Rosca Meets Formal Credit Market*

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Abstract

Rotating Savings and Credit Association (Rosca) is an important informal financial institution in many parts of the world used by participants to share income risks. What is the role of Rosca when formal credit market is introduced? We develop a model in which risk-averse participants attempt to hedge against their private income shocks with access to both Rosca and the formal credit and investigate their interactions. Using the gap of the borrowing and saving interest rates as a measure of the imperfectness of the credit market, we compare three cases: (i) Rosca without credit market; (ii) Rosca with a perfect credit market; (iii) Rosca with an imperfect credit market. We show that a perfect credit market completely crowds out the role of Rosca. However, when credit market is present but imperfect, we show that Rosca and the formal credit market can complement each other in improving social welfare. Interestingly, we find that the social welfare in an environment with both Rosca and formal credit market does not necessarily increase monotonically as the imperfectness of the credit market converges to zero.

Keywords: Roscas, Informal Credit Institutions, Insurance, Private Information

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1 Introduction

Rotating savings and credit associations (Roscas) are important informal financial institutions around the world. Most common in developing countries, they are also popular among immigrant groups in the United States.[1] All Roscas share the following common feature: a group of individuals, most likely closely connected in some social networks, commit to contribute a fixed sum of money into a "pot" in each of the equally-spaced periods during the life of the Rosca; in each period, the "pot" is then allocated to one of its members through some mechanism. The mechanism through which the "pot" is allocated is one of the key dimensions of the many variations of how Roscas operate around the world, as documented by the classic anthropological studies of Roscas by Geertz (1962) and Ardener (1964).

Two main varieties are random Rosca and bidding Rosca.[2] In a random Rosca, the pot in each period is allocated to one of the members determined by the draw of lots, with earlier winners excluded from latter draws until each member of the Rosca has received the pot once. In a bidding Rosca, auctions are used to determine the priority of accessing the pot. In each period, the individual who bids more than the competitors in the form of a pledge of higher contributions to the Rosca is chosen to access the pot. As in the random Rosca, the earlier winners are excluded from latter auctions. In a seminal paper, Besley, Coate and Loury (1993) provide rigorous comparisons of random and bidding Roscas in an environment where individuals save for the purchase of indivisible durable goods where the agents cannot save or borrow by their own. Besley, Coate and Loury (1994) further compare the efficiency properties of allocations achieved by random and bidding Roscas and those achieved with a credit market, and with efficient allocations more generally. They find that neither form of Rosca is efficient.[3]

So far almost all existing studies on Rosca have assumed that Rosca participants do *not* have access to *formal credit markets*. By formal credit markets, we mean that an individual can borrow and save at interest rates that are independent of the realizations of her income and the income of the fellow members in their social network. If borrowing and saving interest rates are identical, we say that the credit market is "*perfect*;"[4] and if the borrowing interest rate is higher than the saving interest rate, we say that the financial market is "*imperfect;*" moreover, we use the borrowing-saving

[1]Roscas are called by many different names around the world. They are called *Hui* in China, *chit funds* in India, *Susu* in Ghana, *tontines* in Senegal, *cheetu* in Sri Lanka, to name a few examples. Light (1972) and Bonnett (1981) respectively documented the use of Roscas among Korean and West India immigrants in the United States. See also Adams and Canavesi de Sahonero (1989), Aleem (1990), Kimuyu (1999) for the use of Roscas in Bolivia, Pakistan and East Africa respectively.

[2]Other forms of allocation mechanisms are described in Ardener (1964), but random and bidding Roscas are the two most common mechanisms.

[3]See also Kovsted and Lyk-Jesen (1999) who study Rosca as a form of savings for lumpy investment.

[4]The notion of "perfect credit market" used in our paper differs from the concept of "complete market" which means that there are Arrow-Debru securities for every contingency, and accordingly interest rates will differ by the state of nature.

interest rate gap as a measure of the "imperfectness" of the credit market: the larger the gap, the more imperfect is the credit market.

Indeed, it is a somewhat accepted wisdom in the literature that Roscas, as informal financial institutions, exist because of the lack of formal credit markets; moreover, it is expected that the development of formal credit markets will lead to the eventual disappearance of Roscas. While it is undoubtedly true that Roscas are more commonly observed in developing countries with underdeveloped formal credit markets, theoretically it remains unclear whether the economic role of Roscas will vanish when individuals can access the formal credit market. In reality, even in some developing countries, Rosca participants are not completely isolated from the formal credit market and they may have some, albeit limited, access to the formal credit market. For example, Zhang (2001), based on surveys of active Rosca participants in Southeastern China, found that more than 35 percent of Rosca participants who experienced adverse incomes shocks and were thus unable to pay their pledged contribution to the Roscas borrowed from formal credit markets to fulfil their Rosca obligations; moreover, 40 percent of Rosca participants in those areas reported lending some of their Rosca winnings to the formal credit market. In other words, for Rosca participants in those areas there are active interactions between Roscas and formal credit markets.

These observations raise some important questions. How can we rationalize active Rosca operations in the presence of a formal credit market? What is the economic role of Roscas when individuals can both lend and borrow in a formal credit market? Does the importance of Rosca decrease monotonically with the "perfectness" of the formal credit market? Does social welfare increase monotonically with the "perfectness" of the formal credit market, given the presence of Rosca?

In this paper, we provide a comprehensive investigation on the *interactions* between the Rosca and the formal credit market. We identify two competing forces in such interactions. On the one hand, in the absence of formal credit market, although Rosca – with a proper size – is *ex ante* welfare improving, it introduces a new risk induced by the uncertainty in the allocation of the money pot. Since the formal credit market enables Rosca participants to hedge the risks of winning or losing the Rosca, it may improve welfare by reducing the cost of participating in Rosca. In this sense, the access to the formal credit market complements Rosca participation.

On the other hand, having access to the formal credit market increases the value of outside option, making it more difficult to satisfy the incentive compatibility and the participation constraints for Rosca. Indeed, we show that in the extreme case when the credit market is perfect, the role of Rosca disappears. However, if the formal credit market is imperfect, i.e. if there is a borrowing-saving interest rate gap, we show that participating in Rosca leads to an improvement, both in the *ex ante* and *ex post* sense, in participants' welfare.

Interestingly, we find that social welfare does *not* monotonically increase as the imperfectness of the credit market is reduced. Specifically, we provide conditions under which social welfare is higher in an environment with a positive borrowing-saving interest rate gap that sustains Rosca participation than that in an environment with a perfect credit market. Our finding implies that the presence of an *imperfect* credit market may facilitate, rather than substitute, Rosca participation because the Rosca participants may use the credit market to smooth the risk associated with losing or winning the money pot in the Rosca bidding.[5]

The idea that Rosca may serve as a form of insurance for its participants is not new. Calomiris and Rajaraman (1998) make a forceful intuitive argument for the insurance role of Rosca. Klonner (2000, 2003) introduces a model of Rosca as an insurance mechanism where individuals privately observe their future incomes, but his model does not consider the possibility of a formal credit market. He shows that individuals will prefer bidding Roscas to random Roscas if the temporal risk aversion is not greater than the static risk aversion.

There are also several papers that analyze Rosca bidding games with private information. Kuo (1993) analyzes the equilibrium of a bidding Rosca where myopic participants' private information is their discount factors; moreover, he assumes that their discount factors are independently redrawn in every period. Thus in Kuo (1993), Rosca biddings are effectively reduced to a sequence of independent auctions. Kovsted and Lyk-Jenson (1999) study the role of Roscas in financing indivisible investment expenditures where individuals differ in their privately observed rate of returns. They only allow individuals to access the formal credit markets when there is a gap between the money pot won in the Rosca and the necessary lumpy investment. Besley and Levenson (1996a, 1996b) and Handa and Kirton (1999) present empirical evidence for the role of the Rosca in saving for durable good purchases and indivisible investment. Yu (2014) investigates how well different Rosca designs serve to accelerate investment in a context where participants with different entrepreneurial skills use Rosca to fund investment projects. He did not allow the entrepreneurs to access formal credit market. Using both reduced-form and structural methods, Kaboski and Townsend (2012a, 2012b) studied the impact of the introduction of an exogenous, large-scale, microcredit intervention program, the Thai Million Baht Village Fund program on the villagers' short-term credit, consumption, agricultural investment, income and asset growth.

Finally, our theoretical model of Rosca bidding in the presence of credit markets also contributes to the auction literature. Three important features of Rosca bidding differentiate our model from the standard auction model. First, in Rosca bidding there is no pre-assigned "buyers" and "sellers"; rather, the roles of "sellers" (i.e., "lenders") and "buyers" (i.e., "borrowers") are determined endogenously. Second, in standard private value auctions a bidder cares about the bids of others only to the extent that the opponents' bids affect her probability of winning; in Rosca bidding, however, the opponents' bid will affect the money in the pot in subsequent rounds and thus directly affect the bidder's continuation value even if she loses the bidding in the current round. Third, because Rosca participants can access the formal credit market every period, in this model bidders' valuations are endogenously derived from the value function of an optimal consumption/saving

[5]In a different setting, Krueger and Perri (2011) study the interaction between public income insurance (through progressive income taxation) and incomplete private risk sharing. They show that the fundamental friction that limits private risk sharing in the first place is crucial in determining whether the provision of public insurance improves or reduces social welfare.

problem.[6]

The remainder of the paper is structured as follows. In Section 2 we present the basic model and discuss two benchmark cases, one without access to any credit market and another with access to a perfect credit market. In Section 4 we investigate the performance of Rosca with an imperfect formal credit market, i.e., where the borrowing interest rate is higher than the saving interest rate. We show that in this environment, Rosca is welfare improving. In Section 5 we show that *ex ante* welfare is non-monotonic with respect to the imperfectness of the formal credit market. In Section 6, we conclude and provide some additional discussions. All proofs are contained in the Appendix.

2 The Model

In this section, we first describe a two-period model of bidding Rosca in the presence of a formal credit market where individuals can borrow and lend at interest rates r_b and r_s respectively.

Environment. Consider two risk averse agents, indexed by $i = 1, 2$, with time-additive utility function

$$u(c_1^i) + \delta \mathrm{E} u(C_2^i),$$

where $c_t^i, t = 1, 2$, is agent i's period-t consumption, $\delta \in (0, 1)$ is the discount factor.[7] We assume that $u(\cdot)$ is twice differentiable, strictly increasing, strictly concave, and satisfies Inada conditions. The income process for individual i in period t, denoted by Y_t^i, consists of a risk-free component $\bar{y} > 0$, and with probability $1 - p$, a negative income shock X_t^i drawn from CDF $F(\cdot)$. Formally,

$$Y_t^i = \bar{y} - X_t^i,$$

where with probability $p \in (0, 1)$, $X_t^i = 0$; and with probability $1 - p$, $X_t^i \sim F(\cdot)$ on the support $(\underline{x}, \bar{x}]$ with $\underline{x} \geq 0$. We assume that $F(\cdot)$ is associated with a continuous density $f(\cdot)$, and that the income shock realizations are independent across agents and periods.

Credit Market and Rosca. In the absence of Rosca, each individual has access to a credit market where they can borrow at interest rate r_b and lend at interest rate $r_s \leq r_b$.

The two agents may also form a Rosca. If they do, they need to decide its *size* m, which is the amount of money each has to contribute to the pot each period. We assume that the Rosca will allocate the priority of using the pot through a *premium-bidding* auction. More specifically, a premium-bidding auction is organized as follows. Each agent, after the realization of her income in period 1, submits a bid indicating how much premium she is willing to add to the mandatory

[6] The first two features also appear in Klonner (2003), but the third feature is unique in this paper. Hubbard, Paarsch and Wright (2013) also study the equilibrium of a discount bidding Rosca where participants have private investment returns as a sequential first-price, sealed-bid auction within the risk-neutral symmetric independent private-values paradigm. As in all existing literature, they assume agents do not have access to formal credit market.

[7] E denotes expectation, and as a notational convention, all random variables are capitalized.

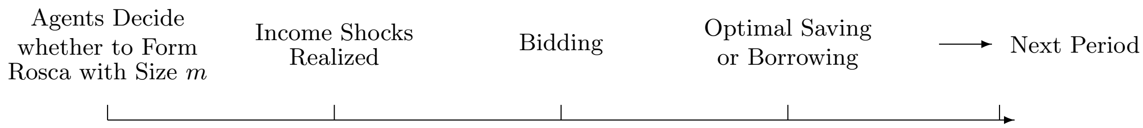

Figure 1: The Time Line of the Model

size m in period 2 if she wins the right to use the pot in period 1. The one who submits the higher premium bid wins the right to use the pot. The winner has to honor her bid in the second period. It is important to emphasize that the credit market is always accessible to the agents even after the formation of a Rosca.

Timing. The timing of the game is depicted in Figure 1. In the beginning of period 1 and before the realizations of period-1 income shocks, the two agents decide whether to form a Rosca, and if so, the size m of the Rosca. Then each agent observes her period-1 income shock, which is her private information. If no Rosca is formed, then the two agents will only use the credit market to insure their income risks; if a Rosca is formed, then the agents can use both the Rosca and the credit market to insure their income risks. In a Rosca, the agents submit their premium bid after realizing their period-1 income shocks, and the winner is able to use the pot money in period 1. Agents make consumption and saving decisions after the Rosca bidding. Then time moves to the second period when the period-2 income shocks are realized. The winner of the period-1 Rosca bidding contributes m plus her winning premium bid to the pot, which is received by the loser of the period-1 bidding in period 2. Both agents then make period-2 consumption decisions.

Cases We Consider in this Paper. In the paper we will discuss three cases of the above model to investigate the interactions between Rosca and formal credit markets based on variations of r_s and r_b. The first case is the benchmark case of *no formal credit markets*, which is captured by $r_s = -1$ and $r_b \to \infty$; i.e., there is no borrowing and saving. Note that to justify the statement that $r_s = -1$ in the case of no formal credit markets, we assume that the "income" is in the form of non-storable perishable goods prior to the introduction of the formal credit market. The second case is another benchmark case where there is *a perfect credit market,* which is represented by $r_b = r_s = r > 0$, i.e., agents can borrow and save at the same interest rates. These two benchmark cases will be analyzed in Section 3. The third case is the general case of imperfect formal credit markets, namely, $r_b > r_s \geq 0$, which we will analyze in Section 4. Table 1 summarizes these cases.

Case I:	No Formal Credit Market	$r_s = -1, r_b \to \infty$
Case II:	Perfect Credit Market	$r_b = r_s = r > 0$
Case III:	Imperfect Credit Market	$r_b > r_s \geq 0$

Table 1: Summary of the Cases Considered in the Paper.

3 Benchmark Cases

3.1 Case I: Rosca without a Formal Credit Market

We first consider a benchmark case of Rosca without access to any formal credit market, where the Rosca allocation is determined through *premium* bidding. That is, at any round the participant who bids to contribute the highest premium payment beyond the agreed-upon size m in latter rounds of the Rosca is allocated the right to access the money pot in this round.

3.1.1 The Bidding Equilibrium

If an individual does not participate in Rosca, then without access to any formal credit market, the consumption decision of a representative agent (say, individual 1), following the realization of the first period's income y_1, is simply, $c_1^* = y_1 = \bar{y} - x_1$.[8] Her utility without Rosca is:[9]

$$V_I^n(y_1) = u(y_1) + \delta \mathrm{E} u(Y_2), \tag{1}$$

where the superscript "n" is mnemonic for "*no* Rosca."

If individual 1 participates in a Rosca with size m, and wins the period-1 pot by bidding b, her expected utility is

$$V_I^w(b; y_1) = u(y_1 + m) + \delta \mathrm{E} u(Y_2 - m - b). \tag{2}$$

If she participates in the Rosca and loses the bidding in the first period, her expected utility given her opponent's bid $\hat{b}$, is:[10]

$$V_I^f(\hat{b}; y_1) = u(y_1 - m) + \delta \mathrm{E} u(Y_2 + m + \hat{b}). \tag{3}$$

We now characterize the equilibrium bidding strategy. Without loss of generality, let us consider bidder 1. Suppose that her opponent follows a bidding strategy $b_I(\cdot)$ that satisfies $b_I'(\cdot) > 0$. Consider the revelation mechanism for bidder 1 with type $x > \underline{x}$. The expected utility for bidder 1 of type-x from reporting $\tilde{x}$ is given by:

$$\begin{aligned} U_I(x, \tilde{x}) \quad = \quad & \left\{\Pr\left(X^j = 0\right) + \left[1 - \Pr(X^j = 0)\right] \Pr\left(\underline{x} < X^j \leq \tilde{x}\right)\right\} V_I^w\left(b_I(\tilde{x}); \bar{y} - x\right) \\ + \quad & \left[1 - \Pr(X^j = 0)\right] \Pr\left(X^j > \tilde{x}\right) \mathrm{E}\left[V_I^f\left(b_I\left(X^j\right); \bar{y} - x\right) \middle| X^j > \tilde{x}\right], \end{aligned} \tag{4}$$

[8] For notational simplicity, we omit the superscript i for agent i whenever doing so would not cause confusion.

[9] We use subscript "I" in value functions, bidding functions etc., to denote this is for case I. Similar subscripts are used for cases II and III below.

[10] The superscripts "w" and "f" are mnemonic for "winning the Rosca bidding" and "failing to win the Rosca bidding" respectivey.

where X^j denotes the period-1 income shock of agent $1'$s opponent (i.e., agent 2). To understand this expression, note that the first term is the expected utility for a type-x bidder 1 from reporting $\tilde{x}$ and win the period 1 bidding. Such mis-reporting will lead the mechanism to submit a bid of $b_I(\tilde{x})$ for her and this bid will win against her opponent with type less than $\tilde{x}$, which occurs with the probability $\Pr\left(X^j = 0\right) + \left[1 - \Pr(X^j = 0)\right]\Pr\left(\underline{x} < X^j \leq \tilde{x}\right)$, and winning with a premium bid of $b_I(\tilde{x})$ will yield a continuation value of $V_I^w(b_I(\tilde{x}); \bar{y} - x)$. The second term can be similarly understood: with probability $\left[1 - \Pr(X^j = 0)\right]\Pr\left(X^j > \tilde{x}\right)$ the opponent's type will be drawn from $(\tilde{x}, \overline{x}]$ and bidder 1 will lose the auction when submitting the bid $b_I(\tilde{x})$. However, bidder 1's continuation value from losing the auction depends on the premium bid submitted by her opponent $b_I\left(X^j\right)$, thus bidder 1's expected continuation value from losing is given by $\mathrm{E}[V_I^f(b_I\left(X^j\right); \bar{y} - x)\Big| X^j > \tilde{x}]$. Using the distribution of X^j, we can rewrite (4) as:

$$U_I(x, \tilde{x}) = [p + (1-p)F(\tilde{x})]\, V_I^w(b_I(\tilde{x}); \bar{y} - x) + (1-p)\int_{\tilde{x}}^{\bar{x}} V_I^f\left(b_I\left(x^j\right); \bar{y} - x\right) dF(x^j). \tag{5}$$

The first-order condition for incentive compatibility (IC) requires that

$$\frac{\partial}{\partial \tilde{x}} U_I(x, \tilde{x})\Big|_{\tilde{x}=x} = 0,$$

which yields

$$b_I'(x) = \frac{(1-p)f(x)}{p + (1-p)F(x)} \frac{V_I^w\left(b_I(x); \bar{y} - x - V_I^f b_I(x); \bar{y} - x\right)}{\delta \mathrm{E} u'\left(Y_2 - m - b_I(x)\right)}. \tag{6}$$

For type-0 bidder, if she reports her type truthfully, her *interim* payoff is given by:

$$\begin{aligned} U_I(0,0) &= \frac{1}{2}\Pr\left(X^j = 0\right)\left[V_I^w(b_I(0); \bar{y}) + V_I^f(b(0); \bar{y})\right] + \left[1 - \Pr(X^j = 0)\right]\mathrm{E}\left[V_I^f(b_I\left(X^j\right); \bar{y})\Big| X^j > \underline{x}\right] \\ &= \frac{1}{2}p\left[V_I^w(b_I(0); \bar{y}) + V_I^f(b_I(0); \bar{y})\right] + (1-p)\int_{\underline{x}}^{\bar{x}} V_I^f\left(b_I(x^j); \bar{y}\right) dF(x^j), \end{aligned} \tag{7}$$

where the first term reflects the probability of tying when her opponent is also of type-0 (occurring with probability $\Pr\left(X^j = 0\right)$) in which case she will obtain $V_I^w(b_I(0); \bar{y})$ and $V_I^f(b_I(0); \bar{y})$ with equal probability; the second term is the expected payoff in the case when her opponent's type is higher than $\underline{x}$. If type-0 bidder deviates to some $\tilde{x} > \underline{x}$, the expression $U_I(0, \tilde{x})$ is analogous to (5).

The following proposition characterizes the equilibrium:

Proposition 1 *Suppose that there exists* $b_0 \geq 0$ *such that*

$$V_I^w(b_0; \bar{y}) \leq V_I^f(b_0; \bar{y}) \tag{8}$$

and

$$V_I^w(b_0; \bar{y} - \underline{x}) \geq V_I^f(b_0; \bar{y} - \underline{x}). \tag{9}$$

Then the solution to differential equation (6) with boundary condition $b_I(0) = \lim_{x \to \underline{x}} b_I(x) = b_0$ *specifies a symmetric Bayesian Nash Equilibrium (BNE) in the Rosca bidding in period 1. Moreover, the equilibrium bidding function* $b_I(\cdot)$ *is non-negative and increasing.*

Condition (8) means that the agent without shocks prefers not to win with bid b_0; while condition (9) means that the agent with the smallest negative shock prefers to win with bid b_0. In particular, when $\underline{x} = 0$, the two conditions (8) and (9) collapse into one, namely, there is a $b_0 \geq 0$ such that:

$$V_I^w (b_0; \bar{y}) = V_I^f (b_0; \bar{y}) . \tag{10}$$

3.1.2 Welfare Evaluations

The *ex ante* welfare gain from participating in a Rosca with size m relative to no Rosca participation, denoted by $\Delta W_I (m)$, is given by:

$$\Delta W_I (m) = p\Delta U_I (0; m) + (1 - p)\mathrm{E} [\Delta U_I(X; m)| X > \underline{x}] , \tag{11}$$

where

$$\Delta U_I (x; m) = U_I (x, x) - V_I^n (\bar{y} - x)$$

is the interim welfare gain of type-x bidder from participating in the Rosca relative to no participation.[11]

Proposition 2 *If there is no access to a credit market and* $0 < \delta < u'(\bar{y})/\, \mathrm{E}u'(Y_2)$*, then*

$$\left.\frac{d\Delta W_I (m)}{dm}\right|_{m=0} > 0.$$

An immediate corollary of Proposition 2 is that a Rosca with an optimally chosen size $m^* > 0$ can improve the participants' *ex ante* welfare. The intuition for the *ex ante* welfare gain from a Rosca is as follows. The potential benefit from Rosca is to smooth the income risk by transferring money m from the higher income state to the lower one. Taking $p = 0$ as an example, the potential benefit from participating in Rosca of size m in the first period is:

$$\int_{\underline{x}}^{\bar{x}} F(x)u(\bar{y} - x + m)dF(x) + \int_{\underline{x}}^{\bar{x}} [1 - F(x)]u(\bar{y} - x - m)dF(x) - \int_0^{\bar{x}} u(\bar{y} - x)dF(x), \tag{12}$$

where, if we recall the explanation underlying (4), the first term is the expected first period's utility if she wins (with probability $F(x)$, she gets utility $u(\bar{y} - x + m)$), the second term is the expected first period's utility if she loses (with probability $1 - F(x)$, she gets utility $u(\bar{y} - x - m)$), and the third term is the expected utility without Rosca participation. The marginal potential benefits from Rosca in the first period, at m sufficiently close to 0, can be obtained by taking derivative of (12) with respect to m, and is given by:

$$\int_{\underline{x}}^{\bar{x}} u'(\bar{y} - x)dF(x)^2 - \int_{\underline{x}}^{\bar{x}} u'(\bar{y} - x)dF(x) > 0,$$

[11] Recall that $U_I (x, x)$ as given by Eq. (5) is type-x agent's expected welfare from participating in a Rosca with size m, and $V_I^n (\bar{y} - x)$ as given by Eq. (1) is type-x agent's expected welfare with no Rosca participation.

which is the difference between the expectation of the first-order statistic of marginal utility and the expected marginal utility.

However, there are potential costs from Rosca in the second period. The second period's *ex ante* expected utility gain (or loss) from Rosca is given by

$$\begin{aligned} & \int_{\underline{x}}^{\bar{x}} \left[F(x)\delta \mathrm{E}u\left(Y_2 - m - b_I(x)\right)\right] dF(x) + \int_{\underline{x}}^{\bar{x}} \left[\int_{x}^{\bar{x}} \delta \mathrm{E}u\left(Y_2 + m + b_I(x^j)\right) dF(x^j)\right] dF(x) - \delta \mathrm{E}u(Y_2) \\ = & \int_{\underline{x}}^{\bar{x}} F(x)\left[\delta \mathrm{E}u\left(Y_2 - m - b_I(x)\right) + \delta \mathrm{E}u\left(Y_2 + m + b_I(x)\right)\right] dF(x) - \delta \mathrm{E}u(Y_2), \end{aligned} \tag{13}$$

where the first term in the first line is the expected second period's utility if she wins (with probability $F(x)$, she gets discounted expected utility $\delta \mathrm{E}u(Y_2 - m - b_I(x))$), the second term is the expected second period's utility if she loses (with probability $1 - F(x)$, she gets discounted conditional expected utility $\int_x^{\bar{x}} \delta \mathrm{E}u\left(Y_2 + m + b_I(x^j)\right) dF(x^j) / [1 - F(x)]$), and the third term is the expected utility without participating Rosca; and the equality follows from an integration by parts. By the concavity of $u(\cdot)$, the second period's expected utility gain is always negative for any $m > 0$, which reflects the cost due to the extra second-period consumption volatility when participating in the Rosca, since both the allocation and the amount of the money pot are random depending on the opponent's first period income shock realization. The bigger the size, the higher the volatility. The expected marginal cost from Rosca in the second period, at m close to 0, is zero:

$$\frac{d}{dm}\left[\int_{\underline{x}}^{\bar{x}} F(x)[\delta \mathrm{E}u(Y_2 - m - b_I(x) + \delta \mathrm{E}u(Y_2 + m + b_I(x))]dF(x)\right]\Bigg|_{m=0} = 0.$$

Therefore, the *ex ante* welfare gain is positive for a Rosca with optimal size $m^* > 0$ as Proposition 2 shows.

3.2 Case II: Rosca and a Perfect Formal Credit Market

Now we consider the other extreme case where Rosca participants can borrow and lend at an interest $r > 0$ in a perfect formal credit market, and examine whether Rosca may still improve participants' welfare. We show that having access to a perfect formal credit market will crowd out the role of Rosca.

3.2.1 Indirect Utility

First, we derive the indirect utility of a representative agent. With a perfect credit market where the interest rates $r_b = r_s = r$, given the first period's income y_1, a representative agent's problem is

$$V(y_1; r) \equiv \max_{\{c_1\}} \left[u(c_1) + \delta \mathrm{E}u\left(\frac{y_1 + \rho Y_2 - c_1}{\rho}\right)\right], \tag{14}$$

where $\rho \equiv 1/(1+r)$.

The value function $V(\cdot;r)$ can be shown to be increasing and concave.[12] Without participating in a Rosca, the agent's utility is given by

$$V_{II}^{n}(y_1;r) = V(y_1;r). \tag{15}$$

Note that under a perfect credit market, a Rosca with size m and winning bid $b_{II}(x)$, depending on the winner's realized shock x, is equivalent to a transfer (in net present value) of:

$$T(x) = \rho[mr - b_{II}(x)] \tag{16}$$

from the loser to the winner. Therefore, the winner's indirect utility is given by

$$V_{II}^{w}(b_{II}(x);\bar{y}-x,r) = V\left(\bar{y}-x+T(x);r\right), \tag{17}$$

and the loser's (random) indirect utility, depending on the winner's income-shock X^j, is given by

$$V_{II}^{f}(b_{II}(X^j);\bar{y}-x,r) = V(\bar{y}-x-T(X^j);r). \tag{18}$$

In what follows, we use these indirect utilities to derive the bidding strategy and then evaluate the social welfare.

3.2.2 An Impossibility Result

Using strategies similar to those in the derivation of expression (5), and plugging in indirect utilities V_{II}^{w} and V_{II}^{f} as given by (17) and (18), we can derive the expected utility for agent 1 of type $x > \underline{x}$ from reporting $\tilde{x} > \underline{x}$ as follows:

$$U_{II}(x,\tilde{x};r) = [p+(1-p)F(\tilde{x})]V(\bar{y}-x+T(\tilde{x});r) + (1-p)\int_{\tilde{x}}^{\bar{x}} V(\bar{y}-x-T(x^j);r)dF(x^j), \tag{19}$$

where the explanation of each term is similar to (5) and is thus omitted here. The first order condition for optimal strategy must satisfy $\frac{\partial}{\partial\tilde{x}}U_{II}(x,\tilde{x};r)\big|_{\tilde{x}=x} = 0$, thus we obtain

$$T'(x) = -\frac{(1-p)f(x)}{p+(1-p)F(x)}\frac{V\left(\bar{y}-x+T(x);r\right)-V(\bar{y}-x-T(x);r)}{\frac{\partial}{\partial y}V(\bar{y}-x+T(x);r)}. \tag{20}$$

Since $T(\cdot) \geq 0$, it follows immediately $T'(\cdot) \leq 0$, i.e., the transfer is decreasing in income shock.

For $x = 0$, $U_{II}(0,0;r)$ is similar to (7) with V_I^w and V_I^f replaced by V_{II}^w and V_{II}^f respectively.

We can prove the following result regarding the social welfare in the presence of a perfect formal credit market:

Proposition 3 *If there is a perfect credit market, then Rosca cannot improve the* ex ante *welfare.*

[12] This is a special case of Lemma 1 in Section 4 where the formal statement and proof are provided.

Propositions 2 and 3 for the two benchmark cases help to identify the key properties of Rosca that are important in understanding its interactions with formal credit markets. There are two competing effects from having access to formal credit markets. On the positive side, having access to formal financial markets can potentially improve individuals' welfare over the immediate outcome of the Rosca bidding. To see this, we note that, if access to a credit market is available, the loser of the Rosca bidding could achieve a higher expected utility by utilizing the credit market, specifically, she receives:

$$\max_{\{c_1\}}\left[u(c_1)+\delta \mathrm{E}u\left(\frac{y_1-m+\rho\left(Y_2+m+\hat{b}\right)-c_1}{\rho}\right)\right]\geq u(y_1-m)+\delta\mathrm{E}u\left(Y_2+m+\hat{b}\right),$$

where the right hand side is the loser's utility without access to the credit market. Similarly, the winner could also be better off by utilizing the formal credit market in response to the Rosca winning.

However, the above potential benefit changes the incentive structure because the presence of formal credit opportunities invites incentives for arbitrage, making it *more difficult* to satisfy the incentive compatibility constraint in Rosca bidding. Recall from (16) that Rosca is equivalent to a transfer of first-period income from the loser to the winner, with the expected net transfer $\pi(\cdot)$ given by

$$\pi(x)\equiv\begin{cases}\frac{1}{2}pT(0)-\left[\frac{1}{2}pT(0)+(1-p)\int_{\underline{x}}^{\bar{x}}T(x^j)dF(x^j)\right] & \text{if } x=0,\\ [p+(1-p)F(x)]T(x)-(1-p)\int_{x}^{\bar{x}}T(x^j)dF(x^j) & \text{if } x>\underline{x},\end{cases}$$

where in each case the first term is the expected in-transfer (conditional on her winning the auction) and the second term is the expected out-transfer (conditional on her losing the auction). By Jensen's inequality, we know that for all x,

$$U_{II}(x,x;r)\leq V(\bar{y}-x+\pi(x);r), \tag{21}$$

where for type $x>\underline{x}$, $U_{II}(x,x;r)$ is the *interim* utility defined as in (19), with $\tilde{x}$ replaced by x and for type $x=0$, $U_{II}(0,0;r)$ is similar to (7) with V_I^w and V_I^f replaced by V_{II}^w and V_{II}^f respectively. The interpretation is that the transfer mechanism underlying the Rosca is no better than just giving an expected net transfer $\pi(\cdot)$ to the participant due to the concavity of $V(\cdot;r)$. Note that the incentive compatibility also requires

$$U_{II}(x,x;r)\geq U_{II}(x,\bar{x};r)=V(\bar{y}-x+T(\bar{x});r)\geq V(\bar{y}-x;r), \tag{22}$$

since it is feasible for a bidder to submit the highest bid $b(\bar{x})\leq mr$ by which she can ensure herself an interim utility no less than $V(\bar{y}-x;r)$. Inequalities (21) and (22) imply that, for all x, we must have $V(\bar{y}-x+\pi(x);r)\geq V(\bar{y}-x;r)$, which in turn implies that $\pi(x)\geq 0$, for all x. The conflicts then arise from the fact that the budget balance constraint $\mathrm{E}\pi(X)=0$ does not allow for $\pi(X)>0$ with a positive probability given $\pi(\cdot)$ is nonnegative. Therefore, the only possibility is $\pi(\cdot)\equiv 0$. Then $\mathrm{E}U_{II}(X,X;r)\leq \mathrm{E}V_{II}^n(\bar{y}-X;r)$, which implies that there is no Rosca that is both incentive compatible and *ex ante* welfare improving, in contrast to Proposition 2 where Rosca is *ex ante* beneficial.

4 Rosca with Imperfect Credit Market

In this section, we consider an imperfect formal credit market where the individuals are allowed to borrow and save, but the borrowing interest rate r_b is greater than the saving interest rate r_s. As described in Table 1 we refer to this as Case III.

4.1 The Case without Rosca

Without Rosca, the problem for a representative agent with income y_1 is:

$$V_{III}^{n}(y_1; r_b, r_s) \equiv \max_{\{c_1, C_2\}} [u(c_1) + \delta \mathrm{E} u(C_2)] \tag{23}$$

$$\text{s.t. } c_1 + \rho_s C_2 \leq y_1 + \rho_s Y_2, \tag{24}$$

$$c_1 + \rho_b C_2 \leq y_1 + \rho_b Y_2, \tag{25}$$

where $\rho_s \equiv 1/(1 + r_s)$ and $\rho_b \equiv 1/(1 + r_b)$, and (24) and (25) are the budget constraints for a saver and a borrower respectively.[13]

We can obtain several useful properties regarding the value function $V_{III}^{n}(y_1; r_b, r_s)$ and optimal consumption policy $c_1^*(y_1; r_b, r_s)$ for Problem (23):

Lemma 1 ***[Properties of** $V_{III}^{n}(\cdot; r_b, r_s)$ **and** $c_1^*(\cdot; r_b, r_s)$]*

(i) *The value function $V_{III}^{n}(\cdot; r_b, r_s)$ is continuous, differentiable, increasing and concave in y_1;*

(ii) *The optimal consumption policy $c_1^*(\cdot; r_b, r_s)$ is increasing in y_1;*

(iii) *$y_1 - c_1^*(y_1; r_b, r_s)$ is increasing in y_1.*

Lemma 1 generalizes the standard result on the intertemporal consumption with linear budget constraint. In particular, Part (iii) says that if an agent with income y_1 is a borrower, then for an agent with income less than y_1, she must borrow more. Also if an income-y_1 agent is a saver, then for an agent with income more than y_1, she must save more. As we will see below, Lemma 1 is useful in terms of assessing the social welfare and delineating a Rosca participant's role (saver or borrower) in the presence of an imperfect credit market.

4.2 Rosca, Indirect Value Functions and the Interaction with an Imperfect Credit Market

Now suppose that there is a Rosca with size m in addition to the access to an imperfect credit market. The value of bidder 1 with income y_1 from winning the auction to access the pot at the end

[13]The two constraints (24) and (25) are equivalent to the constraint $C_2 \leq Y_2 + \min\{\frac{y_1 - c_1}{\rho_s}, \frac{y_1 - c_1}{\rho_b}\}$. We write it as two explicit constraints, (24) for a saver and (25) for a borrower, to facilitate the application of the Lagrangian approach.

of period 1 with a premium bid of b, denoted by $V_{III}^{w}(b; y_1, r_b, r_s)$, is the solution to the following problem of consumption and saving:

$$V_{III}^{w}(b; y_1, r_b, r_s) \equiv \max_{\{c_1, C_2\}} [u(c_1) + \delta \mathrm{E} u(C_2)] \tag{26}$$

$$\text{s.t. } c_1 + \rho_s C_2 \leq y_1 + m + \rho_s(Y_2 - m - b), \tag{27}$$

$$c_1 + \rho_b C_2 \leq y_1 + m + \rho_b(Y_2 - m - b), \tag{28}$$

where in constraints (27) and (28), the first period's income $y_1 + m$ reflects her access to the additional amount m from winning the auction, and the second period's income $Y_2 - m - b$ reflects her paying additional premium b in excess of the normal Rosca payment m in the second period.

Similarly, the value of bidder 1 with income y_1 from losing the period 1 auction while her opponent wins with a premium bid of $\hat{b}$, denoted by $V_{III}^{f}\left(\hat{b}; y_1, r_b, r_s\right)$, is the solution to the following problem of consumption and saving:

$$V_{III}^{f}\left(\hat{b}; y_1, r_b, r_s\right) \equiv \max_{\{c_1, C_2\}} [u(c_1) + \delta \mathrm{E} u(C_2)] \tag{29}$$

$$\text{s.t. } c_1 + \rho_s C_2 \leq y_1 - m + \rho_s(Y_2 + m + \hat{b}), \tag{30}$$

$$c_1 + \rho_b C_2 \leq y_1 - m + \rho_b(Y_2 + m + \hat{b}) \tag{31}$$

where in constraints (30) and (31), the first period's income $y_1 - m$ reflects the fact that bidder 1 has to contribute m to the pot because she loses the bidding, and the second period's income $Y_2 + m + \hat{b}$ shows that she gets back m and the promised premium of $\hat{b}$ from her opponent in period 2.

We now discuss how losing or winning the money pot in the period-1 Rosca auction changes the participant's role of being a borrower or a saver in the credit market. Recall that $c_1^*(y_1; r_b, r_s)$, introduced in Lemma 1, is the optimal consumption in period one without Rosca. And let $c_1^{*w}(y_1, b; r_b, r_s)$ be the Rosca winner's optimal consumption in period 1, when her income is y_1 and the winning bid is b, i.e., the solution to Problem (26); and let $c_1^{*f}(y_1, \hat{b}; r_b, r_s)$ be the counterpart of the loser when her opponent's winning bid is $\hat{b}$, i.e., the solution to Problem (29). We have the following important observations:

Lemma 2 ***[Properties of Consumption/Saving Decisions for Rosca Winners and Losers]***

(i) *If $y_1 - m \geq c_1^*(y_1; r_b, r_s)$, then $V_{III}^{f}(\hat{b}; y_1, r_b, r_s) \geq V_{III}^{n}(y_1; r_b, r_s) \geq V_{III}^{w}(\hat{b}; y_1, r_b, r_s)$ and $y_1 - m \geq c_1^{*f}(y_1, \hat{b}; r_b, r_s)$ for any $\hat{b} \in [mr_s, mr_b]$;*

(ii) *If $c_1^*(y_1; r_b, r_s) \geq m + y_1$, then $V_{III}^{w}(b; y_1, r_b, r_s) \geq V_{III}^{n}(y_1; r_b, r_s) \geq V_{III}^{f}(b; y_1, r_b, r_s)$ and $c_1^{*w}(y_1, b; r_b, r_s) \geq y_1 + m$ for any $b \in [mr_s, mr_b]$.*

The intuition behind Lemma 2 can be stated as follows. Part (i) says that, for a saver in the absence of the Rosca (which is implied by $y_1 - m \geq c_1^*(y_1; r_b, r_s)$), if she saves money in the credit market, she receives a saving interest rate r_s; but if she contributes money through Rosca given her opponent's bid $\hat{b} \in [mr_s, mr_b]$, the implicit interest rate is higher than r_s and thus she is better off with the Rosca. Similarly, Part (ii) says that, for a borrower in the absence of Rosca (which is implied by $c_1^*(y_1; r_b, r_s) \geq m + y_1$), if she borrows money from the credit market, she would have to pay a borrowing interest rate r_b; but if she can get the money through winning the Rosca bidding with any bid $b \in [mr_s, mr_b]$, her implicit borrowing interest rate is lower and thus she is better off with Rosca. Lemma 2 establishes a link between the agents' roles in the credit market before and after Rosca participation. We will use Lemma 2 to solve the Rosca bidding equilibrium when the size of Rosca is properly chosen.

4.3 Bidding Equilibrium and Welfare Analysis

Now we characterize the bidding equilibrium of Rosca with size m. Similar to formula (4), the expected utility for bidder 1 of type-x ($x > \underline{x}$) from reporting type $\tilde{x}$ ($\tilde{x} > \underline{x}$) is given by:

$$\begin{aligned} U_{III}(x, \tilde{x}; r_b, r_s) &= \left\{\Pr\left(X^j = 0\right) + \left[1 - \Pr(X^j = 0)\right] \Pr\left(\underline{x} < X^j \leq \tilde{x}\right)\right\} V_{III}^w\left(b_{III}(\tilde{x}); \bar{y} - x, r_b, r_s\right) \\ &+ \left[1 - \Pr(X^j = 0)\right] \Pr\left(X^j > \tilde{x}\right) \mathrm{E}[V_{III}^f(b_{III}\left(X^j\right); \bar{y} - x, r_b, r_s) \Big| X^j > \tilde{x}], \end{aligned} \tag{32}$$

where $V_{III}^w\left(b_{III}(\tilde{x}); \bar{y} - x, r_b, r_s\right)$ and $V_{III}^f(b_{III}\left(X^j\right); \bar{y} - x, r_b, r_s)$ are defined as in (26) and (29), respectively.

The explanation of (32) is similar to (4). Using the distribution of X, we rewrite (32) as:

$$\begin{aligned} U_{III}(x, \tilde{x}; r_b, r_s) &= [p + (1-p)F(\tilde{x})]\, V_{III}^w\left(b_{III}(\tilde{x}); \bar{y} - x, r_b, r_s\right) \\ &\quad + (1-p) \int_{\tilde{x}}^{\bar{x}} V_{III}^f\left(b_{III}(x^j); \bar{y} - x, r_b, r_s\right) dF(x^j). \end{aligned} \tag{33}$$

Note that Lemma 1 implies that $V_{III}^w\left(\cdot; \bar{y} - x, r_b, r_s\right)$ and $V_{III}^f\left(\cdot; \bar{y} - x, r_b, r_s\right)$ are differentiable. Using the same technique used to derive (6), the equilibrium bidding function $b_{III}(\cdot)$ must satisfy:

$$b_{III}'(x) = -\frac{(1-p)f(x)}{p + (1-p)F(x)} \frac{V_{III}^w\left(b_{III}(x); \bar{y} - x, r_b, r_s\right) - V_{III}^f\left(b_{III}(x); \bar{y} - x, r_b, r_s\right)}{\frac{\partial V_{III}^w(b_{III}(x); \bar{y} - x, r_b, r_s)}{\partial b}}. \tag{34}$$

We should point out that equation (34) only specifies the necessary first-order condition. We need some technical conditions to insure the global optimality of $b_{III}(\cdot)$ as characterized by (34). First, we see from (34) that, to ensure that $b_{III}(\cdot)$ is monotonically increasing, any type-x agent needs to be better off being a winner of in the period-1 Rosca bidding with a bid $b_{III}(x)$ than being a loser with the same bid from her opponent. To guarantee this, we need to impose some restrictions on the size of the Rosca m, namely m can not be too large: if m is too large, it is possible that the winner of the Rosca bidding ends being a saver, while the loser ends up being a borrower, causing a violation of the monotonicity of $b_{III}(\cdot)$. It turns out that a sufficient condition is that the Rosca size $m > 0$ is bounded above by $\overline{m}(r_b, r_s)$ as given by:[14]

$$m \leq \overline{m}(r_b, r_s) \equiv \min\{c_1^*(\bar{y} - \underline{x}; r_b, r_s) - (\bar{y} - \underline{x}), \bar{y} - c_1^*(\bar{y}; r_b, r_s)\}. \tag{35}$$

[14]The existence of $\bar{m}(r_b, r_s)$ is in turn guaranteed by:

$$u'(\bar{y}) < \delta(1 + r_s)\mathrm{E}u'(Y_2) \text{ and } u'(\bar{y} - \underline{x}) > \delta(1 + r_b)\mathrm{E}u'(Y_2).$$

By Lemma 2, if $m \leq \bar{y} - c_1^*(\bar{y}; r_b, r_s)$, then the type-0 bidder will not be a borrower regardless of her wining or losing the auction; and if $m \leq c_1^*(\bar{y} - \underline{x}; r_b, r_s) - (\bar{y} - \underline{x})$, then the type-$\underline{x}$ bidder will not be a saver regardless of her wining or losing the auction. Second, we also need a technical condition that the utility function has weakly decreasing absolute risk aversion and $u''''(.) \geq 0$, where the latter implies increasing absolute prudence.[15]

Lemma 3 *Consider an imperfect credit market with borrowing and saving interest rate r_b and r_s respectively, with $0 \leq r_s < r_b$. Suppose that $u(\cdot)$ has weakly decreasing absolute risk aversion and that $u'''' \geq 0$, and the size of Rosca m is bounded by $\bar{m}(r_b, r_s)$ as given by (35), then there exists a bidding equilibrium in period-1 Rosca auction such that:*

(i) *$b_{III}(0) = mr_s$;*

(ii) *for type $x > \underline{x}$, $b_{III}(x)$ is characterized by the differential equation*

$$b'_{III}(x) = \frac{(1-p)f(x)}{p + (1-p)F(x)} \frac{V(\bar{y} - x + \rho_b(mr_b - b_{III}(x); r_b) - V(\bar{y} - x - \rho_b(mr_b - b_{III}(x)); r_b)}{\rho_b V'(\bar{y} - x + \rho_b(mr_b - b_{III}(x)); r_b)}, \quad (36)$$

with an initial condition $b_{III}(\underline{x}) = mr_s$, where $V(\cdot; r)$ is defined as in (14) and $V'(y_1; r)$ denotes its derivative with respect to y_1.

If the equilibrium is characterized by Lemma 3, the welfare property is given by:

Proposition 4 *Assume $r_b > r_s \geq 0$ and that the conditions for Lemma 3 hold. Then under the equilibrium bidding strategy specified in Lemma 3, Rosca improves participants'* ex ante *welfare.*

The key observation here is that the interest rate gap limits the agents' arbitrage between Rosca and the credit market. With the gap between the borrowing and saving interest rates, a saver will be self-disciplined to submit a bid with an implied interest rate lower than the saving interest rate in the credit market. In equilibrium, the borrower will realize that the saver will not bid higher than the saving interest rate, and thus may submit a bid with an implied interest rate lower than the borrowing interest rate. Since the bids are between the two credit market interest rates, both saver and borrower are better off. In the Corollary 1 below, we explicitly characterize the bidding equilibrium for the case of Constant Absolute Risk Aversion (CARA) utility functions:

Corollary 1 *Under the CARA utility function $u(c) = [1 - \exp(-\beta c)]/\beta$, the equilibrium bidding function is given by:*

$$b_{III}(x) = \begin{cases} mr_s, & \text{if } x = 0 \\ mr_b + \left(\frac{2+r_b}{2\beta}\right) \ln\left\{1 - \left[1 - \exp\left(-\frac{2\beta m(r_b - r_s)}{2+r_b}\right)\right]\left[\frac{p}{p+(1-p)F(x)}\right]^2\right\}, & \text{if } x > \underline{x}, \end{cases} \quad (37)$$

Under the above condition, the agent with no income shock will be a saver, i.e., $\bar{y} > c_1^*(\bar{y}; r_b, r_s)$, and the agent with the income shok $\underline{x}$ will be a borrower, i.e., $c_1^*(\bar{y} - \underline{x}; r_b, r_s) > (\bar{y} - \underline{x})$.

[15] In a seminal paper, Kimball (1990) shows that the intensity of saving also depends on the absolute prudence $\gamma(\cdot) \equiv -\frac{u'''(\cdot)}{u''(\cdot)}$. He shows that the assumption $u''''(\cdot) \geq 0$ implies that the absolute prudence $\gamma(\cdot)$ is increasing.

where Rosca size m is bounded by

$$\overline{m}\left(r_b, r_s\right) = \min\left\{\frac{1}{2+r_b}\left(-\frac{\ln[(1+r_b)\delta \mathrm{E}\exp(-\beta Y_2)]}{\beta} - (\bar{y}-\underline{x})\right), \frac{1}{2+r_s}\left(\bar{y}+\frac{\ln[(1+r_s)\delta \mathrm{E}\exp(-\beta Y_2)]}{\beta}\right)\right\}. \tag{38}$$

5 Welfare Comparisons

In the previous sections, we have investigated the possibility for Rosca to improve the welfare, under a variety of credit market conditions. Now we discuss the change of welfare when credit market conditions improve. In the absence of Rosca, it is straightforward to show that, for any y_1, the following inequalities hold:

$$V_I^n(y_1) \leq V_{III}^n(y_1; r_b, r_s) \leq \min\{V_{II}^n(y_1; r_s), V_{II}^n(y_1; r_b)\} = \min\{V_{III}^n(y_1; r_s, r_s), V_{III}^n(y_1; r_b, r_b)\}, \tag{39}$$

where $V_I^n(y_1)$ is defined as in (1), $V_{II}^n(y_1; r)$ is defined as in (15) and $V_{III}^n(y_1; r_b, r_s)$ is defined as in (23). We summarize the above inequalities by the following proposition:

Proposition 5 ***(Welfare and Credit Market Imperfectness in the Absence of Rosca)*** *In the absence of Rosca, a reduction in the credit market imperfectness, as measured by the borrowing/savings interest rate gap, increases the social welfare.*

The next proposition addresses the following question: are agents better off in an environment where they form a Rosca with size m, but do not have access to credit market, or in an environment with a perfect formal credit market with interest rate r with no Rosca? The answer is affirmatively the latter, if the interest rate r in the perfect credit market is not too high.

Proposition 6 ***(Perfect Credit Market without Rosca vs. Rosca without Credit Market)*** *Assume $p = 0$. If the Rosca size m is such that $\int_{\underline{x}}^{\bar{x}} b_I(x)dF(x)^2/m \geq r$, we have*

$$\mathrm{E}U_I(X, X) \leq \mathrm{E}V_{II}^n(\bar{y} - X; r).$$

In particular, $\int_{\underline{x}}^{\bar{x}} b_I(x)dF(x)^2/m \geq r$ holds if $\rho = 1/(1+r) \geq \delta$.

The intuition for Proposition 6 is as follows. In a traditional society where there is no credit market but a continuum of *ex ante* identical residents who are randomly paired to form Rosca with size m. The average winning bid in this economy is exactly $\int_{\underline{x}}^{\bar{x}} b_I(x)dF(x)^2$,[16] which results in an average implied interest rate $\int_{\underline{x}}^{\bar{x}} b_I(x)dF(x)^2/m$. Under the stated condition that $\int_{\underline{x}}^{\bar{x}} b_I(x)dF(x)^2/m$ higher than the interest rate r prevalent in the perfect credit market, agents in the modern society with perfect credit market will experience less volatile consumption as that in the traditional society

[16] The average winning bid has a c.d.f. $F(x)^2$ instead of $F(x)$ because it is the highest order statistic of two random variables.

Parameter	Value
CARA Parameter: β	0.065
Borrowing Interest Rate: r_b	0.27
Discount Factor: δ	0.77
Top Income: $\bar{y}$	72.5
Fraction with No Negative Income Shocks: p	0.63
Income Shock Distribution $F(x)$: Uniform $(\underline{x}, \bar{x}]$	$\underline{x} = 91$,$\bar{x} = 95$
Rosca size: m	5.96

Table 2: Parameters for the Numerical Example.

with Rosca only. Interestingly, if the interest rate r is determined by a Walrasian equilibrium where $\int_{\underline{x}}^{\bar{x}}[\bar{y} - x - c_1^*(\bar{y} - x; r, r)]dF(x) = 0$, then under Constant Absolute Risk Averse (CARA) utility function, we indeed have $\rho \geq \delta$.

Finally, we consider traditional societies with the presence of traditional Rosca and a formal credit market, and ask the following question: in the presence of Rosca, is the social welfare monotonically increasing as the credit market imperfectness, as measured by $r_b - r_s$, converges to zero? Interestingly and somewhat surprisingly, the answer is *no*, at least for CARA utility, as the following proposition states:

Proposition 7 ***(Non-Monotonicity of Welfare with respect to Credit Market Imperfectness in the Presence of Rosca)*** *Assume that the utility is CARA. For any fixed r_b, when the size of Rosca $m \leq \min_{r_s \in [0, r_b]} \overline{m}(r_b, r_s)$ where $\overline{m}(r_b, r_s)$ is given by (38) satisfies*

$$\frac{\bar{y} - c_1^*(\bar{y}; r_b, r_b)}{m} < (1-p)^2 + p(1-p) \int_{\underline{x}}^{\bar{x}} \exp\left[\frac{\beta(1+r_b)}{2+r_b} x\right] dF(x), \tag{40}$$

then for some $r_s < r_b$, we have

$$\mathrm{E}U_{III}(X, X; r_b, r_s) > \mathrm{E}U_{III}(X, X; r_b, r_b). \tag{41}$$

To illustrate Proposition 7, we provide a numerical example where the parameters that define the environment are described in Table 2. We consider the changes in welfare when the saving interest rate r_s varies from 0 to $r_b = 0.27$. We first verify that the conditions for Proposition 7 are satisfied in the example. It can be shown that the type-0 agent will be a saver in period 1 regardless of r_s and her optimal period-1 saving, $\bar{y} - c_1^*(\bar{y}; r_b, r_s)$ where $c_1^*(\bar{y}; r_b, r_s)$ is give by (A18), is given by

$$\bar{y} - c_1^*(\bar{y}; r_b, r_s) = \frac{1}{\beta(2+r_s)} \left\{ \ln[(1+r_s)\delta] + \ln\left[p + (1-p)\int_{\underline{x}}^{\bar{x}} \exp(\beta x)\, dF(x)\right] \right\},$$

which can be shown to be a decreasing function of r_s and is no smaller than (approximately) 34.12, achieved at $r_s = r_b = 0.27$.[17] It can also be verified that agents with income shock $x > \underline{x}$ will be a

[17]Details of the calculations are available in an online appendix available at http://www.econ.upenn.edu/~hfang.

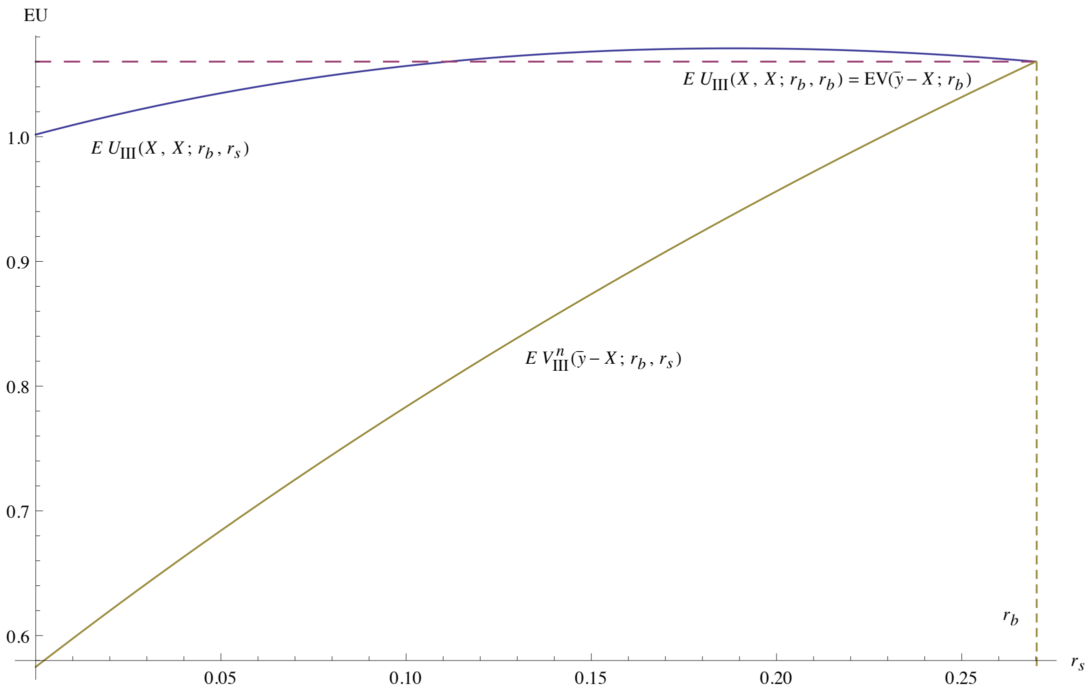

Figure 2: Non-Monotonicity of Social Welfare in the Imperfectness of Formal Credit Market When Rosca is Also Present.

Notes: $EU_{III}(X, X; r_b, r_s)$ denotes the *ex ante* expected welfare in an environment with Rosca and an impefect credit market; $EV^n_{III}(\bar{y} - X; r_b, r_s)$ is the *ex ante* expected welfare in an imperfect credit market but without Rosca; $EV(\bar{y} - X; r_b)$ is the *ex ante* expected welfare in an environment with perfect credit market of interest rate $r_s = r_b$. Note at $r_s = r_b, EU_{III}(X, X; r_b, r_b) = EV^n_{III}(\bar{y} - X; r_b, r_b) = EV(\bar{y} - X; r_b)$.

borrower in period 1, with the optimal borrowing for type-$\underline{x}$ given by:

$$c_1^*(\bar{y}-\underline{x}; r_b, r_s)-(\bar{y}-\underline{x}) = \frac{1}{\beta(2+r_b)}\left\{\beta\underline{x} - \ln\left[(1+r_b)\delta\right] - \ln\left[p + (1-p)\int_{\underline{x}}^{\bar{x}} \exp(\beta x)\, dF(x)\right]\right\} = 5.97.$$

Therefore the Rosca size $m = 5.96$ indeed satisfies that $m \leq \min_{r_s \in [0, r_b]}\{\bar{y} - c_1^*(\bar{y}; r_b, r_s), c_1^*(\bar{y} - \underline{x}; r_b, r_s) - (\bar{y} - \underline{x})\} = 5.97$. Finally, inequality (40) in Proposition 7 is satisfied because

$$\frac{\bar{y} - c_1^*(\bar{y}; r_b, r_b)}{m} = \frac{34.12}{5.96} = 5.72 < (1-p)^2 + p(1-p)\int_{\underline{x}}^{\bar{x}} \exp\left[\frac{\beta(1+r_b)}{2+r_b}x\right] dF(x) = 7.00.$$

Figure 2 demonstrates the numerical results. As we have shown in Proposition 4, Rosca results in a social welfare gain under an imperfect market, i.e., the curve $\text{E}U_{III}(X, X; r_b, \cdot)$ is above the curve $\text{E}V^n_{III}(\bar{y} - X; r_b, \cdot)$, for all levels of r_s. There are two important observations. First, the welfare gain of the Rosca vanishes as the market becomes perfect, i.e., $r_s \to r_b$, which is shown by the impossibility result with a general utility $u(\cdot)$ (see Proposition 3). Second, in this numerical example, as r_s approaches r_b, $\text{E}U_{III}(X, X; r_b, \cdot)$ is not monotonically increasing. The reason is that as $r_s \to r_b$, it becomes more difficult to satisfy the incentive compatibility constraint in the Rosca mechanism, which drives the bid up, and reduces welfare gain from Rosca. Moreover, the decrease of welfare gain from Rosca will dominate the increase of welfare gain from the improvement of the credit market. Finally, as $r_s = r_b$, we have shown that the perfect credit market will eliminate any welfare gain from Rosca, $\mathrm{E}U_{III}(X, X; r_b, r_b) = \mathrm{E}V(\bar{y} - X, r_b)$. The non-monotonicity shows that the perfectness of the credit market is not always desirable since it may inhibit the performance of the self-insurance mechanism such as Rosca.

Moreover, it can be seen that inequality (40) is more likely to hold if the expected marginal disutility of income shocks $\int_{\underline{x}}^{\bar{x}} \exp\left[\frac{\beta(1+r_b)}{2+r_b} x\right] dF(x)$ is higher. Moreover, the fact that the right hand side of (40) is inversely U-shaped in p indicates that for the non-monotonicity to occur, p should not be too small or too large.

6 Conclusion

In this paper, we develop an auction model of risk averse Rosca participants facing income shocks to investigate the interaction between the credit market and Rosca, which fills a hole in the existing literature. We uncover two competing forces regarding the interaction between a Rosca and a formal credit market. On the one hand, the formal credit market may facilitate (rather than substitute) the participation of Rosca through the smoothing of the risk introduced by the uncertainty in the Rosca bidding; on the other hand, the incentive compatibility constraint for Rosca participants becomes more difficult to satisfy due to the arbitrage incentives resulting from the access to the formal credit market.

Using the gap of the borrowing and saving interest rates as a measure of the imperfectness of the credit market, we compare three cases: (i) Rosca without credit market; (ii) Rosca with a perfect credit market; (iii) Rosca with an imperfect credit market. We show that a perfect credit market completely crowds out the role of Rosca. However, when credit market is present but imperfect, we show that Rosca and the formal credit market can complement each other in improving social welfare. Interestingly, we find that the social welfare in an environment with both Rosca and formal credit market does not necessarily increase monotonically as the imperfectness of the credit market converges to zero.

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A Appendix: Proofs

Proof of Proposition 1:

Suppose that $b_0 \geq 0$ satisfies (8) and (9). We prove Proposition 1 in two steps.

(**Step 1.**) We show that any solution to Eq. (6) with boundary condition $b_I(0) = b_I(\underline{x}) = b_0$ satisfies $b_I'(\cdot) > 0$ and $b_I(\cdot) \geq 0$.

First, we show that there exists some $x^* > \underline{x}$ such that $b_I'(x) > 0$ for $x \in (\underline{x}, x^*)$. Suppose not. There must exist some $\varepsilon > 0$ such that $b_I'(x) \leq 0$ for any $x \in (\underline{x}, \underline{x} + \varepsilon)$. Then, for $x \in (\underline{x}, \underline{x} + \varepsilon)$ we have:

$$V_I^w(b_I(x); \bar{y}-x) - V_I^f(b_I(x); \bar{y}-x) > V_I^w(b_I(x); \bar{y}-\underline{x}) - V_I^f(b_I(x); \bar{y}-\underline{x}) \geq V_I^w(b_I(\underline{x}); \bar{y}-\underline{x}) - V_I^f(b_I(\underline{x}); \bar{y}-\underline{x}) \geq 0,$$

where the first inequality follows from the strict concavity of $u(\cdot)$; and the second inequality follows from the fact that $V_I^w(\cdot; \bar{y} - \underline{x})$ decreases in b and $V_I^f(\cdot; \bar{y} - \underline{x})$ increases in b; and the last inequality follows from the assumption that $b_I(\underline{x}) = b_0$ satisfying inequality (9). Therefore, by Eq. (6), the above inequality in turn implies $b_I'(x) > 0$, a contradiction.

Next we show that $b_I'(\cdot) > 0$ for all $x > \underline{x}$. Suppose not. Given that $b_I'(x) > 0$ for x close to $\underline{x}$ as we have shown, there must exist x^* such that $b_I'(x^*) = 0$, $b_I(x^* + \varepsilon) < b_I(x^*)$ and $b_I'(x^* + \varepsilon) < 0$ for some $\varepsilon > 0$. If so, we have

$$\begin{aligned} & V_I^w(b_I(x^* + \varepsilon); \bar{y} - (x^* + \varepsilon)) - V_I^f(b_I(x^* + \varepsilon); \bar{y} - (x^* + \varepsilon)) \\ > \; & V_I^w(b_I(x^*); \bar{y} - (x^* + \varepsilon)) - V_I^f(b_I(x^*); \bar{y} - (x^* + \varepsilon)) \\ > \; & V_I^w(b_I(x^*); \bar{y} - x^*) - V_I^f(b_I(x^*); \bar{y} - x^*) = 0, \end{aligned}$$

where the first inequality follows from the fact that $V_I^w(\cdot; \bar{y} - x) - V_I^f(\cdot; \bar{y} - x)$ decreases in b; the second inequality follows from the strict concavity of $u(\cdot)$; and the last equality follows from the hypothesis that $b_I'(x^*) = 0$ (through Eq. (6)). Therefore, by Eq. (6) again, the above inequality implies that $b_I'(x^* + \varepsilon) > 0$, a contradiction. Thus, for all $x > \underline{x}$, $b_I'(\cdot) > 0$ and $b_I(\cdot) \geq 0$.

(**Step 2.**) We show that $b_I(\cdot)$ that that solves Eq. (6) with boundary condition $b_I(0) = b_I(\underline{x}) = b_0$ is a symmetric BNE of the Rosca bidding game.

First, we show that there is no incentive for type $x > \underline{x}$ to mimic any $\tilde{x} \neq x$ such that $\tilde{x} > \underline{x}$. Note that expression (4) and the concavity of $u(\cdot)$ imply that:

$$\frac{\partial^2 U_I(x, \tilde{x})}{\partial \tilde{x} \partial x} = (1 - p) f(\tilde{x})[u'(\bar{y} - x - m) - u'(\bar{y} - x + m)] \geq 0. \tag{A1}$$

Together with the first order condition that $\partial U_I(x, \tau)/\partial \tau \mid_{\tau = x} = 0$, (A1) implies that, for any $x, \tilde{x} > \underline{x}$,

$$U_I(x, x) - U_I(x, \tilde{x}) = \int_{\tilde{x}}^{x} \frac{\partial U_I(x, \tau)}{\partial \tau} d\tau \geq 0. \tag{A2}$$

Therefore, there is no incentive for any type $x > \underline{x}$ to mimic any $\tilde{x} \neq x$ such that $\tilde{x} > \underline{x}$.

We next show that there is no incentive for type $x > \underline{x}$ to mimic type 0. Since $b_I(0) = b_I(\underline{x}) = b_0$, if a type-$x$ bidder were to mimic type-0 agent and bid b_0, her gain from the deviation is:

$$\begin{aligned} U_I(x, 0) - U_I(x, x) \quad & \leq \quad U_I(x, 0) - U_I(x, \underline{x}) = \frac{1}{2} p [V_I^f(b_I(\underline{x}); \bar{y} - x) - V_I^w(b_I(\underline{x}); \bar{y} - x)] \\ & \leq \quad \frac{1}{2} p [V_I^f(b_I(x); \bar{y} - x) - V_I^w(b_I(x); \bar{y} - x)] \leq 0, \end{aligned}$$

where the first inequality is due to $U_I(x,x) \geq U_I(x,\underline{x})$, which follows from (A2); the equality simply uses the definition of $U_I(x,0)$ and $U_I(x,\underline{x})$; and the third step uses $b_I(\underline{x}) \leq b_I(x)$ and that $V_I^w(\cdot;\bar{y}-x) - V_I^f(\cdot;\bar{y}-x)$ decreases in b.

Lastly, we use condition (8) to show that there is no incentive for type-0 agent to deviate from bidding $b_I(0) = b_0$. Note that any type $x > \underline{x}$ will bid higher than $b_I(\underline{x}) = b_0$ because $b_I'(\cdot) > 0$. Given the opponent's equilibrium bidding strategy $b_I(\cdot)$, a type-0 bidder's expected gain of deviation is:

$$\begin{aligned} & U_I(x,0) - U_I(0,0) \\ = \;& p\left\{V_I^w(b_I(x);\bar{y}) - \frac{1}{2}[V_I^w(b_I(0);\bar{y}) + V_I^f(b_I(0);\bar{y})]\right\} + (1-p)\left[F(x)V_I^w(b_I(x);\bar{y}) - \int_{\underline{x}}^{x} V_I^f(b_I(x^j);\bar{y})dF(x^j)\right] \\ \leq \;& \frac{1}{2}p\left[V_I^w(b_I(0);\bar{y}) - V_I^f(b_I(0);\bar{y})\right] + (1-p)F(x)\left[V_I^w(b_I(0);\bar{y}) - V_I^f(b_I(0);\bar{y})\right] \leq 0, \end{aligned}$$

where the first inequality follows from the fact that $V_I^w(\cdot;\bar{y})$ decreases in b and $V_I^f(\cdot;\bar{y})$ increases in b; and the second inequality follows from the assumption that b_0 satisfies inequality (8). Thus, type-0 bidder have no incentive to mimic $x \in (\underline{x},\bar{x}]$. ■

Proof of Proposition 2:

We first prove that, under the stated condition that $\delta \in (0, u'(\bar{y})/\mathrm{E}u'(Y_2))$, the bidding equilibrium as characterized by Proposition 1 exists when m is sufficiently small. To see this, note that, if $\delta \in (0, u'(\bar{y})/\mathrm{E}u'(Y_2))$, then

$$\left.\frac{\partial\left[V_I^w(0;\bar{y}) - V_I^f(0;\bar{y})\right]}{\partial m}\right|_{m=0} = 2\left[u'(\bar{y}) - \delta \mathrm{E}u'(Y_2)\right] > 0.$$

Thus, for sufficiently small $m > 0$, we have $V_I^w(0;\bar{y}) > V_I^f(0;\bar{y})$. Since $V_I^w(\cdot;\bar{y}) - V_I^f(\cdot;\bar{y})$ is continuous and decreasing in b, there exists some $b_0 > 0$ such that $V_I^w(b_0;\bar{y}) = V_I^f(b_0;\bar{y})$, which satisfies condition (8). Further, note that $V_I^w(b_0;\bar{y}-x) - V_I^f(b_0;\bar{y}-x)$ is increasing in x, $V_I^w(b_0;\bar{y}) = V_I^f(b_0;\bar{y})$ implies $V_I^w(b_0;\bar{y}-\underline{x}) \geq V_I^f(b_0;\bar{y}-\underline{x})$ since $\underline{x} \geq 0$. Thus, b_0 also satisfies condition (9). Therefore, Proposition 1 applies and a bidding equilibrium as characterized by Proposition 1 exists.

Now, the *ex ante* expected utility from participating in Rosca with size m conditional on $x \geq \underline{x}$, $\int_{\underline{x}}^{\bar{x}} U_I(x,x)dF(x)$, where $U_I(x,x)$ is defined in (5) with $\tilde{x}$ being replaced by x, can be written as:

$$\begin{aligned} \int_{\underline{x}}^{\bar{x}} U_I(x,x)dF(x) &= \int_{\underline{x}}^{\bar{x}} \left\{[p + (1-p)F(x)]V_I^w(b_I(x);\bar{y}-x) + (1-p)\int_{x}^{\bar{x}} V_I^f(b_I(x^j);\bar{y}-x)dF(x^j)\right\} dF(x) \\ &= \int_{\underline{x}}^{\bar{x}} [p + (1-p)F(x)][u(\bar{y}-x+m) + \delta \mathrm{E}u(Y_2 - m - b_I(x))]dF(x) \\ &\quad + (1-p)\left[\int_{\underline{x}}^{\bar{x}} [(1-F(x))u(\bar{y}-x-m)dF(x) + \delta \mathrm{E}u(Y_2 + m + b_I(x))F(x)]\, dF(x)\right] \quad \text{(A3)} \end{aligned}$$

where the second equality uses the fact that we can use integrating by parts to obtain:

$$\begin{aligned} & \int_{\underline{x}}^{\bar{x}} \left(\int_{x}^{\bar{x}} V_I^f(b_I(x^j);\bar{y}-x)dF(x^j)\right) dF(x) \\ = \;& \int_{\underline{x}}^{\bar{x}} \int_{x}^{\bar{x}} \left[u(\bar{y}-x-m) + \delta \mathrm{E}u(Y_2 + m + b_I(x^j))\right] dF(x^j)dF(x) \\ = \;& \int_{\underline{x}}^{\bar{x}} (1-F(x))u(\bar{y}-x-m)dF(x) + \int_{\underline{x}}^{\bar{x}} \delta \mathrm{E}u(Y_2 + m + b_I(x))F(x)dF(x). \end{aligned}$$

Using the definition of $\Delta U_I(x;m)$ as given by (11), and taking derivative of (A3) with respect to m, we have:

$$\left.\frac{d\mathrm{E}\left[\Delta U_I(X;m)|\, X > \underline{x}\right]}{dm}\right|_{m=0} = \int_{\underline{x}}^{\overline{x}} \left[p + (1-p)F(x) - (1-p)(1-F(x))\right] u'(\bar{y}-x)dF(x)$$
$$-p\delta \mathrm{E}u'(Y_2)\int_{\underline{x}}^{\overline{x}} \left[\left.\frac{\partial b_I(x)}{\partial m} + 1]\right|_{m=0}\right] dF(x). \tag{A4}$$

Similarly, we can rewrite the *ex ante* expected utility for type-0 agent from participating in Rosca with size m, using expression (7), as

$$\begin{aligned} U_I(0,0) \;\; &= \;\; \frac{p}{2}\left[u(\bar{y}+m) + \delta \mathrm{E}u(Y_2 - m - b_I(0)) + u(\bar{y}-m) + \delta \mathrm{E}u(Y_2 + m + b_I(0))\right] \quad \text{(A5)} \\ &+(1-p)\int_{\underline{x}}^{\bar{x}} \left[u(\bar{y}-m) + \delta \mathrm{E}u(Y_2 + m + b_I(x))\right] dF(x). \end{aligned}$$

Taking derivative of (A5) with respect to m, we obtain:

$$\left.\frac{d\Delta U_I(0;m)}{dm}\right|_{m=0} = (1-p)\delta \mathrm{E}u'(Y_2)\int_{\underline{x}}^{\overline{x}} \left.\left[\frac{\partial b_I(x)}{\partial m} + 1\right]\right|_{m=0} dF(x) - (1-p)u'(\bar{y}). \tag{A6}$$

Combining (A4) and (A6), we have:

$$\begin{aligned} \left.\frac{d\Delta W_I(m)}{dm}\right|_{m=0} &= p\left.\frac{d\Delta U_I(0;m)}{dm}\right|_{m=0} + (1-p)\left.\frac{d\mathrm{E}\left[\Delta U_I(X;m)|\, X > 0\right]}{dm}\right|_{m=0} \\ &= (1-p)\left[\int_{\underline{x}}^{\overline{x}} \left[p + (1-p)F(x) - (1-p)(1-F(x))\right] u'(\bar{y}-x)dF(x) - pu'(\bar{y})\right] \\ &> (1-p)^2\int_{\underline{x}}^{\bar{x}} \left[2F(x) - 1\right] u'(\bar{y}-x)dF(x) \\ &> (1-p)^2\int_{\underline{x}}^{\bar{x}} \left[2F(x) - 1\right] u'(\bar{y}-\hat{x})dF(x) \\ &= (1-p)^2 u'(\bar{y})\int_{\underline{x}}^{\bar{x}} \left[2F(x) - 1\right] dF(x) \\ &= (1-p)^2 u'(\bar{y})\left.\left[F(x)^2 - F(x)\right]\right|_{\underline{x}}^{\bar{x}} = 0, \end{aligned}$$

where the first inequality follows from the fact that $u'(\bar{y}-x) > u'(\bar{y})$, and in the second inequality $\hat{x}$ is defined by $F(\hat{x}) = 1/2$, and it follows from the fact that $u'(\bar{y}-x)$ is increasing in x. Therefore, $\left.\frac{d\Delta W_I(m)}{dm}\right|_{m=0} > 0$. ■

Proof of Proposition 3:

The proof is provided in the text. ■

Proof of Lemma 1:

Using the Lagrangian method (see Luenberger, 1969) for concave functional $\mathrm{E}u(\cdot)$, we can rewrite the representative agent's problem (23) as:

$$V^n_{III}(y_1; r_b, r_s) = \min_{\lambda_b \geq 0, \lambda_s \geq 0}\max_{\{c_1, C_2\}} L^n_{III}(c_1, C_2; \lambda_b, \lambda_s; y_1, Y_2) \tag{A7}$$

where

$$L^n_{III}(c_1, C_2; \lambda_b, \lambda_s; y_1, Y_2) = u(c_1) + \delta \mathrm{E}\left\{u(C_2) + \lambda_b[y_1 - c_1 + \rho_b(Y_2 - C_2)] + \lambda_s[y_1 - c_1 + \rho_s(Y_2 - C_2)]\right\} \tag{A8}$$

is the Lagrangian functional and $\lambda_b \geq 0$ and λ_s are respectively the Lagrangian multiplier for constraints (25) and (24).[18] Because $u(\cdot)$ is concave, and the constraint is linear in the control variables, both the solution and the multiplier are unique. The first-order conditions are given by:

$$u'(c_1) = \lambda_s + \lambda_b \text{ and } \delta \mathrm{E} u'(C_2) = \rho_s \lambda_s + \rho_b \lambda_b. \tag{A9}$$

(i) Given that $L^n_{III}(c_1, C_2; \lambda_b, \lambda_s; y_1, Y_2)$ is continuous and differentiable in each argument and concave in (c_1, C_2), by a generalized envelope theorem (e.g., Milgrom and Segal, 2002), $\max_{\{c_1,C_2\}} L^n_{III}(c_1, C_2; \lambda_b, \lambda_s; y_1, Y_2)$ is continuous and differentiable in y_1. Then,

$$\frac{\partial}{\partial y_1}\left[\max_{\{c_1,C_2\}} L^n_{III}(c_1, C_2; \lambda_b, \lambda_s; y_1, Y_2)\right]$$

is continuous in $(\lambda_b, \lambda_s, y_1)$. Therefore, the value $V^n_{III}(y_1; r_b, r_s)$ is also differentiable in y_1, given that $\max_{\{c_1,C_2\}} L^n_{III}(c_1, C_2; \lambda_b, \lambda_s; y_1, Y_2)$ is convex in (λ_b, λ_s). By the envelope theorem, $\frac{\partial}{\partial y_1} V^n_{III}(y_1; r_b, r_s) = \lambda_b + \lambda_s > 0$, so $V^n_{III}(\cdot; r_b, r_s)$ is increasing.

To show the concavity of $V^n_{III}(\cdot; r_b, r_s)$, observe that the Lagrangian $L^n_{III}(c_1, C_2; \lambda_b, \lambda_s; y_1, Y_2)$ is linear in y_1. Thus for $y_1 \neq y_1'$ and $\alpha \in [0, 1]$, we have:

$$\begin{aligned}
& V^n_{III}(\alpha y_1 + (1-\alpha) y_1'; r_b, r_s) \\
= \;& \min_{\lambda_b \geq 0, \lambda_s \geq 0} \max_{\{c_1,C_2\}} \left[\alpha L^n_{III}(c_1, C_2; \lambda_b, \lambda_s; y_1, Y_2) + (1-\alpha) L^n_{III}(c_1, C_2; \lambda_b, \lambda_s; y_1', Y_2)\right] \\
\geq \;& \alpha \min_{\lambda_b \geq 0, \lambda_s \geq 0} \max_{\{c_1,C_2\}} L^n_{III}(c_1, C_2; \lambda_b, \lambda_s; y_1, Y_2) + (1-\alpha) \min_{\lambda_b \geq 0, \lambda_s \geq 0} \max_{\{c_1,C_2\}} L^n_{III}(c_1, C_2; \lambda_b, \lambda_s; y_1, Y_2) \\
= \;& \alpha V^n_{III}(y_1; r_b, r_s) + (1-\alpha) V^n_{III}(y_1'; r_b, r_s),
\end{aligned}$$

thus $V^n_{III}(\cdot; r_b, r_s)$ is concave. In particular, $V(\cdot; r)$ as defined in (14) is precisely $V^n_{III}(\cdot; r_b, r_s)$ when $r_b = r_s = r$; thus it is concave.

(ii) By the envelope theorem and the first order condition (A9), we have

$$\frac{\partial V^n_{III}(y_1; r_b, r_s)}{\partial y_1} = \lambda_b + \lambda_s = u'(c_1^*(y_1; r_b, r_s)).$$

Since we have shown in (i) that $V(\cdot; r_b, r_s)$ is concave, it follows that $u'(c_1^*(y_1; r_b, r_s))$ is decreasing in y. Thus, $c_1^*(y_1; r_b, r_s)$ is increasing in y_1.

(iii) First consider the case $c_1^*(y_1; r_b, r_s) > y_1$, i.e. when the agent is a borrower in the first period. This implies that $\lambda_s = 0$. From the first order condition (A9), we have

$$u'(c_1^*(y_1; r_b, r_s)) = \lambda_b = \frac{\delta}{\rho_b} \mathrm{E} u'\left(Y_2 + \frac{y_1 - c_1^*(y_1; r_b, r_s)}{\rho_b}\right).$$

[18] The complementarity slackness condition applies, i.e., $\lambda_k = 0$, $k = b, s$, if the corresponding constraint is slack.

Since $u'(c_1^*(y_1; r_b, r_s))$ is decreasing in y_1 and the above first order condition holds for any y_1, then $y_1 - c_1^*(y_1; r_b, r_s)$ is increasing in y_1.

Similarly, for the case $c_1^*(y_1; r_b, r_s) < y_1$, which implies that $\lambda_b = 0$, we have the first order condition

$$u'(c_1^*(y_1; r_b, r_s)) = \lambda_s = \frac{\delta}{\rho_s} \mathrm{E}u' \left(Y_2 + \frac{y_1 - c_1^*(y_1; r_b, r_s)}{\rho_s} \right).$$

Similar reasoning as above implies that $y_1 - c_1^*(y_1; r_b, r_s)$ is increasing in y_1. ■

Proof of Lemma 2:

Using the Lagrangian method we can rewrite the problems $V_{III}^w (b; y_1, r_b, r_s)$ and $V_{III}^f \left(\hat{b}; y_1, r_b, r_s \right)$ as given by (26) and (29) respectively as:

$$\begin{aligned} V_{III}^w (b; y_1, r_b, r_s) &= \min_{\lambda_b \geq 0, \lambda_s \geq 0} \max_{\{c_1, C_2\}} L_{III}^w (c_1, C_2; \lambda_b, \lambda_s; y_1 + m, Y_2 - m - b), && \text{(A10)} \\ V_{III}^f \left(\hat{b}; y_1, r_b, r_s \right) &= \min_{\lambda_b \geq 0, \lambda_s \geq 0} \max_{\{c_1, C_2\}} L_{III}^f (c_1, C_2; \lambda_b, \lambda_s; y_1 - m, Y_2 + m + \hat{b}), && \text{(A11)} \end{aligned}$$

where the Lagrangians $L_{III}^w (\cdot)$ and $L_{III}^f (\cdot)$ are defined analogous to (A8). Since both $L_{III}^w (\cdot)$ and $L_{III}^f (\cdot)$ are linear in (m, b), we can use the same argument as that in the proof of Part (i) of Lemma 1 to get:

$$V_{III}^n (y_1; r_b, r_s) \geq \frac{1}{2} \left[V_{III}^f (b; y_1, r_b, r_s) + V_{III}^w (b; y_1, r_b, r_s) \right]. \tag{A12}$$

(i) We first show that $y_1 - m \geq c_1^*(y_1; r_b, r_s)$ implies $y_1 - m \geq c_1^{*f}(y_1, mr_s; r_b, r_s)$. Suppose to the contrary, $y_1 - m < c_1^{*f}(y_1, mr_s; r_b, r_s)$, that is, an agent with income y_1 who loses the Rosca bidding in period 1 and expects to receive from the opponent a premium bid mr_s in second period will switch to become a borrower in period 1. This implies that:

$$\begin{aligned} u' \left(c_1^*(y_1; r_b, r_s) \right) &> u' \left(c_1^{*f}(y_1, mr_s; r_b, r_s) \right) = \frac{\delta}{\rho_b} \mathrm{E}u' \left(Y_2 + m + mr_s + \frac{y_1 - m - c_1^{*f}(y_1, mr_s; r_b, r_s)}{\rho_b} \right) \\ &> \frac{\delta}{\rho_s} \mathrm{E}u' \left(Y_2 + \frac{m}{\rho_s} \right) \geq \frac{\delta}{\rho_s} \mathrm{E}u' \left(Y_2 + \frac{y_1 - c_1^*(y_1; r_b, r_s)}{\rho_s} \right) = u' \left(c_1^*(y_1; r_b, r_s) \right), \end{aligned}$$

where the first inequality follows from the supposition $c_1^*(y_1; r_b, r_s) \leq y_1 - m < c_1^{*f}(y_1, mr_s; r_b, r_s)$; the first equality follows from the first order condition for a borrower since $y_1 - m < c_1^{*f}(y_1, mr_s; r_b, r_s)$; the second inequality follows from $\rho_s > \rho_b$ and $y_1 - m < c_1^{*f}(y_1, mr_s; r_b, r_s)$; the third inequality follows from $y_1 - m \geq c_1^*(y_1; r_b, r_s)$ immediately; and the last equality follows from the first order condition for a saver with income y_1 and the definition of $c_1^* (y_1; r_b, r_s)$. This is a contraction. Thus, $c_1^{*f}(y_1, mr_s; r_b, r_s) \leq y_1 - m$.

We next show that $y_1 - m \geq c_1^*(y_1; r_b, r_s)$ implies that $c_1^{*f}(y_1, \hat{b}; r_b, r_s) \leq y_1 - m$ for any $\hat{b} \in (mr_s, mr_b]$. As we have shown above, when $\hat{b} = mr_s$ the loser of the period-one Rosca bidding will remain a saver, hence the two problems, (23) and (29) with $\hat{b} = mr_s$, are identical because both the objective functions and constraints are identical; therefor,

$$V_{III}^f (mr_s; y_1, r_b, r_s) = V_{III}^n (y_1; r_b, r_s).$$

Since $V_{III}^{f}(\hat{b}; y_1, r_b, r_s)$ is increasing in $\hat{b}$, then for any $\hat{b} \in (mr_s, mr_b]$, we have

$$V_{III}^{f}(\hat{b}; y_1, r_b, r_s) > V_{III}^{f}(mr_s; y_1, r_b, r_s) = V_{III}^{n}(y_1; r_b, r_s). \tag{A13}$$

We now argue that inequality (A13) implies $c_1^{*f}(y_1, \hat{b}; r_b, r_s) \leq y_1 - m$. To see this, suppose to the contrary that $c_1^{*f}(y_1, \hat{b}; r_b, r_s) > y_1 - m$. This implies that $\lambda_s^* = 0$ at the optimal solution for problem (A11). Thus, we have:

$$\begin{aligned} L_{III}^{f}(c_1^*, C_2^*; \lambda_b^*, \lambda_s^*; y_1 - m, Y_2 + m + \hat{b}) &= L_{III}^{f}(c_1^*, C_2^*; \lambda_b^*, \lambda_s^*; y_1, Y_2) - \lambda_b^* \rho_b (mr_b - \hat{b}) \\ &\leq L_{III}^{n}(c_1, C_2; \lambda_b^*, \lambda_s^*; y_1, Y_2), \end{aligned}$$

which implies that $V_{III}^{f}(\hat{b}; y_1, r_b, r_s) \leq V_{III}^{n}(y_1; r_b, r_s)$, a contradiction.

Finally, using inequality (A12), we have $V_{III}^{f}(\hat{b}; y_1, r_b, r_s) \geq V_{III}^{f}(mr_s; y_1, r_b, r_s) = V_{III}^{n}(y_1; r_b, r_s) \geq V_{III}^{w}(\hat{b}; y_1, r_b, r_s)$.

(ii) We would like to show that $c_1^*(y_1; r_b, r_s) \geq y_1 + m$ implies that $c_1^{*w}(y_1, b; r_b, r_s) \geq y_1 + m$. Suppose to the contrary that $c_1^{*w}(y_1, b; r_b, r_s) < y_1 + m$. Then we obtain the following contradiction:

$$\begin{aligned} u'\left(c_1^*(y_1; r_b, r_s)\right) &< u'\left(c_1^{*w}(y_1, b; r_b, r_s)\right) = \frac{\delta}{\rho_s} \mathrm{E}u'\left(Y_2 - m - b + \frac{y_1 + m - c_1^{*w}(y_1, b; r_b, r_s)}{\rho_s}\right) \\ &< \frac{\delta}{\rho_b} \mathrm{E}u'\left(Y_2 - \frac{m}{\rho_b}\right) \leq u'\left(c_1^*(y_1; r_b, r_s)\right) \end{aligned}$$

where the first step follows from the supposition $c_1^{*w}(y_1, b; r_b, r_s) < y_1 + m \leq c_1^*(y_1; r_b, r_s)$; the second step follows from the first order condition for a "saver" since $c_1^{*w}(y_1, b; r_b, r_s) < y_1 + m$; the third step follows from $\rho_s > \rho_b, y_1 + m > c_1^{*w}(y_1, b; r_b, r_s)$, and $b \leq mr_b$; and the last step uses the first order condition for the case $y_1 + m \leq c_1^*(y_1; r_b, r_s)$. This is a contraction. Thus, $c_1^*(y_1; r_b, r_s) \geq y_1 + m$ implies that $c_1^{*w}(y_1, b; r_b, r_s) \geq y_1 + m$ for any $b \in [mr_s, mr_b]$.

We next show that if $c_1^*(y_1; r_b, r_s) \geq y_1 + m$, then $V_{III}^{w}(b; y_1, r_b, r_s) \geq V_{III}^{n}(y_1; r_b, r_s)$ for any $b \in [mr_s, mr_b]$. Since we have shown that a winner of the period-one Rosca bidding will remain a borrower, for $b = mr_b$, the two problems, (23) and (26) with $b = mr_b$, are in fact identical because both the objective function and the constraints are identical. Thus, we have $V_{III}^{w}(mr_b; y_1, r_b, r_s) = V_{III}^{n}(y_1; r_b, r_s)$. Since $V_{III}^{w}(b; y_1, r_b, r_s)$ is decreasing in b, we have

$$V_{III}^{w}(b; y_1, r_b, r_s) \geq V_{III}^{w}(mr_b; y_1, r_b, r_s) = V_{III}^{n}(y_1; r_b, r_s) \geq V_{III}^{f}(b; y_1, r_b, r_s)$$

where the last inequality follows from (A12). ■

Proof of Lemma 3:

(**Step 1**.) *We show that, under the restriction on Rosca size $m \leq \overline{m}(r_b, r_s)$, any agent with type $x > \underline{x}$ will be a borrower, regardless of winning or losing the period-one Rosca bidding.*

Note that the size restriction on m implies that $0 < m \leq c_1^*(\bar{y} - \underline{x}; r_b, r_s) - (\bar{y} - \underline{x})$, which in turn, by Part (iii) of Lemma 1, implies $m \leq c_1^*(y_1; r_b, r_s) - y_1$ for any $y_1 < \bar{y} - \underline{x}$.

That the winner of the period-one Rosca bidding will remain a borrower follows from Part (ii) of Lemma 2, which states that $m+y_1 \leq c_1^*(y_1;r_b,r_s)$ implies $c_1^{*w}(y_1,b;r_b,r_s) \geq y_1+m$ for any $b \in [mr_s, mr_b]$.

We now show that the loser of the period-one Rosca bidding will also be a borrower. Suppose to the contrary, then $c_1^{*f}(y_1,b;r_b,r_s) < y_1 - m$. Then, we have:

$$\begin{aligned} u'\left(c_1^*(y_1;r_b,r_s)\right) &< u'\left(c_1^{*f}(y_1,b;r_b,r_s)\right) \leq \frac{\delta}{\rho_b}\mathrm{E}u'\left(Y_2+m+b+\frac{y_1-m-c_1^{*f}(y_1,b;r_b,r_s)}{\rho_s}\right) \\ &\leq \frac{\delta}{\rho_b}\mathrm{E}u'\left(Y_2+\frac{y_1-c_1^*(y_1;r_b,r_s)}{\rho_b}\right) = u'\left(c_1^*(y_1;r_b,r_s)\right), \end{aligned}$$

where the first step follows from the supposition $c_1^*(y_1;r_b,r_s) \geq y_1 - m > c_1^{*f}(y_1,b;r_b,r_s)$; the second step follows from the first order condition for being a saver; the third step is by $m+b+\frac{y_1-m-c_1^{*f}(y_1,b;r_b,r_s)}{\rho_s} \geq 0 \geq \frac{y_1-c_1^*(y_1;r_b,r_s)}{\rho_b}$; and last step comes from the first order condition for a borrower when $c_1^*(y_1;r_b,r_s) < y_1$.

Then the indirect utility functions $V_{III}^w(\cdot;y_1,r_b,r_s)$ and $V_{III}^f(\cdot;y_1,r_b,r_s)$, as defined by (26) and (29) respectively, can be simplified as

$$V_{III}^w(b;y_1,r_b,r_s) = V\left(y_1+\rho_b(mr_b-b);r_b\right) \tag{A14}$$

$$V_{III}^f(b;y_1,r_b,r_s) = V(y_1-\rho_b(mr_b-b);r_b), \tag{A15}$$

where $V(\cdot;r)$ is defined as in (14). These simplifications allow us to rewrite the first order condition (34) as (36).

(**Step 2.**) *We show that the incentive compatibility for any type $x > \underline{x}$ is satisfied by the postulated bidding equilibrium.*

First, we show that any type $x > \underline{x}$ will not deviate to bid $b_{III}(0) = b_{III}(\underline{x}) = mr_s$. Since $b_{III}'(x) \geq 0$ by $V_{III}^w(b(x);\overline{y}-x,r_b,r_s) \geq V_{III}^f(b(x);\overline{y}-x,r_b,r_s)$ (see Part (ii) of Lemma 2), we have:

$$\begin{aligned} & U_{III}\left(x,x;r_b,r_s\right) - U_{III}\left(x,0;r_b,r_s\right) \\ = \; & p\left\{V_{III}^w(b(x);\overline{y}-x,r_b,r_s) - \frac{1}{2}\left[V_{III}^f(b(0);\overline{y}-x,r_b,r_s) + V_{III}^w(b(0);\overline{y}-x,r_b,r_s)\right]\right\} \\ & +(1-p)\left[F(x)V_{III}^w(b(x);\overline{y}-x,r_b,r_s) - \int_{\underline{x}}^{x} V_{III}^f(b(x^j);\overline{y}-x,r_b,r_s)dF(x^j)\right] \\ \geq \; & p[V_{III}^w(b(x);\overline{y}-x,r_b,r_s) - V_{III}^n(\overline{y}-x,r_b,r_s)] \\ & +(1-p)F(x)[V_{III}^w(b(x);\overline{y}-x,r_b,r_s) - V_{III}^f(b(x);\overline{y}-x,r_b,r_s)] \\ \geq \; & 0, \end{aligned}$$

where the first step follows from the expression of $U_{III}\left(x,\tilde{x};r_b,r_s\right)$ as in (33); the second step is by inequality (A12) and $b(x^j) \leq b(x)$, for $x^j \leq x$; and the last step is by $V_{III}^w(b(x);\overline{y}-x,r_b,r_s) \geq V_{III}^n(\overline{y}-x;r_b,r_s) \geq V_{III}^f(b(x);\overline{y}-x,r_b,r_s)$ that is implied by inequality (A12) again.

We next show that there is no incentive for type $x > \underline{x}$ to mimic type $\tilde{x} \neq x > \underline{x}$. By the same argument as in the proof of Proposition 1, it suffices to show $\partial^2 U_{III}\left(x,\tilde{x};r_b,r_s\right)/\partial x \partial \tilde{x} \geq 0$.

For notational convenience, denote $T_{III}(x) = \rho_b [mr_b - b_{III}(x)]$. Substituting (A14) into (33), and taking derivatives with respect to x and $\tilde{x}$, we have:

$$\frac{\partial^2 U_{III}(x, \tilde{x}; r_b, r_s)}{\partial \tilde{x} \partial x} = (1-p) f(\tilde{x}) \left[\begin{array}{c} V'(y_1 - T_{III}(\tilde{x}); r_b) - V'(y_1 + T_{III}(\tilde{x}); r_b) \\ +V''(y_1 + T_{III}(\tilde{x}); r_b) \frac{V(\tilde{y}_1 + T_{III}(\tilde{x}); r_b) - V(\tilde{y}_1 - T_{III}(\tilde{x}); r_b)}{V'(\tilde{y}_1 + T_{III}(\tilde{x}); r_b)} \end{array} \right],$$

where $y_1 = \bar{y} - x, \tilde{y}_1 = \bar{y} - \tilde{x}$; V' and V'' respectively denote the first and second order derivative with respect to y_1. In the supplemental appendix we show that the technical conditions that u exhibits decreasing absolute risk aversion and $u'''' \geq 0$ are sufficient to ensure that $\partial^2 U_{III}(x, \tilde{x}; r_b, r_s) / \partial x \partial \tilde{x} \geq 0$.[19]

(**Step 3.**) *We show that the incentive compatibility for type-0 agent is satisfied in the postulated equilibrium.*

Lemma 2 shows that $m \leq \bar{y} - c_1^*(\bar{y}; r_b, r_s)$ implies that $c_1^{*f}(\bar{y}, \hat{b}; r_b, r_s) \leq \bar{y} - m$ for any $\hat{b} \in [mr_s, mr_b]$ and that $V_{III}^f(\hat{b}; \bar{y}, r_b, r_s) \geq V_{III}^n(\bar{y}; r_b, r_s) \geq V_{III}^w(\hat{b}; \bar{y}, r_b, r_s)$. Therefore, type-0 bidder is a saver regardless of winning or losing the period-one Rosca bidding and it does not have strict incentives to deviate to bid any $\tilde{b} > mr_s = b_{III}(\underline{x})$. ■

Proof of Proposition 4:

We showed in the Proof of Lemma 3 that the size restriction on $m \leq \overline{m} \equiv \min\{c_1^*(\bar{y} - \underline{x}) - (\bar{y} - \underline{x}), \bar{y} - c_1^*(\bar{y}; r_b, r_s)\}$ implies that, regardless of winning or losing the period-one Rosca bidding, the bidder will be a saver if $x = 0$ and a borrower if $x > \underline{x}$. We have, for type-0 agent,

$$\begin{aligned} & U_{III}(0, 0; r_b, r_s) = \frac{p}{2} \left[V_{III}^f(b_{III}(0); \bar{y}, r_b, r_s) + V_{III}^w(b_{III}(0); \bar{y}, r_b, r_s)) \right] \\ & +(1-p) \int_{\underline{x}}^{\bar{x}} V_{III}^f(b_{III}(x^j); \bar{y}, r_b, r_s) dF(x^j) \\ > \; & \frac{p}{2} \left[V_{III}^f(b_{III}(0); \bar{y}, r_b, r_s) + V_{III}^w(b_{III}(0); \bar{y}, r_b, r_s)) \right] + (1-p) V_{III}^f(b_{III}(0); \bar{y}, r_b, r_s) \quad \text{(A16)} \\ = \; & pV(\bar{y}; r_s) + (1-p) V(\bar{y}; r_s) = V(\bar{y}; r_s) = V_{III}^n(\bar{y}; r_b, r_s), \end{aligned}$$

where the first step is simply the definition of $U_{III}(0, 0; r_b, r_s)$, and the second step follows from the fact that $b'_{III}(\cdot) > 0$ and $V_{III}^f(\cdot; \bar{y}, r_b, r_s)$ is increasing; the third step uses the fact that $b_{III}(0) = mr_s$ implies that $V_{III}^f(b_{III}(0); \bar{y}, r_b, r_s) = V_{III}^w(b_{III}(0); \bar{y}, r_b, r_s)) = V(\bar{y}; r_s)$; and the last step follows the fact that type-0 agent is a saver in the absence of Rosca.

For agents with type-$x > \underline{x}$, we have:

$$\begin{aligned} U_{III}(x, x; r_b, r_s) \; & \geq \; U_{III}(x, \bar{x}; r_b, r_s) = V_{III}^w(b_{III}(\bar{x}); \bar{y} - x, r_b, r_s) = V(\bar{y} - x + \rho_b(mr_b - b_{III}(\bar{x})); r_b) \\ & \geq \; V(\bar{y} - x; r_b) = V_{III}^n(\bar{y} - x; r_b, r_s), \quad \text{(A17)} \end{aligned}$$

where the first step follows from the incentive compatibility constraint that type-x bidder does not have strict incentive to pretend as type-$\bar{x}$; the second step follows from the fact that type$-\bar{x}$ agent will always win the period-one Rosca bidding because $b'_{III} > 0$; the third step follows from the fact

[19] The supplemental appendix is available at http://www.econ.upenn.edu/˜hfang.

that type-$\bar{x}$ agent will be a borrower despite winning the Rosa bidding; the fourth step uses the fact that $b_{III}(\bar{x}) \leq mr_b$; and the last step follows from the fact that type-$\bar{x}$ agent is a borrower in the absence of Rosca.

Inequalities (A16) and (A17) show that under the equilibrium bidding strategy specified in Lemma 3, agents of all types are interim individually rational. The *ex ante* welfare gain $\mathrm{E}U_{III}(X, X; r_b, r_s)$ $- \mathrm{E}V_{III}^n(\bar{y} - X; r_b, r_s) > 0$ immediately follows. ■

Proof of Corollary 1:

For CARA utility, the first order condition for the optimal consumption $c_1^*(\cdot)$ in Problem (14) is given by:

$$e^{-\beta c_1} = \delta(1+r)\mathrm{E}\exp\left[-\beta\left(Y_2 + \frac{y_1 - c_1}{\rho}\right)\right],$$

which, after some algebra, yields:

$$c_1^*(y_1) = \frac{1+r}{2+r}y_1 - \frac{\ln\left[(1+r)\,\delta K\right]}{\beta(2+r)}, \tag{A18}$$

where $K \equiv E\exp\left(-\beta Y_2\right)$ is a constant that equals to the expected marginal utility of period-2's income. Therefore, the value function $V\left(y_1; r\right)$ is:

$$V\left(y_1; r\right) = \frac{1+\delta}{\beta} - \frac{1}{\beta}\eta(r)\exp\left[-\frac{\beta\left(1+r\right)}{2+r}y_1\right], \tag{A19}$$

where for notational convenience we denote

$$\eta(r) \equiv \frac{2+r}{1+r}\left[(1+r)\,\delta K\right]^{\frac{1}{2+r}}. \tag{A20}$$

Plugging (A19) into the differential equation (36) in Lemma 3, we obtain:

$$\frac{\beta b_{III}'(x)}{2+r_b} = \frac{(1-p)\,f\left(x\right)\left\{\exp\left[\frac{2\beta[mr_b - b_{III}(x)]}{2+r_b}\right] - 1\right\}}{p+(1-p)F(x)}. \tag{A21}$$

With the following change of variables:

$$h(x) \equiv \frac{2\beta\left[mr_b - b_{III}\left(x\right)\right]}{2+r_b}, \tag{A22}$$

we can re-write the differential equation (A21) as:

$$h'\left(x\right) = \frac{2\left(1-p\right)f\left(x\right)}{p+(1-p)F(x)}\left\{1 - \exp\left[h\left(x\right)\right]\right\}, \tag{A23}$$

The above differential equation has a general solution:

$$h\left(x\right) = -\ln\left\{1 - \omega\left[\frac{p}{p+\left(1-p\right)F\left(x\right)}\right]^2\right\}, \tag{A24}$$

where ω is an integration constant that is determined from the boundary condition $b_{III}(\underline{x}) = mr_s$, which requires

$$h(\underline{x}) = \frac{2\beta m(r_b - r_s)}{2 + r_b}. \tag{A25}$$

Thus the integration constant ω is given by:

$$\omega = 1 - \exp[-h(\underline{x})] = 1 - \exp\left(-\frac{2\beta m(r_b - r_s)}{2 + r_b}\right). \tag{A26}$$

We can then solve $b_{III}(x)$ from (A24) to obtain (37). The necessary size restriction $\bar{m}$ for this example follows immediately from $c_1^*(\cdot)$. We verify the sufficiency of the first order condition for the bidding equilibrium in the supplemental appendix.[20] ■

Proof of Proposition 5:

The inequalities directly follow from inspecting the admissible sets defined by inequalities in Problem (23), and the fact that all the value functions considered are the solutions to the special cases of the general Problem (23). ■

Proof of Proposition 6:

Recall the expression for $\mathrm{E}[U_I(X,X)\,|X > \underline{x}]$ as given by (A3). Under the assumption that $p = 0$, (A3) can be simplified as:

$$\begin{aligned} \mathrm{E}[U_I(X,X)\,|X > \underline{x}] &= \int_{\underline{x}}^{\overline{x}} F(x)\left[u(\bar{y} - x + m) + \delta \mathrm{E}u(Y_2 - m - b_I(x))\right] dF(x) \\ &\quad + \int_{\underline{x}}^{\overline{x}} [1 - F(x)]u(\bar{y} - x - m)dF(x) + \int_{\underline{x}}^{\overline{x}} F(x)\delta \mathrm{E}u(Y_2 + m + b_I(x))dF(x). \end{aligned} \tag{A27}$$

By Jensen's inequality, we have

$$\begin{aligned} \int_{\underline{x}}^{\overline{x}} F(x)\delta \mathrm{E}u(Y_2 + m + b_I(x))dF(x) &= \frac{1}{2}\delta \int_{\underline{x}}^{\overline{x}} \mathrm{E}u\left(Y_2 + m + b_I(x)\right) dF(x)^2 \qquad \text{(A28)} \\ &\leq \frac{1}{2}\delta \mathrm{E}u\left(Y_2 + m + \int_{\underline{x}}^{\overline{x}} b_I(x)dF(x)^2\right), \\ \int_{\underline{x}}^{\overline{x}} F(x)\delta \mathrm{E}u(Y_2 - m - b_I(x))dF(x) &= \frac{1}{2}\delta \int_{\underline{x}}^{\overline{x}} \mathrm{E}u\left(Y_2 - m - b_I(x)\right) dF(x)^2 \qquad \text{(A29)} \\ &\leq \frac{1}{2}\delta \mathrm{E}u\left(Y_2 - m - \int_{\underline{x}}^{\overline{x}} b_I(x)dF(x)^2\right). \end{aligned}$$

Concavity of $u(\cdot)$ and the assumption that $mr \leq \int_{\underline{x}}^{\overline{x}} b_I(x)\,dF(x)^2$ implies

$$\mathrm{E}u\left(Y_2 + m + \int_{\underline{x}}^{\overline{x}} b_I(x)dF(x)^2\right) + \mathrm{E}u\left(Y_2 - m - \int_{\underline{x}}^{\overline{x}} b_I(x)dF(x)^2\right) \leq \mathrm{E}u(Y_2 + m + mr) + \mathrm{E}u(Y_2 - m - mr). \tag{A30}$$

[20] The supplemental appendix is available at http://www.econ.upenn.edu/~hfang.

Applying inequalities (A28)-(A30) to (A27), we obtain:

$$\begin{aligned}
\mathrm{E}[U_I(X,X)\,|X>\underline{x}] &\leq \int_{\underline{x}}^{\overline{x}} F(x)\left[u(\bar{y}-x+m)+\delta \mathrm{E}u(Y_2-m-mr)\right]dF(x) \\
&\quad +\int_{\underline{x}}^{\overline{x}} [1-F(x)]\left[u(\bar{y}-x-m)+\delta \mathrm{E}u(Y_2+m+mr)\right]dF(x) \\
&= \int_{\underline{x}}^{\overline{x}} \left\{F(x)V_I^w(mr;\bar{y}-x)+[1-F(x)]V_I^f(mr;\bar{y}-x)\right\}dF(x) \\
&\leq \int_{\underline{x}}^{\overline{x}} V(\bar{y}-x;r)dF(x)=\mathrm{E}V_{II}^n\left(\bar{y}-X;r\right),
\end{aligned}$$

where the last inequality follows from $\max\left\{V_I^w(mr;\bar{y}-x),V_I^f(mr;\bar{y}-x)\right\}\leq V(\bar{y}-x;r)$. Thus,

Now we show that $\rho=1/\left(1+r\right)\geq\delta$ is a sufficient condition for $mr\leq\int_{\underline{x}}^{\overline{x}} b_I\left(x\right)dF(x)^2$. We first show that the bidding equilibrium $b_I\left(\cdot\right)$ satisfies:

$$\int_{\underline{x}}^{\overline{x}} b_I\left(x\right)dF(x)\geq\frac{1-\delta}{\delta}m. \tag{A31}$$

Inequality (A31), together with the assumption that $\rho=1/\left(1+r\right)\geq\delta$, implies that:

$$\int_{\underline{x}}^{\overline{x}} b_I\left(x\right)dF(x)^2>\int_{\underline{x}}^{\overline{x}} b_I\left(x\right)dF(x)\geq\frac{1-\delta}{\delta}m\geq mr,$$

as desired.

We prove (A31) by contradiction. Suppose, to the contrary, that $\int_{\underline{x}}^{\overline{x}} b_I\left(x\right)dF(x)<\left(1-\delta\right)m/\delta$. Now multiply $F(x)(1-F(x))\delta\mathrm{E}u'\left(Y_2-m-b_I(x)\right)$ on both sides of Eq. (6), and integrate both sides over $\left[\underline{x},\bar{x}\right]$, we obtain

$$\int_{\underline{x}}^{\overline{x}} F(x)[1-F(x)]\delta\mathrm{E}u'(Y_2-m-b_I(x))b_I'(x)dx=\int_{\underline{x}}^{\overline{x}}[1-F(x)][V_I^w(b_I(x);\bar{y}-x)-V_I^f(b_I(x);\bar{y}-x)]dF(x). \tag{A32}$$

Using integration by parts, the left hand side of (A32) becomes:

$$\int_{\underline{x}}^{\overline{x}} F(x)[1-F(x)]\delta\mathrm{E}u'(Y_2-m-b_I(x))b_I'(x)dx=\int_{\underline{x}}^{\overline{x}}\delta\mathrm{E}u(Y_2-m-b_I(x))[1-2F(x)]dF(x).$$

Plugging in the expressions for $V_I^w(b_I(x);\bar{y}-x)$ and $V_I^f(b_I(x);\bar{y}-x)$ as give by (2) and (3), (A32) can be simplified as:

$$\begin{aligned}
&\int_{\underline{x}}^{\overline{x}}\delta\mathrm{E}u\left(Y_2-m-b_I(x)\right)F(x)dF(x) \qquad \text{(A33)}\\
=&\int_{\underline{x}}^{\overline{x}}\delta\mathrm{E}u(Y_2+m+b_I(x))[1-F(x)]dF(x)-\int_{\underline{x}}^{\overline{x}}[1-F(x)][u(\bar{y}-x+m)-u(\bar{y}-x-m)]dF(x).
\end{aligned}$$

Plugging (A33) into (A27), we have

$$
\begin{aligned}
& \mathrm{E}[U_I(X,X)\,|X>\underline{x}] \\
= \ & \int_{\underline{x}}^{\overline{x}} \delta \mathrm{E}u\left(Y_2+m+b_I(x)\right) dF(x) + \int_{\underline{x}}^{\overline{x}} 2[1-F(x)]u(\bar{y}-x-m)dF(x) + \int_{\underline{x}}^{\overline{x}} [2F(x)-1]u(\bar{y}-x+m)dF(x) \\
< \ & \underbrace{\delta \mathrm{E}u(Y_2+\frac{m}{\delta}) + \int_{\underline{x}}^{\overline{x}} 2[1-F(x)]u(\bar{y}-x-m)dF(x) + \int_{\underline{x}}^{\overline{x}} [2F(x)-1]u(\bar{y}-x+m)dF(x)}_{\varphi(m)}, \qquad \text{(A34)}
\end{aligned}
$$

where the last inequality follows from Jensen's inequality

$$\int_{\underline{x}}^{\overline{x}} \mathrm{E}u\left(Y_2+m+b_I(x)\right) dF(x) \le \mathrm{E}u\left(Y_2+m+\int_{\underline{x}}^{\overline{x}} b_I\left(x\right) dF(x)\right)$$

and the supposition that $\int_{\underline{x}}^{\overline{x}} b_I\left(x\right) dF(x) < \left(1-\delta\right) m/\delta$.

Now consider the term $\varphi(m)$ as defined in (A34). Note that $\varphi(0) = \mathrm{E}V_I^n(\bar{y}-X)$; and recall from Proposition 2 that $d\Delta W^I(m)/dm\big|_{m=0} > 0$, which implies that $\varphi(\varepsilon) > \varphi(0)$ for a small $\varepsilon > 0$. However, we have the following two observations:

$$\varphi'(0) = \mathrm{E}u'(Y_2) - \int_{\underline{x}}^{\overline{x}} u'(\bar{y}-x)dF(x) = 0$$

where the second equality follows from the fact that $Y_2 = \bar{y} - X$ by i.i.d. assumption; and

$$\varphi''(0) = \frac{1}{\delta}\mathrm{E}u''(Y_2) + \int_{\underline{x}}^{\overline{x}} u''(\bar{y}-x)dF(x) < 0.$$

This is a contradiction to $\varphi(\varepsilon) > \varphi(0)$. Thus we establish (A31) and the result follows. ■

Proof of Proposition 7:

First, we derive an expression for $U_{III}(x,x;r_b,r_s)$ with $x > \underline{x}$. Under the restrictions on Rosca size, we know that any agent with type $x > \underline{x}$ agent is a borrower regardless of losing or winning the first period Rosca bidding. Thus we have, for $x > \underline{x}$,

$$
\begin{aligned}
& U_{III}(x,x;r_b,r_s) \overset{(1)}{=} [p+(1-p)F(x)]V\left(\bar{y}-x+\rho_b(mr_b-b_{III}(x));r_b\right) \\
& +(1-p)\int_{x}^{\bar{x}} V\left(\bar{y}-x-\rho_b(mr_b-b_{III}(x^j));r_b\right) dF(x^j) \\
& \overset{(2)}{=} \frac{1+\delta}{\beta} - \frac{1}{\beta}\eta(r_b)\exp\left[-\frac{\beta\left(1+r_b\right)}{2+r_b}(\bar{y}-x)\right] \times \\
& \left\{[p+(1-p)F(x)]\exp\left[-\frac{\beta(mr_b-b_{III}(x))}{2+r_b}\right] + (1-p)\int_{x}^{\bar{x}} \exp\left[-\frac{\beta(mr_b-b_{III}(x^j))}{2+r_b}\right] dF(x^j)\right\} \\
& \overset{(3)}{=} \frac{1+\delta}{\beta} - \frac{1}{\beta}\eta(r_b)\exp\left[-\frac{\beta\left(1+r_b\right)}{2+r_b}(\bar{y}-x)\right] \times \\
& \left\{[p+(1-p)F(x)]\exp\left[-\frac{h\left(x\right)}{2}\right] + (1-p)\int_{x}^{\bar{x}} \exp\left[-\frac{h\left(x^j\right)}{2}\right] dF(x^j)\right\} \\
& \overset{(4)}{=} \frac{1+\delta}{\beta} - \frac{1}{\beta}\eta(r_b)\exp\left[-\frac{\beta\left(1+r_b\right)}{2+r_b}(\bar{y}-x)\right]\sqrt{1-\omega p^2} \qquad \text{(A35)}
\end{aligned}
$$

where the first equality follows from the definition of $U_{III}(x, x; r_b, r_s)$ and the fact that type-x is always a borrower under the assumed restriction on Rosca size m; the second equality follows from substituting the expression $V(y; r)$ for CARA utility function as defined in formula (A19); the third equality simply uses the definition of the change of variable function $h(x)$ as defined in (A22); finally, the fourth equality is a result of the following two calculations. The first calculation is a direct result of (A24):

$$\exp\left[-\frac{h(x)}{2}\right] = \sqrt{1 - \omega\left[\frac{p}{p + (1-p)F(x)}\right]^2}, \tag{A36}$$

where ω a constant given in (A26). The second calculation is:

$$\begin{aligned}(1-p)\int_{\underline{x}}^{\bar{x}} \exp\left[-\frac{h(x^j)}{2}\right] dF(x^j) &= (1-p)\int_{\underline{x}}^{\bar{x}} \sqrt{1 - \omega\left[\frac{p}{p + (1-p)F(x^j)}\right]^2} dF(x^j) \\ &= \int_{[p+(1-p)F(x)]^2}^{1} \frac{d\tau}{2\sqrt{\tau - \omega p^2}} \\ &= \sqrt{\tau - \omega p^2}\Big|_{[p+(1-p)F(x)]^2}^{1} = \sqrt{1-\omega p^2} - \sqrt{[p + (1-p)F(x)]^2 - \omega p^2},\end{aligned}$$

where the second equality follows from a change-of-variable

$$\tau \equiv [p + (1-p)F(x^j)]^2. \tag{A37}$$

We next derive an expression for $U_{III}(0, 0; r_b, r_s)$. Under the assumed restriction on Rosca size m, a type-0 agent with income $\bar{y}$ will be a saver regardless of winning or losing the period one Rosca bidding, and as shown in Corollary 1, $b_{III}(0) = mr_s$; thus we have:

$$U_{III}(0, 0; r_b, r_s) = pV(\bar{y}; r_s) + (1-p)\int_{\underline{x}}^{\bar{x}} V(\bar{y} + \rho_s(b_{III}(x) - mr_s); r_s)dF(x). \tag{A38}$$

Evaluating the integral in the second term of the above expression, we get:

$$\begin{aligned}&\int_{\underline{x}}^{\bar{x}} V(\bar{y} + \rho_s(b_{III}(x) - mr_s); r_s)dF(x) \\ &\stackrel{(1)}{=} \frac{1+\delta}{\beta} - \frac{1}{\beta}\eta(r_s)\exp\left[-\frac{\beta(1+r_s)}{2+r_s}\bar{y}\right]\int_{\underline{x}}^{\bar{x}} \exp\left[-\frac{\beta(b_{III}(x) - mr_s)}{2+r_s}\right] dF(x) \\ &\stackrel{(2)}{=} \frac{1+\delta}{\beta} - \frac{1}{\beta}\eta(r_s)\exp\left[-\frac{\beta(1+r_s)}{2+r_s}\bar{y}\right]\exp\left[\frac{\beta m(r_s - r_b)}{2+r_s}\right]\int_{\underline{x}}^{\bar{x}} \exp\left[-\frac{2+r_b}{2+r_s}\frac{\beta(b_{III}(x) - mr_b)}{2+r_b}\right] dF(x) \\ &\stackrel{(3)}{=} \frac{1+\delta}{\beta} - \frac{1}{\beta}\eta(r_s)\exp\left[-\frac{\beta(1+r_s)}{2+r_s}\bar{y}\right]\exp\left[\frac{\beta m(r_s - r_b)}{2+r_s}\right]\int_{\underline{x}}^{\bar{x}} \exp\left[-\frac{h(x)}{2}\right]^{-\frac{2+r_b}{2+r_s}} dF(x) \\ &\stackrel{(4)}{=} \frac{1+\delta}{\beta} - \frac{1}{\beta}\eta(r_s)\exp\left[-\frac{\beta(1+r_s)}{2+r_s}\bar{y}\right]\exp\left[\frac{\beta m(r_s - r_b)}{2+r_s}\right]\int_{\underline{x}}^{\bar{x}} \left\{1 - \omega\left[\frac{p}{p+(1-p)F(x)}\right]^2\right\}^{-\frac{2+r_b}{2(2+r_s)}} dF(x) \\ &\stackrel{(5)}{=} \frac{1+\delta}{\beta} - \frac{1}{\beta(1-p)}\eta(r_s)\exp\left[-\frac{\beta(1+r_s)}{2+r_s}\bar{y}\right]\exp\left[\frac{\beta m(r_s - r_b)}{2+r_s}\right]\int_{p}^{1} \left[1 - \omega\left(\frac{p}{\xi}\right)^2\right]^{-\frac{2+r_b}{2(2+r_s)}} d\xi,\end{aligned} \tag{A39}$$

where the first equality follows from substituting the expression $V(y;r)$ for CARA utility function as defined in formula (A19); the second equality follows from re-arranging terms; the third equality simply uses the definition of the change of variable function $h(x)$ as defined in (A22); the fourth equality uses the expression (A36); and the last equality uses the change of variable

$$\xi \equiv p + (1-p)F(x).$$

Plugging (A39) into (A38), we obtain:

$$\begin{aligned} U_{III}(0,0;r_b,r_s) &= \frac{1+\delta}{\beta} - \frac{p}{\beta}\eta(r_s)\exp\left[-\frac{\beta(1+r_s)}{2+r_s}\bar{y}\right] \\ &-\frac{1}{\beta}\eta(r_s)\exp\left[-\frac{\beta(1+r_s)}{2+r_s}\bar{y}\right]\exp\left[\frac{\beta m(r_s-r_b)}{2+r_s}\right]\int_p^1\left[1-\omega\left(\frac{p}{\xi}\right)^2\right]^{-\frac{2+r_b}{2(2+r_s)}}d\xi. \end{aligned} \tag{A40}$$

Using (A35) and (A40), we can write the *ex ante* expected welfare of agents in the presence of the Rosca and imperfect credit market, $\mathrm{E}U_{III}(X,X;r_b,r_s)$, as follows:

$$\begin{aligned} &\mathrm{E}U_{III}(X,X;r_b,r_s) = pU_{III}(0,0;r_b,r_s) + (1-p)\mathrm{E}[U_{III}(X,X;r_b,r_s)|\, X > \underline{x}] \\ = \; &\frac{1+\delta}{\beta} - \frac{p^2}{\beta}\underbrace{\eta(r_s)\exp\left[-\frac{\beta(1+r_s)}{2+r_s}\bar{y}\right]}_{\text{Term A}} \\ &-\frac{p}{\beta}\underbrace{\eta(r_s)\exp\left[-\frac{\beta(1+r_s)}{2+r_s}\bar{y}\right]}_{\text{Term A}}\underbrace{\exp\left[\frac{\beta m(r_s-r_b)}{2+r_s}\right]\int_p^1\left[1-\omega\left(\frac{p}{\xi}\right)^2\right]^{-\frac{2+r_b}{2(2+r_s)}}d\xi}_{\text{Term B}} \\ &-\frac{1-p}{\beta}\eta(r_b)\underbrace{\sqrt{1-\omega p^2}}_{\text{Term C}}\int_{\underline{x}}^{\bar{x}}\exp\left[-\frac{\beta(1+r_b)}{2+r_b}(\bar{y}-x)\right]dF(x). \end{aligned} \tag{A41}$$

We now evaluate

$$\lim_{r_s\to r_b}\frac{\partial}{\partial r_s}\mathrm{E}U_{III}(X,X;r_b,r_s) \tag{A42}$$

and show that under the stated conditions in the proposition, it is negative. To evaluate (A42), we first note that, in (A41), r_s shows up in what we labeled as "Term A", "Term B" and "Term C" (via the term ω as shown in (A26)).

For Term A, we can show by simple but tedious algebra that:

$$\frac{\partial}{\partial r_s}(\text{Term A}) = -\frac{1}{2+r_s}\eta(r_s)\exp\left[-\frac{\beta(1+r_s)}{2+r_s}\bar{y}\right]\left\{\frac{\beta\bar{y}}{2+r_s}+\frac{\ln\left[(1+r_s)\,\delta K\right]}{2+r_s}\right\}. \tag{A43}$$

For Term C, we have:

$$\begin{aligned} \lim_{r_s\to r_b}\frac{\partial}{\partial r_s}(\text{Term C}) &= \lim_{r_s\to r_b}\left[-\frac{1}{2}\frac{p^2}{\sqrt{1-\omega p^2}}\frac{\partial\omega}{\partial r_s}\right] \\ &= \frac{\beta m p^2}{2+r_b}. \end{aligned} \tag{A44}$$

where the second equality follows from [recalling of the definition of ω as given in (A26)],

$$\begin{aligned}
\lim_{r_s \to r_b} \omega &= 0; \\
\lim_{r_s \to r_b} \frac{\partial \omega}{\partial r_s} &= \lim_{r_s \to r_b} \frac{\partial}{\partial r_s} \left[1 - \exp\left(-\frac{2\beta m(r_b - r_s)}{2 + r_b} \right) \right] \qquad \text{(A45)} \\
&= \lim_{r_s \to r_b} \left[-\frac{2\beta m}{2 + r_b} \exp\left(-\frac{2\beta m(r_b - r_s)}{2 + r_b} \right) \right] \\
&= -\frac{2\beta m}{2 + r_b}. \qquad \text{(A46)}
\end{aligned}$$

For Term B, we have

$$\lim_{r_s \to r_b} (\text{Term B}) = \int_p^1 d\xi = 1 - p; \qquad \text{(A47)}$$

moreover,

$$\begin{aligned}
&\lim_{r_s \to r_b} \frac{\partial\,(\text{Term B})}{\partial r_s} \overset{(1)}{=} \lim_{r_s \to r_b} \frac{\partial}{\partial r_s} \left\{ \exp\left[\frac{\beta m(r_s - r_b)}{2 + r_s} \right] \int_p^1 \left[1 - \omega \left(\frac{p}{\xi} \right)^2 \right]^{-\frac{2+r_b}{2(2+r_s)}} d\xi \right\} \\
&\overset{(2)}{=} \lim_{r_s \to r_b} \frac{\partial}{\partial r_s} \exp\left[\frac{\beta m(r_s - r_b)}{2 + r_s} \right] \cdot \lim_{r_s \to r_b} \int_p^1 \left[1 - \omega \left(\frac{p}{\xi} \right)^2 \right]^{-\frac{2+r_b}{2(2+r_s)}} d\xi \\
&+ \lim_{r_s \to r_b} \exp\left[\frac{\beta m(r_s - r_b)}{2 + r_s} \right] \cdot \lim_{r_s \to r_b} \frac{\partial}{\partial r_s} \left\{ \int_p^1 \left[1 - \omega \left(\frac{p}{\xi} \right)^2 \right]^{-\frac{2+r_b}{2(2+r_s)}} d\xi \right\} \\
&\overset{(3)}{=} \frac{\beta m(1-p)}{2 + r_b} + \int_p^1 \left\{ \lim_{r_s \to r_b} \left[1 - \omega \left(\frac{p}{\xi} \right)^2 \right]^{-\frac{2+r_b}{2(2+r_s)}} \right\} \cdot \left\{ \lim_{r_s \to r_b} \frac{\partial}{\partial r_s} \left(-\frac{2 + r_b}{2(2 + r_s)} \ln \left[1 - \omega \left(\frac{p}{\xi} \right)^2 \right] \right) \right\} d\xi \\
&\overset{(4)}{=} \frac{\beta m(1-p)}{2 + r_b} - \frac{\beta m p(1-p)}{2 + r_b} \\
&\overset{(5)}{=} \frac{\beta m(1-p)^2}{2 + r_b}. \qquad \text{(A48)}
\end{aligned}$$

where the third equality uses the facts that

$$\begin{aligned}
\lim_{r_s \to r_b} \frac{\partial}{\partial r_s} \exp\left[\frac{\beta m(r_s - r_b)}{2 + r_s} \right] &= \frac{\beta m}{2 + r_b}, \\
\lim_{r_s \to r_b} \int_p^1 \left[1 - \omega \left(\frac{p}{\xi} \right)^2 \right]^{-\frac{2+r_b}{2(2+r_s)}} d\xi &= \int_p^1 d\xi = 1 - p;
\end{aligned}$$

and the fourth equality follows from

$$\begin{aligned}
&\int_p^1 \left\{ \lim_{r_s \to r_b} \frac{\partial}{\partial r_s} \left(-\frac{2 + r_b}{2(2 + r_s)} \ln \left[1 - \omega \left(\frac{p}{\xi} \right)^2 \right] \right) \right\} d\xi \\
&= \int_p^1 \left[\frac{1}{2} \left(\frac{p}{\xi} \right)^2 \left(\lim_{r_s \to r_b} \frac{\partial \omega}{\partial r_s} \right) \right] d\xi \\
&= -\frac{\beta m}{2 + r_b} \int_p^1 \left(\frac{p}{\xi} \right)^2 d\xi = -\frac{\beta m p(1-p)}{2 + r_b}.
\end{aligned}$$

Applying equalities (A43)-(A48), we obtain

$$
\begin{aligned}
& \lim_{r_s \to r_b} \frac{\partial}{\partial r_s} \mathrm{E}U_{III}(X, X; r_b, r_s) \\
= \; & -\frac{p^2}{\beta}\left[\lim_{r_s \to r_b} \frac{\partial}{\partial r_s}(\text{Term A})\right] - \frac{p}{\beta}\left[\lim_{r_s \to r_b} \frac{\partial}{\partial r_s}(\text{Term A})\right]\left[\lim_{r_s \to r_b}(\text{Term B})\right] \\
& -\frac{p}{\beta}\left[\lim_{r_s \to r_b}(\text{Term A})\right]\left[\lim_{r_s \to r_b} \frac{\partial}{\partial r_s}(\text{Term B})\right] \\
& -\frac{1-p}{\beta}\eta(r_b)\left[\lim_{r_s \to r_b} \frac{\partial}{\partial r_s}(\text{Term C})\right]\int_{\underline{x}}^{\bar{x}} \exp\left[-\frac{\beta(1+r_b)}{2+r_b}(\bar{y}-x)\right] dF(x) \\
= \; & \frac{p^2}{\beta(2+r_b)}\eta(r_b)\exp\left[-\frac{\beta(1+r_b)}{2+r_b}\bar{y}\right]\left(\frac{\beta\bar{y}}{2+r_b} + \frac{\ln[(1+r_b)\delta K]}{2+r_b}\right) \\
& +\frac{p(1-p)}{\beta(2+r_b)}\eta(r_b)\exp\left[-\frac{\beta(1+r_b)}{2+r_b}\bar{y}\right]\left(\frac{\beta\bar{y}}{2+r_b} + \frac{\ln[(1+r_b)\delta K]}{2+r_b}\right) \\
& -\frac{p}{\beta}\eta(r_b)\exp\left[-\frac{\beta(1+r_b)}{2+r_b}\bar{y}\right]\left[\frac{\beta m}{(2+r_b)}(1-p)^2\right] \\
& -\frac{1-p}{\beta}\eta(r_b)\frac{p^2\beta m}{2+r_b}\int_{\underline{x}}^{\bar{x}} \exp\left[-\frac{\beta(1+r_b)}{2+r_b}(\bar{y}-x)\right] dF(x) \\
= \; & \frac{\eta(r_b)pm}{2+r_b}\exp\left[-\frac{\beta(1+r_b)}{2+r_b}\bar{y}\right] \times \\
& \left\{\frac{\frac{\bar{y}}{2+r_b} + \frac{\ln[(1+r_b)\delta K]}{\beta(2+r_b)}}{m} - \left[(1-p)^2 + (1-p)p\int_{\underline{x}}^{\bar{x}} \exp\left[\frac{\beta(1+r_b)}{2+r_b}x\right] dF(x)\right]\right\} \\
= \; & \frac{\eta(r_b)pm}{2+r_b}\exp\left[-\frac{\beta(1+r_b)}{2+r_b}\bar{y}\right] \times \\
& \left\{\frac{\bar{y} - c_1^*(\bar{y}, r_b)}{m} - \left[(1-p)^2 + (1-p)p\int_{\underline{x}}^{\bar{x}} \exp\left[\frac{\beta(1+r_b)}{2+r_b}x\right] dF(x)\right]\right\},
\end{aligned}
$$

where the last equality follows from (A18). Therefore, $\lim_{r_s \to r_b} \frac{\partial}{\partial r_s} \mathrm{E}U_{III}(X, X; r_b, r_s) < 0$ if (40) holds. ■

B Supplemental Appendix

B.1 Verifying the Sufficiency of the First Order Condition in Lemma 3

In this supplemental appendix, we show that the technical assumptions in Lemma 3 guarantees the sufficiency of the first order condition. It suffices to show the following lemma:

Lemma B1 *Assume that $-u''(\cdot)/u'(\cdot)$ is weakly decreasing and $u'''' \geq 0$, then $\partial^2 U_{III}(x,\tilde{x};r_b,r_s)/\partial\tilde{x}\partial x \geq 0$ for any $x > \underline{x}$ and $\tilde{x} > \underline{x}$.*

Substituting (A14) into (33), and taking derivatives with respect to x and $\tilde{x}$, we have:

$$\frac{\partial^2 U_{III}(x,\tilde{x};r_b,r_s)}{\partial\tilde{x}\partial x} = (1-p)f(\tilde{x})\left[\begin{array}{c} V'\left(y_1 - T_{III}(\tilde{x});r_b\right) - V'\left(y_1 + T_{III}(\tilde{x});r_b\right) \\ +V''\left(y_1 + T_{III}(\tilde{x});r_b\right)\frac{V(\tilde{y}_1+T_{III}(\tilde{x});r_b)-V(\tilde{y}_1-T_{III}(\tilde{x});r_b)}{V'(\tilde{y}_1+T_{III}(\tilde{x});r_b)}\end{array}\right],$$

where $y_1 = \overline{y} - x, \tilde{y}_1 = \overline{y} - \tilde{x}$; V' and V'' respectively denote the first and second order derivative with respect to y_1. For convenience, we also suppress argument r_b in the value function $V(y;r_b)$, so $V(\cdot)$ is a shortcut for $V(\cdot;r_b)$. To show $\partial^2 U_{III}(x,\tilde{x};r_b,r_s)/\partial\tilde{x}\partial x \geq 0$, it suffices to show

$$V'\left(y_1 - T_{III}(\tilde{x})\right) - V'\left(y_1 + T_{III}(\tilde{x})\right) + V''\left(y_1 + T_{III}(\tilde{x})\right)\frac{V\left(\tilde{y}_1 + T_{III}(\tilde{x})\right) - V\left(\tilde{y}_1 - T_{III}(\tilde{x})\right)}{V'\left(\tilde{y}_1 + T_{III}(\tilde{x})\right)} \geq 0,$$

which can be rewritten as

$$\frac{V'\left(y_1 - T_{III}(\tilde{x})\right) - V'\left(y_1 + T_{III}(\tilde{x})\right)}{V''\left(y_1 + T_{III}(\tilde{x})\right)} + \frac{V\left(\tilde{y}_1 + T_{III}(\tilde{x})\right) - V\left(\tilde{y}_1 - T_{III}(\tilde{x})\right)}{V'\left(\tilde{y}_1 + T_{III}(\tilde{x})\right)} \leq 0, \tag{B49}$$

since $V''\left(y_1 + T_{III}(\tilde{x})\right) \leq 0$ by the concavity of $V(\cdot)$.

We show inequality (B49) in two steps: first, we show a sufficient condition based on the properties of $V(\cdot)$; second, we show that under the stated conditions for the lemma, the sufficient condition in the first step is satisfied.

Step 1. We show that (B49) holds if $-\frac{V''(\cdot)}{V'(\cdot)}$ and $-V'''(\cdot)$ are weakly decreasing.

Note that if $-\frac{V''(\cdot)}{V'(\cdot)}$ is weakly decreasing, then $V'''(\cdot) \geq 0$; hence $V'(\cdot)$ is convex. We thus have

$$\begin{aligned} & V\left(\tilde{y}_1 + T_{III}(\tilde{x})\right) - V\left(\tilde{y}_1 - T_{III}(\tilde{x})\right) = \int_{\tilde{y}_1 - T_{III}(\tilde{x})}^{\tilde{y}_1 + T_{III}(\tilde{x})} V'(\tau)d\tau \\ \leq\ & \frac{1}{2}\left[V'\left(\tilde{y}_1 + T_{III}(\tilde{x})\right) + V'\left(\tilde{y}_1 - T_{III}(\tilde{x})\right)\right]\left[\tilde{y}_1 + T_{III}(\tilde{x}) - \left(\tilde{y}_1 - T_{III}(\tilde{x})\right)\right] \\ =\ & T_{III}(\tilde{x})[V'\left(\tilde{y}_1 + T_{III}(\tilde{x})\right) + V'\left(\tilde{y}_1 - T_{III}(\tilde{x})\right)], \end{aligned}$$

which implies that

$$\frac{V\left(\tilde{y}_1 + T_{III}(\tilde{x})\right) - V\left(\tilde{y}_1 - T_{III}(\tilde{x})\right)}{V'\left(\tilde{y}_1 + T_{III}(\tilde{x})\right)} \leq T_{III}(\tilde{x})\left[1 + \frac{V'\left(\tilde{y}_1 - T_{III}(\tilde{x})\right)}{V'\left(\tilde{y}_1 + T_{III}(\tilde{x})\right)}\right]. \tag{B50}$$

Similarly, note that if $-V'''(\cdot)$ is weakly decreasing, then $V''(\cdot)$ is convex. Together with the fact that $V''(\cdot) < 0$, we have

$$\begin{aligned}
& V'(y_1 + T_{III}(\tilde{x})) - V'(y_1 - T_{III}(\tilde{x})) = \int_{y_1 - T_{III}(\tilde{x})}^{y_1 + T_{III}(\tilde{x})} V''(\tau) d\tau \\
\leq \ & \frac{1}{2}\left[V''(y_1 + T_{III}(\tilde{x})) + V''(y_1 - T_{III}(\tilde{x}))\right]\left[y_1 + T_{III}(\tilde{x}) - (y_1 - T_{III}(\tilde{x}))\right] \\
= \ & T_{III}(\tilde{x})[V''(y_1 + T_{III}(\tilde{x})) + V''(y_1 - T_{III}(\tilde{x}))],
\end{aligned}$$

which implies, by dividing both size by $-V''(y_1 + T_{III}(\tilde{x})) > 0$,

$$\frac{V'(y_1 - T_{III}(\tilde{x})) - V'(y_1 + T_{III}(\tilde{x}))}{V''(y_1 + T_{III}(\tilde{x}))} \leq -T_{III}(\tilde{x})\left[1 + \frac{V''(y_1 - T_{III}(\tilde{x}))}{V''(y_1 + T_{III}(\tilde{x}))}\right]. \tag{B51}$$

Combining the inequalities (B50) and (B51), we have

$$\begin{aligned}
& \frac{V'(y_1 - T_{III}(\tilde{x})) - V'(y_1 + T_{III}(\tilde{x}))}{V''(y_1 + T_{III}(\tilde{x}))} + \frac{V(\tilde{y}_1 + T_{III}(\tilde{x})) - V(\tilde{y}_1 - T_{III}(\tilde{x}))}{V'(\tilde{y}_1 + T_{III}(\tilde{x}))} \\
\leq \ & T_{III}(\tilde{x})\left[\frac{V'(\tilde{y}_1 - T_{III}(\tilde{x}))}{V'(\tilde{y}_1 + T_{III}(\tilde{x}))} - \frac{V''(y_1 - T_{III}(\tilde{x}))}{V''(y_1 + T_{III}(\tilde{x}))}\right].
\end{aligned}$$

Since $T_{III}(\tilde{x}) \geq 0$, to show (B49), it suffices to show

$$\frac{V'(\tilde{y}_1 - T_{III}(\tilde{x}))}{V'(\tilde{y}_1 + T_{III}(\tilde{x}))} - \frac{V''(y_1 - T_{III}(\tilde{x}))}{V''(y_1 + T_{III}(\tilde{x}))} \leq 0. \tag{B52}$$

Consider the left hand side of inequality (B52). It can be show by simple algebra that the first term $\frac{V'(\tilde{y}_1 - T_{III}(\tilde{x}))}{V'(\tilde{y}_1 + T_{III}(\tilde{x}))}$ is decreasing in $\tilde{y}_1$ due to the assumption that $-\frac{V''(\cdot)}{V'(\cdot)}$ is weakly decreasing; and the second term $-\frac{V''(y_1 - T_{III}(\tilde{x}))}{V''(y_1 + T_{III}(\tilde{x}))}$ is decreasing in y_1 if $-\frac{V'''(\cdot)}{V''(\cdot)}$ is increasing, which follows from the stated assumption that $-V'''$ is weakly decreasing (i.e. $V'''' \geq 0$).

We now prove inequality (B52). Consider two possible cases. (i). If $\tilde{y}_1 \geq y_1$, we have $\frac{V'(\tilde{y}_1 - T_{III}(\tilde{x}))}{V'(\tilde{y}_1 + T_{III}(\tilde{x}))} \leq \frac{V'(y_1 - T_{III}(\tilde{x}))}{V'(y_1 + T_{III}(\tilde{x}))}$; hence

$$\frac{V'(\tilde{y}_1 - T_{III}(\tilde{x}))}{V'(\tilde{y}_1 + T_{III}(\tilde{x}))} - \frac{V''(y_1 - T_{III}(\tilde{x}))}{V''(y_1 + T_{III}(\tilde{x}))} \leq \frac{V'(y_1 - T_{III}(\tilde{x}))}{V'(y_1 + T_{III}(\tilde{x}))} - \frac{V''(y_1 - T_{III}(\tilde{x}))}{V''(y_1 + T_{III}(\tilde{x}))} \leq 0,$$

where the last inequality holds because $-\frac{V''(\cdot)}{V'(\cdot)}$ is decreasing. (ii). If $\tilde{y}_1 < y_1$, we have $-\frac{V''(y_1 - T_{III}(\tilde{x}))}{V''(y_1 + T_{III}(\tilde{x}))} \leq -\frac{V''(\tilde{y}_1 - T_{III}(\tilde{x}))}{V''(\tilde{y}_1 + T_{III}(\tilde{x}))}$; hence

$$\frac{V'(\tilde{y}_1 - T_{III}(\tilde{x}))}{V'(\tilde{y}_1 + T_{III}(\tilde{x}))} - \frac{V''(y_1 - T_{III}(\tilde{x}))}{V''(y_1 + T_{III}(\tilde{x}))} \leq \frac{V'(\tilde{y}_1 - T_{III}(\tilde{x}))}{V'(\tilde{y}_1 + T_{III}(\tilde{x}))} - \frac{V''(\tilde{y}_1 - T_{III}(\tilde{x}))}{V''(\tilde{y}_1 + T_{III}(\tilde{x}))} \leq 0,$$

where the last inequality holds because $-\frac{V''(\cdot)}{V'(\cdot)}$ is decreasing.

Step 2. We show that $\left(-\frac{u''}{u'}\right)' \leq 0$ *and* $u'''' \geq 0$ *implies that* $-\frac{V''(\cdot)}{V'(\cdot)}$ *and* $-V'''(\cdot)$ *are both weakly decreasing.*

For convenience, we use short notation $c_1^{*\prime} = \frac{\partial c_1^*}{\partial y_1}$ and suppress y_1 in case of no confusion and simply use r instead of r_b. First we show that $-\frac{V''(\cdot)}{V'(\cdot)}$ is weakly decreasing, which is equivalent to show that $V'''V' - V''^2 \geq 0$. Note that the first order condition for optimal consumption in Problem (14), $u'(c_1^*) = \delta(1+r)\mathrm{E}u'(C_2^*)$, yields

$$c_1^{*\prime} = \frac{(1+r)^2\delta \mathrm{E}u''(C_2^*)}{u''(c_1^*) + (1+r)^2\delta \mathrm{E}u''(C_2^*)} = 1 - \frac{u''(c_1^*)}{u''(c_1^*) + (1+r)^2\delta \mathrm{E}u''(C_2^*)}. \tag{B53}$$

Thus we have

$$0 < c_1^{*\prime} < 1.$$

Taking derivative of $c_1^{*\prime}$ with respect to y_1, we obtain after some algebra,

$$\begin{aligned} c_1^{*\prime\prime} &= -(1+r)^2\delta \frac{u'''(c_1^*)c_1^{*\prime}\mathrm{E}u''(C_2^*) - u''(c_1^*)\mathrm{E}u'''(C_2^*)\frac{1}{\rho}(1-c_1^{*\prime})}{[u''(c_1^*) + (1+r)^2\delta \mathrm{E}u''(C_2^*)]^2} \\ &= -\frac{u'''(c_1^*)c_1^{*\prime}\mathrm{E}u''(C_2^*) - u''(c_1^*)\mathrm{E}u'''(C_2^*)\frac{1}{\rho}(1-c_1^{*\prime})}{(1+r)^2\delta[\mathrm{E}u''(C_2^*)]^2} c_1^{*\prime 2}. \end{aligned} \tag{B54}$$

Also we recall from the proof of Lemma 1,

$$V'' = u''(c_1^*)c_1^{*\prime} < 0.$$

Therefore, we have

$$\begin{aligned} V''' &= u'''(c_1^*)(c_1^{*\prime})^2 + u''(c_1^*)c_1^{*\prime\prime} \\ &= \left[u'''(c_1^*) - u''(c_1^*)\frac{u'''(c_1^*)c_1^{*\prime}\mathrm{E}u''(C_2^*) - u''(c_1^*)\mathrm{E}u'''(C_2^*)\frac{1}{\rho}(1-c_1^{*\prime})}{(1+r)^2\delta[\mathrm{E}u''(C_2^*)]^2}\right] c_1^{*\prime 2} \\ &= \left[u'''(c_1^*)c_1^{*\prime} + \frac{u''(c_1^*)^2\mathrm{E}u'''(C_2^*)\frac{1}{\rho}(1-c_1^{*\prime})}{(1+r)^2\delta[\mathrm{E}u''(C_2^*)]^2}\right] c_1^{*\prime 2} \\ &\geq 0. \end{aligned} \tag{B55}$$

where the second step uses (B54), the third step uses (B53), and the last step is due to the fact that $u''' \geq 0$ and $0 < c_1^{*\prime} < 1$.

Using the expression (B55), by some algebra, we have

$$V'''V' - V''^2 = \left\{u'(c_1^*)u'''(c_1^*) - u''(c_1^*)^2 + u'(c_1^*)(1-c_1^{*\prime})\left[\frac{u''(c_1^*)^2\mathrm{E}u'''(C_2^*)\frac{1}{\rho}}{(1+r)^2\delta[\mathrm{E}u''(C_2^*)]^2} - u'''(c_1^*)\right]\right\} c_1^{*\prime 2}.$$

Since $c_1^{*\prime 2} > 0$, to show $V'''V' - V''^2 \geq 0$, it is equivalent to show

$$u'(c_1^*)u'''(c_1^*) - u''(c_1^*)^2 + u'(c_1^*)(1-c_1^{*\prime})\left[\frac{u''(c_1^*)^2\mathrm{E}u'''(C_2^*)\frac{1}{\rho}}{(1+r)^2\delta[\mathrm{E}u''(C_2^*)]^2} - u'''(c_1^*)\right] \geq 0. \tag{B56}$$

To this end, the following reasoning is key:

$$\begin{aligned}
\mathrm{E}u'(C_2^*)\mathrm{E}u'''(C_2^*) &\geq \mathrm{E}u'(C_2^*)\mathrm{E}\frac{u''(C_2^*)^2}{u'(C_2^*)} = \mathrm{E}\left[\sqrt{u'(C_2^*)}\right]^2 \mathrm{E}\left(\frac{u''(C_2^*)}{\sqrt{u'(C_2^*)}}\right)^2 \\
&\geq \left(\mathrm{E}\sqrt{u'(C_2^*)}\frac{u''(C_2^*)}{\sqrt{u'(C_2^*)}}\right)^2 = \left[\mathrm{E}u''(C_2^*)\right]^2.
\end{aligned}$$

where the first step uses $u''' \geq \frac{u''^2}{u'}$, which follows from the assumption that $\left(-\frac{u''}{u'}\right)' \leq 0$, the second step is straightforward, and the third step follows from Cauchy-Schwartz inequality.

Applying the above result $\mathrm{E}u'(C_2^*)\mathrm{E}u'''(C_2^*) \geq [\mathrm{E}u''(C_2^*)]^2$ to the left hand of inequality (B56) and using the first order condition $(1+r)\delta\mathrm{E}u'(C_2^*) = u'(c_1^*)$, we show that the left hand side of inequality (B56) is

$$\begin{aligned}
\text{LHS of } (B56) &\geq u'(c_1^*)u'''(c_1^*) - u''(c_1^*)^2 + u'(c_1^*)(1-c_1^{*\prime})\left[\frac{u''(c_1^*)^2}{(1+r)\delta\mathrm{E}u'(C_2^*)} - u'''(c_1^*)\right] \\
&= u'(c_1^*)u'''(c_1^*) - u''(c_1^*)^2 + (1-c_1^{*\prime})[u''(c_1^*)^2 - u'''(c_1^*)u'(c_1^*)] \\
&= [u'(c_1^*)u'''(c_1^*) - u''(c_1^*)^2]c_1^{*\prime} \geq 0,
\end{aligned}$$

which shows $V'''V' - V''^2 \geq 0$; i.e., $-\frac{V''(\cdot)}{V'(\cdot)}$ is weakly decreasing.

Next, we want to show $V'''' \geq 0$. We use $\frac{1-c_1^{*\prime}}{c_1^{*\prime}} = \frac{u''(c_1^*)}{(1+r)^2\delta\mathrm{E}u''(C_2^*)}$ from (B53) to rewrite V''' in (B55) as:

$$V''' = \left(u'''(c_1^*) + \frac{\frac{1}{\rho}\mathrm{E}u'''(C_2^*)}{[(1+r)^2\delta]^2}\left[\frac{u''(c_1^*)}{\mathrm{E}u''(C_2^*)}\right]^3\right)c_1^{*\prime 3}.$$

Thus, taking the derivative of V''' with respect to y_1 and divide by $c_1^{*\prime 2}$, we have

$$\begin{aligned}
\frac{V''''}{c_1^{*\prime 2}} &= \left(u'''(c_1^*) + \frac{\frac{1}{\rho}\mathrm{E}u'''(C_2^*)}{[(1+r)^2\delta]^2}\left[\frac{u''(c_1^*)}{\mathrm{E}u''(C_2^*)}\right]^3\right)' c_1^{*\prime} + 3\left(u'''(c_1^*) + \frac{\frac{1}{\rho}\mathrm{E}u'''(C_2^*)}{[(1+r)^2\delta]^2}\left[\frac{u''(c_1^*)}{\mathrm{E}u''(C_2^*)}\right]^3\right)c_1^{*\prime\prime} \\
&= \underbrace{u''''(c_1^*)c_1^{*\prime 2} + \frac{\frac{1}{\rho}}{[(1+r)^2\delta]^2}\left[\frac{u''(c_1^*)}{\mathrm{E}u''(C_2^*)}\right]^3 \mathrm{E}u''''(C_2^*)\frac{1}{\rho}(1-c_1^{*\prime})c_1^{*\prime}}_{\text{Term A}} \\
&\quad \underbrace{+\frac{\frac{1}{\rho}\mathrm{E}u'''(C_2^*)c_1^{*\prime}}{[(1+r)^2\delta]^2}3\left[\frac{u''(c_1^*)}{\mathrm{E}u''(C_2^*)}\right]^3\left(\frac{u'''(c_1^*)}{u''(c_1^*)}c_1^{*\prime} - \frac{\mathrm{E}u'''(C_2^*)}{\mathrm{E}u''(C_2^*)}\frac{1}{\rho}(1-c_1^{*\prime})\right)}_{\text{Term B}} + \underbrace{3u'''(c_1^*)c_1^{*\prime\prime}}_{\text{Term C}} \\
&\quad \underbrace{+3\frac{\frac{1}{\rho}\mathrm{E}u'''(C_2^*)}{[(1+r)^2\delta]^2}\left[\frac{u''(c_1^*)}{\mathrm{E}u''(C_2^*)}\right]^3 c_1^{*\prime\prime}}_{\text{Term D}}.
\end{aligned}$$

Term A is non-negative due to the assumption that $u'''' \geq 0$. So we only consider the remaining three items. We merge Terms B and D into

$$\text{Term B+Term D} = \frac{\frac{1}{\rho}}{[(1+r)^2\delta]^2} 3 \left(\frac{u''(c_1^*)}{\mathrm{E}u''(C_2^*)}\right)^3 \mathrm{E}u'''(C_2^*) \underbrace{\left(\frac{u'''(c_1^*)}{u''(c_1^*)} c_1^{*\prime 2} - \frac{\mathrm{E}u'''(C_2^*)}{\mathrm{E}u''(C_2^*)} \frac{1}{\rho}(1-c_1^{*\prime})c_1^{*\prime} + c_1^{*\prime\prime}\right)}_{\text{Term E}}. \tag{B57}$$

Using the expression of $c_1^{*\prime\prime}$ (see (B54)), Term E is simplified as

$$\begin{aligned}
&\text{Term E} = \frac{u'''(c_1^*)}{u''(c_1^*)} c_1^{*\prime 2} - \frac{\mathrm{E}u'''(C_2^*)}{\mathrm{E}u''(C_2^*)} \frac{1}{\rho}(1-c_1^{*\prime})c_1^{*\prime} - \frac{u'''(c_1^*)c_1^{*\prime}\mathrm{E}u''(C_2^*) - u''(c_1^*)\mathrm{E}u'''(C_2^*)\frac{1}{\rho}(1-c_1^{*\prime})}{(1+r)^2\delta[\mathrm{E}u''(C_2^*)]^2} c_1^{*\prime 2} \\
= &\left(\frac{u'''(c_1^*)}{u''(c_1^*)} c_1^{*\prime 2} - \frac{u'''(c_1^*)c_1^{*\prime 3}}{(1+r)^2\delta \mathrm{E}u''(C_2^*)}\right) + \frac{1}{\rho}(1-c_1^{*\prime})c_1^{*\prime} \frac{\mathrm{E}u'''(C_2^*)}{\mathrm{E}u''(C_2^*)} \left(\frac{u''(c_1^*)}{(1+r)^2\delta\mathrm{E}u''(C_2^*)} c_1^{*\prime} - 1\right) \\
= &\frac{u'''(c_1^*)}{u''(c_1^*)} c_1^{*\prime 3} - \frac{1}{\rho}(1-c_1^{*\prime})c_1^{*\prime 2} \frac{\mathrm{E}u'''(C_2^*)}{\mathrm{E}u''(C_2^*)}.
\end{aligned}$$

where the last step applies the fact $\frac{1-c_1^{*\prime}}{c_1^{*\prime}} = \frac{u''(c_1^*)}{(1+r)^2\delta\mathrm{E}u''(C_2^*)}$ from (B53) to both baskets.

Therefore, we have

$$\text{Term B} + \text{Term D} = \frac{\frac{1}{\rho}}{[(1+r)^2\delta]^2} 3 \left(\frac{u''(c_1^*)}{\mathrm{E}u''(C_2^*)}\right)^3 \mathrm{E}u'''(C_2^*) \left\{\frac{u'''(c_1^*)}{u''(c_1^*)} c_1^{*\prime} - \frac{1}{\rho}(1-c_1^{*\prime}) \frac{\mathrm{E}u'''(C_2^*)}{\mathrm{E}u''(C_2^*)}\right\} c_1^{*\prime 2}. \tag{B58}$$

By the expression of $c_1^{*\prime\prime}$ as in (B54), we have

$$\text{Term C} = -3u'''(c_1^*) \frac{u'''(c_1^*)c_1^{*\prime}\mathrm{E}u''(C_2^*) - u''(c_1^*)\mathrm{E}u'''(C_2^*)\frac{1}{\rho}(1-c_1^{*\prime})]}{(1+r)^2\delta[\mathrm{E}u''(C_2^*)]^2} c_1^{*\prime 2}. \tag{B59}$$

Combining (B58) and (B59), we obtain:

$$\begin{aligned}
&\text{Term B} + \text{Term D} + \text{Term C} \\
= \;& 3c_1^{*\prime 2} \left\{ \begin{array}{c} \mathrm{E}u'''(C_2^*)\frac{1}{\rho}(1-c_1^{*\prime}) \frac{u''(c_1^*)}{(1+r)^2\delta[\mathrm{E}u''(C_2^*)]^2} \left(u'''(c_1^*) - \frac{1}{(1+r)\delta}\left[\frac{u''(c_1^*)}{\mathrm{E}u''(C_2^*)}\right]^2 \mathrm{E}u'''(C_2^*)\right) \\ + \frac{1}{(1+r)^2\delta\mathrm{E}u''(C_2^*)} \left(\frac{1}{(1+r)\delta}\left[\frac{u''(c_1^*)}{\mathrm{E}u''(C_2^*)}\right]^2 \mathrm{E}u'''(C_2^*) - u'''(c_1^*)\right) u'''(c_1^*)c_1^{*\prime} \end{array} \right\} \\
= \;& 3c_1^{*\prime 2} \frac{1}{(1+r)^2\delta\mathrm{E}u''(C_2^*)} \left(u'''(c_1^*) - \frac{\mathrm{E}u'''(C_2^*)}{(1+r)\delta}\left[\frac{u''(c_1^*)}{\mathrm{E}u''(C_2^*)}\right]^2\right) \left[\mathrm{E}u'''(C_2^*)\frac{1}{\rho}(1-c_1^{*\prime}) \frac{u''(c_1^*)}{\mathrm{E}u''(C_2^*)} - u'''(c_1^*)c_1^{*\prime}\right] \\
= \;& -3 \frac{c_1^{*\prime 3}}{(1+r)^2\delta\mathrm{E}u''(C_2^*)} \left\{u'''(c_1^*) - \frac{\mathrm{E}u'''(C_2^*)}{(1+r)\delta}\left[\frac{u''(c_1^*)}{\mathrm{E}u''(C_2^*)}\right]^2\right\}^2 \\
\geq \;& 0.
\end{aligned}$$

Therefore, we have the desired result

$$\frac{V''''}{c_1^{*\prime 2}} \geq \text{Term B} + \text{Term C} + \text{Term D} \geq 0. \quad \blacksquare$$

B.2 Verification of Global Optimality for the CARA Case in Corollary 1

To show the global optimality of the bidding function (37) for any $x > \underline{x}$, it suffices to check that condition (B49) is satisfied. Recall from (A19), $V(\cdot; r)$ is exponential for the CARA case. Hence, for any y_1, $\tilde{y}_1$, and $T_{III}(x) = \rho_b(mr_b - b_{III}(x))$, we have

$$
\begin{aligned}
& \frac{V'\left(y_1 - T_{III}(\tilde{x}); r_b\right) - V'\left(y_1 + T_{III}(\tilde{x}); r_b\right)}{V''\left(y_1 + T_{III}(\tilde{x}); r_b\right)} \\
= & -\frac{\frac{(1+r_b)}{2+r_b}\eta(r_b)\exp\left[-\frac{\beta(1+r_b)}{2+r_b}(y_1 - T_{III}(\tilde{x}))\right] - \frac{(1+r_b)}{2+r_b}\eta(r_b)\exp\left[-\frac{\beta(1+r_b)}{2+r_b}(y_1 + T_{III}(\tilde{x}))\right]}{\frac{\beta(1+r_b)}{2+r_b}\frac{(1+r_b)}{2+r_b}\eta(r_b)\exp\left[-\frac{\beta(1+r_b)}{2+r_b}(y_1 + T_{III}(\tilde{x}))\right]} \\
= & -\frac{\exp\left[-\frac{\beta(1+r_b)}{2+r_b}(y_1 - T_{III}(\tilde{x}))\right] - \exp\left[-\frac{\beta(1+r_b)}{2+r_b}(y_1 + T_{III}(\tilde{x}))\right]}{\frac{\beta(1+r_b)}{2+r_b}\exp\left[-\frac{\beta(1+r_b)}{2+r_b}(y_1 + T_{III}(\tilde{x}))\right]}
\end{aligned}
$$

and

$$
\begin{aligned}
& \frac{V\left(\tilde{y}_1 + T_{III}(\tilde{x}); r_b\right) - V\left(\tilde{y}_1 - T_{III}(\tilde{x}); r_b\right)}{V'\left(\tilde{y}_1 + T_{III}(\tilde{x}); r_b\right)} \\
= & \frac{\frac{1}{\beta}\eta(r_b)\exp\left[-\frac{\beta(1+r_b)}{2+r_b}(y_1 + T_{III}(\tilde{x}))\right] - \frac{1}{\beta}\eta(r_b)\exp\left[-\frac{\beta(1+r_b)}{2+r_b}(y_1 - T_{III}(\tilde{x}))\right]}{\frac{(1+r_b)}{2+r_b}\eta(r_b)\exp\left[-\frac{\beta(1+r_b)}{2+r_b}(y_1 + T_{III}(\tilde{x}))\right]} \\
= & \frac{\exp\left[-\frac{\beta(1+r_b)}{2+r_b}(y_1 - T_{III}(\tilde{x}))\right] - \exp\left[-\frac{\beta(1+r_b)}{2+r_b}(y_1 + T_{III}(\tilde{x}))\right]}{\frac{\beta(1+r_b)}{2+r_b}\exp\left[-\frac{\beta(1+r_b)}{2+r_b}(y_1 + T_{III}(\tilde{x}))\right]}.
\end{aligned}
$$

Therefore, condition (B49) is satisfied by

$$
\frac{V'\left(y_1 - T_{III}(\tilde{x}); r_b\right) - V'\left(y_1 + T_{III}(\tilde{x}); r_b\right)}{V''\left(y_1 + T_{III}(\tilde{x}); r_b\right)} + \frac{V\left(\tilde{y}_1 + T_{III}(\tilde{x}); r_b\right) - V\left(\tilde{y}_1 - T_{III}(\tilde{x}); r_b\right)}{V'\left(\tilde{y}_1 + T_{III}(\tilde{x}); r_b\right)} = 0.
$$

Finally, for type-0 bidder, $b_{III}(0) = mr_s$, as we have shown in Lemma 3.