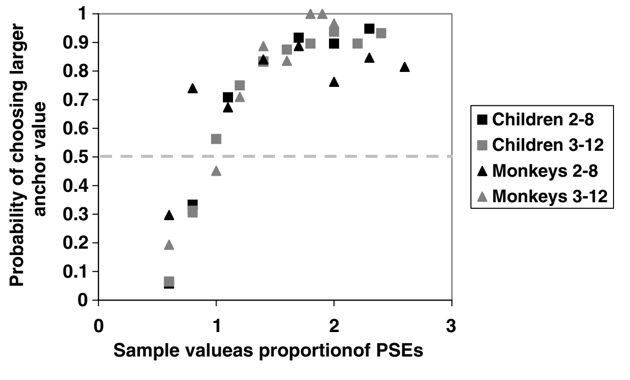

Fig. 5. Probability of choosing the larger anchor value as a function of the sample number (normalized as a proportion of PSEs) for children and rhesus monkeys tested in a numerical bisection task. The plot reflects superimposition of data from 2-to-8 and 3-to-12 scales for each population. Specifically, black squares represent children's data from the 2-to-8 scale, and gray squares represent children's data from the 3-to-12 scale; black triangles represent monkeys' data from the 2-to-8 scale, and gray triangles represent monkeys' data from the 3-to-12 scale. Data from rhesus monkeys were taken from Jordan and Brannon (2006).

As shown in Fig. 5, the psychophysical curves for the two species did not differ significantly for either condition: 2-to-8 condition, $t(6) = 0.677$, $p = .523$; 3-to-12 condition, $t(9) = 0.524$, $p = .616$. The superimposition of these functions when plotted on a logarithmic scale suggests that a common numerical comparison process underlies nonverbal numerical discriminations in 6-year-olds and rhesus monkeys.

Discussion

Three main findings emerged from this experiment. First, as predicted by Weber's law, the PSE was closer to the geometric mean than to the arithmetic mean of the anchor values. Second, when distinct anchor values with the same ratio were used, the psychometric functions superimposed, providing further evidence for Weber's law. Finally, the nonverbal numerical representations of children superimposed with those of rhesus monkeys tested previously on the same task.

No evidence that children used serial verbal counting

Although our sample of children had mastered the basics of the verbal counting system as evidenced by their responses in the posttest, their reaction times to touch the sample suggested that they did not verbally count in our task. Counting is a serial process that adheres to three critical principles: the one-to-one principle, the stable order principle, and the cardinal principle (Gelman & Gallistel, 1978). The one-to-one principle states that there is a one-to-one correspondence between the number of labels applied in a set and the number of to-be-counted elements. The stable order principle states that these labels are applied in a stable order across counting episodes. The cardinal principle states that the last label applied represents the cardinal value of the set. Subitizing, the rapid parallel apprehension of numerical value, has been proposed as a mechanism used by adult humans to determine the value of small numerosities (Kaufman, Lord, Reese, & Volkmann, 1949; Mandler & Shebo, 1982; Trick & Pylyshyn, 1994). Although values in our experiment exceeded the values thought to be within the subitizing range, other parallel processes have been proposed for enumeration of large values (Barth et al., 2003; Dehaene & Changeux, 1993).

Analyses of reaction time to respond to the sample numerosity provided no evidence that children serially enumerated elements when asked to make numerical similarity judgments as quickly as they could. Specifically, reaction time did not increase with sample numerosity, as would be expected if children required more time to verbally enumerate larger values (Fig. 4). This lack of variation in reaction time to sample across numerosities is similar to that found previously in rhesus monkeys tested on this task (Jordan & Brannon, 2006).[2] The pattern of results is also similar to findings by Barth and colleagues (2003) that the reaction times of adult humans comparing numerosities nonverbally are not modulated by absolute set size; that is, participants do not require more time to compare large sets than they require to compare small sets, suggesting that a parallel enumeration mechanism may also underlie these numerical judgments. Similar to the numerical stimuli

[2] Children were approximately three times slower to touch the sample than were rhesus monkeys. This is similar to data from an experiment by Temple and Posner (1998) in which 5-year-olds comparing numerosities were approximately three times slower than adults in their reaction times to make a key press.

presented previously to rhesus monkeys, the stimuli in the current experiment were designed to ensure that number, and not other continuous variables such as individual or cumulative surface area or distance between elements, was a consistently valid cue to matching in this task. Thus, our data suggest that children used a nonverbal parallel process to form numerical representations in this bisection task.

Evidence for a system shared with nonhuman animals and governed by Weber's law

The points of indifference or PSEs in the two bisection conditions were closer to the geometric means than to the arithmetic means of the anchor values. These results suggest that children represented number linearly with scalar variance or logarithmically; the current experiment cannot differentiate between these two possibilities because representing number linearly with a ratio comparison rule and representing number logarithmically both predict ratio dependence and that the PSE should fall at the geometric mean. Many studies with rats, pigeons, monkeys, and adult humans also show that the PSE in nonverbal numerical and temporal bisection tasks is at the geometric mean (Allan & Gibbon, 1991; Church & Deluty, 1977; Fetterman, 1993; Jordan & Brannon, 2006; Meck & Church, 1983; Meck et al., 1985; Platt & Davis, 1983; Roberts, 2005; Stubbs, 1976). However, a previous study testing 5- and 8-year-olds and adults on a nonverbal numerical bisection task in which time and number covaried found that the PSE was closer to the arithmetic mean in all age groups (Droit-Volet et al., 2003). These divergent results may have been due to particular aspects of the task used by Droit-Volet and colleagues (2003) that may have encouraged a serial enumeration strategy. Most important, in their study, the to-be-enumerated stimuli were presented sequentially and so could not have been enumerated by a parallel process. In particular, the stimuli used by Droit-Volet and colleagues were presented for long durations (e.g., the anchor values 2 and 8 had durations of 2 and 8 s, respectively); thus, they may have engendered verbal counting, although the researchers tried to prevent this by requiring participants to engage in the verbal distracter task of producing repetitive speech (i.e., "blabla") as fast as possible. Assuming that participants were representing number nonverbally instead of counting in the bisection experiment conducted by Droit-Volet and colleagues, however, it is clear that participants would have needed to enumerate the stimuli using a serial process because the stimuli were presented sequentially. In contrast, our data suggest that children in the current study used a parallel enumeration strategy. It is possible that this serial nonverbal enumeration process used by participants in Droit-Volet and colleagues' study elicited a more linear format of numerical representation. Finally, the Weber fractions produced by children in our data (.24 and .26) were slightly smaller than the Weber fractions produced by children in the nonverbal numerical conditions of the study conducted by Droit-Volet and colleagues (.32 and .28); thus, the nature of our task may also have caused increased sensitivity to number.

Further investigation is still needed to determine definitively the factors that contribute to the different pattern of results obtained in our study compared with that obtained in Droit-Volet and colleagues' (2003) study. Numerical experience and age previously have been found to influence the type of representational system used (Booth & Siegler, 2006; Siegler & Booth, 2004; Siegler & Opfer, 2003). For example, Siegler and Booth (2004) observed a shift from a logarithmic representation of number to a linear representation of number between 6 and 8 years of age when testing children on this type of task using the range of 0 to 100, and there was also an intermediate period during which a mixture of logarithmic and linear representations was used. Thus, in this type of symbolic numerical estimation task, children possess and use multiple types of numerical representations, but they increasingly rely on a more accurate linear representation with greater age and formal numerical experience with the range of values tested.

A final difference between the current study and the study by Droit-Volet and colleagues (2003) is that we used multiple ranges of numerosities, whereas Droit-Volet and colleagues used only one set of anchor values. Thus, our study provides the first evidence that psychometric functions produced by children in a nonverbal numerical bisection task superimpose when anchor values are varied in absolute numerosity but maintain a constant ratio. Taken together with our finding that the PSE is closer to the geometric mean than to the arithmetic mean of anchor values in both sets of absolute numerosities, and that the psychometric functions of children and monkeys superimpose, these data argue for the control of nonverbal numerical similarity judgments by an analog magnitude representational system adhering to Weber's law.

In sum, this experiment adds to the converging evidence that analog magnitude representations of number in humans and nonhuman animals are governed by Weber's law. Some research is under way, but much research is still necessary to determine the nature of the interaction between the different coexisting systems of numerical representations that children possess and use. For example, how does experimental context influence which numerical representation systems children use spontaneously (e.g., Siegler & Opfer, 2003)? How exactly do children's verbal representations of number map onto their preexisting nonverbal representations (e.g., Lipton & Spelke, 2005)? Can we apply our understanding of children's nonverbal numerical abilities to better help them understand formal, linguistically based mathematics (e.g., Barth et al., 2005)? Solving such problems should provide crucial knowledge about the development of numerical cognition.

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