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On the compassion of time-limited welfare programs

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Abstract

Supporters of recent welfare reforms argue that time limits and other eligibility restrictions serve recipients. We present a simple model of agents with present-biased preferences to investigate the theoretical validity of this claim. We first identify four types of outcome that describe the behavior of a present-biased agent in the absence of time limits. We then show that the behavioral consequences of time limits are contingent on which outcome characterizes the agent's behavior in the absence of time limits. We show that under some conditions the imposition of time limits may improve the well-being of welfare recipients evaluated both in terms of long-run, time-consistent utility and the period-one self's utility. This benefit of time limits may come either from allowing the welfare eligible to start working earlier than they otherwise would or, contrary to the intent of the reforms, from allowing them to postpone working.

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The ultimate issue at stake in serious welfare reform is not just dollars and cents but compassion. It's not compassionate to lead people into a life of drugs, dependency and despair. Real compassion must sometimes take the form of tough love. It's time to get people out of the destructive lifestyle of welfare once and for all.

Senator Robert Dole, Presidential Campaign Speech, May 21, 1996.

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1. Introduction

The most prominent features of the recent U.S. welfare reforms are the introduction of time limits, work requirements and other eligibility restrictions.¹ The sharp decline in welfare program participation since the enactment of major federal welfare reform in 1996, and evidence that the reform, and time limits in particular, are contributing to that decline (see e.g., Grogger, 2002b; Grogger and Michalopoulos, 2002) has further solidified the broad political support for these policy changes.

Proponents of the recent welfare reform, like Senator Dole quoted above, often argued for the new eligibility restrictions in the name of compassion for the poor. Prominent law and opinion makers emphasized the value of welfare time limits and other restriction to *recipients*, rather than to taxpayers or to society at large. They, like Dole, claimed that while perhaps “tough,” time limits and other welfare eligibility restrictions were truly more compassionate than the previous rules. By encouraging the poor to enter the workforce, the restrictions would not just be good for taxpayers, but would also be good for the poor themselves.

It is reasonable to suspect that these “compassion for the poor” arguments merely represent political palliatives intended to disguise the true intention of the reform, namely saving federal tax dollars. The evidence, however, does not support this suspicion. According to U.S. House of Representatives (1998) and U.S. Department of Health and Human Services (1999), the block grants to the states under the reformed system, TANF, were set at least as high as each state’s 1995, pre-reform, spending level. In total, federal spending on TANF in 1998 was \$16.5 billion, an increase of more than 50% over the \$11.4 billion spent in 1996.

How, then, might time limits on welfare eligibility and work requirements benefit the welfare eligible?² It is a fundamental property of standard economic decision problems that adding constraints to an agent’s choice set cannot make her better off. Compassionate time limits are therefore precluded by the standard framework. This truism motivates our departure from the standard setup: We allow agents to have *present-biased* preferences, and thus introduce the potential for both problems of self control and utility gains from restricting choice sets.

We consider a stylized, finite horizon model in which every period an agent with present-biased preferences chooses whether to work in the market, receive welfare benefits if eligible, or stay home without welfare. We investigate when and how time limits could benefit the welfare-eligible by providing them with a commitment to work—a commit-

¹ While work requirements had historical precedents (see Persky, 1997), time limits were largely absent from welfare reform proposals before 1988. During the early 1990s, several U.S. states began experiments with time-limited welfare in small demonstrations. In 1996 President Clinton signed the Personal Responsibility and Work Opportunity Reconciliation Act (PRWORA) imposing a national, 5-year lifetime limit on benefits from the Temporary Assistance to Needy Families Program (TANF). Ellwood (1988) is often credited with giving time limits their first thorough presentation.

² A related paper by Besley and Coate (1992) studied the incentive and deterrent effects of workfare. They assume that the policy maker’s objective is to minimize the cost of the poverty alleviation program.

ment that alleviates problems of self control.³ An important assumption underlying our results is that, if she has no prior work experience, a welfare-eligible woman's *net wage* from working (gross wage net of forgone leisure, child care, etc.) is smaller than her welfare benefits; but as she accumulates work experience, her wage will eventually exceed her welfare benefits. Under this assumption, the choice to work in the labor market involves *immediate costs* and *delayed benefits*. These intertemporal incentives for working, together with present-biased preferences, generate interesting intrapersonal strategic interactions between early and late selves. We partially characterize equilibrium behavior under these circumstances and formalize four patterns of outcomes that may result both from the agent's present-biased time preference, and the sophisticated anticipation of that bias. The most interesting results from the analysis of sophisticated agents are summarized as follows:

- If, in the absence of time limits, an agent chooses welfare because in each period the current self is unable to commit her future selves to work (called the “lack of commitment outcome” in the text), then the imposition of time limits may function as a commitment device providing the necessary incentives for future selves to work. This makes the agent better off than she was without time limits, not only from the perspective of a long-run self whose preferences are time-consistent but also from the perspective of the period-one, present-biased self. In this scenario, the time limit makes the agent better off by allowing her to start working *earlier* than she would have in the absence of a time limit.
- If, in the absence of time limits, an agent chooses work because the only way for her period one self to commit all her future selves to work is to start working right away (called the “now or never outcome” in the text), then the imposition of time limits may *delay* her entry into the work force. In this scenario, time limits make the period-one self better off but the long-run self worse off.
- The behavior of the sophisticated agent in the now or never outcome suggests a novel explanation of the low program take-up rates among the welfare eligible: agents might choose low-wage work, even when welfare offers higher returns, because they recognize that if they do not work now they never will. Anticipation of future self-control problems and a fear of long-term welfare dependence may encourage some of the welfare-eligible to stay out of the program.⁴

Other elements of the recent welfare reform, in particular work requirements, may also serve as imperfect commitment devices. We choose to analyze time limits for their simplicity. In a companion empirical implementation of this paper (Fang and Silverman, 2001) we simulate the effect of work requirements and find that work requirements can

³ To our knowledge, self control problems have not been incorporated in prior academic studies of welfare reform, though anecdotal evidence indicates that they affect the decision of welfare participants. DeParle (1997) and Massachusetts DTA (2000), for example, provide detailed descriptions of how self-struggle can affect decisions to enter and remain in the labor force.

⁴ This contrasts with Moffitt's (1983) explanation of the low welfare take-up rates based on stigma—a disutility from welfare program participation per se.

generate increases in overall labor market participation as effectively as a 5-year time limit.⁵

A few recent papers analyze the effects of time limits on welfare participation. Grogger and Michalopoulos (1999, 2002) provide a model with standard exponential discounting and show that, when faced with time limits, families with younger children are more likely to “bank” their benefits by forgoing welfare than are families with older children. The intuition for this result is that the option value of welfare eligibility is higher for families with younger children than it is for those with older children. Grogger and Michalopoulos’ empirical analysis, using data from Florida’s Family Transition Program, supports this prediction.⁶ In their model, agents obtain independent wage draws each period from the same distribution. Our paper instead assumes that the wage draws will be higher as the agent accumulates more labor market experience. In Grogger and Michalopoulos’ model, a woman’s only intertemporal consideration is the stock of welfare eligibility; while in our model, both her labor market experience and her stock of welfare eligibility are intertemporally important. Because of this, our model is consistent with a wider range of outcomes. We show, in examples (see Section 4.5 for details), that both the “lack of commitment” and the “now or never” outcomes are consistent with the empirical findings that women with younger children are more likely to exit welfare in response to time limits.⁷

The remainder of the paper is structured as follows. In Section 2, we introduce the basic model, along with our assumptions and equilibrium concepts; in Section 3 we characterize the behavior of time-consistent agents, both with and without time limits; in Section 4 we consider the behavior of sophisticated agents with time-inconsistent preferences with and without time limits; and in Section 5 we conclude and describe possible extensions. All the proofs are relegated to Appendix A.

2. The basic model

2.1. Choices and preferences

We consider a discrete time, finite horizon model with periods $t = 1, \dots, T$. We can think of T as the number of years until the agent’s youngest child turns 18—the age of the child above which the mother will not be eligible for welfare benefits. In each period, an agent chooses one and only one of the following options: receive welfare if eligible, work in the labor market, or stay at home without work or welfare. The following describes these options in greater detail; and further discussion of the assumptions of the model is provided in Section 2.2.

⁵ An important complicating factor in the analysis of work requirements is the extent to which the required work may contribute to human capital accumulation while it detracts from leisure and home production time. See Fang and Silverman (2001) for details.

⁶ Two other papers, Grogger (2002a,b), find additional evidence consistent with this prediction using data from the Survey of Income and Program Participation and the March Current Population Survey, respectively.

⁷ Unfortunately, we are unable to provide general conditions for Grogger–Michalopoulos’ result that women with younger children are more likely to conserve their welfare eligibility.

Welfare. We assume that if an agent chooses welfare, she receives a welfare benefit m which represents cash assistance and the monetary value of in-kind aid such as food stamps, housing subsidies and medical insurance. The time limit on welfare eligibility is denoted by $L > 0$.

Work. The wage from work is broadly defined. It includes salary but is net of day-care cost, forgone leisure, added stress from work, etc. We assume that if an agent works her wage depends only on the cumulative number of periods she has ever worked, denoted by τ . The wage experience profile is denoted by $w(\tau)$. We make the following assumption about $w(\tau)$:

Assumption 1. There exists $0 < \tau^* < T$ such that $w(\tau) < m$ if $\tau < \tau^*$, and $w(\tau) \geq m$ if $\tau \geq \tau^*$.

In words, we assume that an agent with no work experience earns net wages less than the welfare benefit m , but as experience accumulates the agent will eventually be able to earn a wage that exceeds the benefit. It is worth emphasizing that Assumption 1 is consistent with *non-monotonic* wage-experience profiles, and its only requirement is that the wage-experience profile crosses the welfare benefit level m only once from below. Fig. 1 in Section 2.2 provides evidence that the net return from working in the labor market is non-monotonic in experience and satisfies Assumption 1. The non-monotonicity of the wage experience profile plays an important role in our examples in Section 4.3.

Home. The third choice is for an agent to stay home without work or welfare. We assume that by staying home an agent can generate endowment income e from home production and family and neighborhood resources.⁸ We will, without loss of generality, normalize e to zero.

We assume the agent consumes all of the income associated with her choice in each period and obtains an instantaneous utility u_t .⁹ An agent in period t is concerned about both her present and future instantaneous utilities. Let $U^t(u_t, u_{t+1}, \dots, u_T)$ represent an agent’s intertemporal preferences from the perspective of period t , where U^t is continuous and increasing in all of its components. We adopt a simple, and now familiar formulation of agents’ potentially time-inconsistent preferences— (β, δ) -preferences (Phelps and Pollak, 1968; Laibson, 1994):

Definition 1. (β, δ) -preferences are preferences that can be represented by:

$$\text{For all } t, U^t(u_t, \dots, u_T) \equiv \delta^t u_t + \beta \sum_{s=t+1}^T \delta^s u_s, \quad \text{where } \beta \in (0, 1], \delta \in (0, 1]. \quad (1)$$

⁸ The choice of staying at home should be interpreted broadly—it is what a woman will be forced to choose when she exhausts welfare eligibility, if working in the labor market is too costly. According to Edin and Lein (1997, p. xi), “almost all poor single mothers supplement their regular income with some combination of off-the-books employment and money from relatives, lovers and the fathers of their children.” Therefore, it is likely that agents still have positive income when they choose to stay at home.

⁹ The assumption of no borrowing or saving appears realistic for low-income single women with children (Edin and Lein, 1997).

The parameter δ represents long-run, time-consistent discounting and is called the *discount factor*. The parameter β represents short-term impatience, and is called the *present bias factor*. Note that when $\beta = 1$, (β, δ) -preferences are the standard, time-consistent preferences. When $\beta \in (0, 1)$, (β, δ) -preferences are time-inconsistent, “quasi-hyperbolic” discounting preferences. We will call an agent with $\beta = 1$, time-consistent (TC); and an agent with $\beta \in (0, 1)$ present-biased (PB).

Following previous studies of time-inconsistent preferences, we will analyze the behavior of an agent by thinking of the decision-maker in each period as a separate *self*. Each period- t self makes her choice to maximize her current preferences $U^t(u_t, \dots, u_T)$, knowing her future selves control their own future behavior. The literature on time-inconsistent preferences distinguishes between *sophisticated* and *naive* agents (Strotz, 1955; Pollak, 1968; O’Donoghue and Rabin, 1999a,b). An agent is *sophisticated* if for every t , the period- t self knows her future selves’ preferences and anticipates her future selves’ behavior when making her own choice. She is *naive* if for every t , the period- t self wrongly believes that her future selves’ preferences are identical to her own.¹⁰ In this paper we focus most of our analysis on sophisticated agents but also examine the behavior of naifs when it helps to distinguish between the effects of the present bias per se and the effects of sophisticated anticipation of that bias.

2.2. Discussion of the assumptions

In this section we discuss the main assumptions of the model:

2.2.1. Wages rise above welfare benefits as experience accumulates

Some participants in the debate over welfare reform have challenged the assumption that welfare recipients’ wages will grow as they gain work experience (see, for example, Burtless, 1995 and Edin and Lein, 1997). However, a recent study by Loeb and Corcoran (2001) disputes this view. Loeb and Corcoran argue that prior studies were focussed only on wage growth with age, and did not examine how wages changed with work experience.¹¹ Since a welfare recipient is often in and out of the labor market, age is not a good measure of her experience. Loeb and Corcoran’s (2001) results suggest that welfare recipients’ wages will grow with full-time work experience at the same rate as those who had not received welfare. Moreover, their estimates show that the returns to experience are non-linear: they obtain coefficient estimates of 0.11 to 0.14 for the linear term and -0.005 to -0.007 for the quadratic term of the log wage-experience function. In our own study (Fang and Silverman, 2001), we follow a structural approach to implement an extension of the current model and find coefficient estimates of 0.115 for the linear term and -0.006 for the quadratic term of the log wage-experience function, remarkably similar to Loeb and Corcoran’s (2001) findings. In Fig. 1, we use the estimates from Fang and Silverman (2001) to simulate the undiscounted returns to *uninterrupted* work versus

¹⁰ O’Donoghue and Rabin (2001) presents a less extreme and perhaps more realistic model of “partial naive” in which agents anticipate but underestimate the present bias of their future selves.

¹¹ We thank an anonymous referee for making us aware of this study.

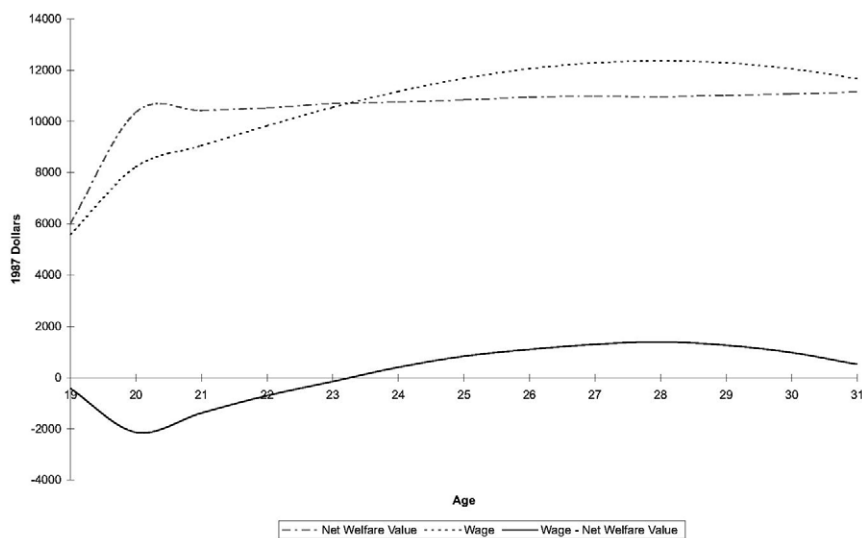


Fig. 1. Simulated undiscounted returns to uninterrupted work versus welfare, Ages 19–31 for woman with 11 years of completed schooling in state with average welfare benefits. *Source:* Authors' simulations in Fang and Silverman (2001).

welfare for women ages 19 to 31 who have 11 years of completed schooling in a state with average level of welfare benefits. In this simulation, net welfare value includes the welfare payment, home leisure's value, and welfare application costs.¹²

The most important feature of Fig. 1 is that wages from uninterrupted work *eventually overtake* welfare's net value. Moreover, it is possible that the difference between wages and net welfare values are *non-monotonic* in experience (which is here isomorphic to age because we assumed work is uninterrupted). Non-monotonicity of wages relative to welfare's value as a function of experience, though not essential to our analysis, introduces interesting dynamic effects into the model. To summarize, Assumption 1 about the relationship between wages and welfare benefits is largely consistent with the most recent evidence: for many of the welfare-eligible there would be immediate net costs to entering the workforce but, eventually, the benefits of accumulated experience would give work a higher immediate return. Finally, it is important to note that if, instead of Assumption 1, working in the labor market always generates higher net value than welfare, or vice versa, then it would be impossible for time limits to make welfare-eligible women better off.

2.2.2. Welfare benefits are time invariant

Our assumption of a time invariant welfare benefit m is counterfactual. It ignores the changes in real benefits due to the birth of additional children, price inflation, and changes

¹² The jump in the net welfare value at age 19 is due to our assumption that the welfare start-up costs (and possibly stigma as well) last for only one period.

in the programs. Adding these more realistic features will, however, complicate analysis without adding insight.

2.2.3. Options such as part-time work while on welfare are assumed away

Edin and Lein (1997) interviewed 214 women receiving welfare and found that just 5% of them worked in the official labor market, while 39% worked in an unofficial but legal market either off the books or using a false identity. From this work, these welfare-reliant women earned on average \$128 per month, two-thirds of which came from unofficial work. This income from work represented on average 15% of the monthly budget for these women; while 64% came from welfare, and 17% from family and friends. Based on this evidence it appears that, in reality, part-time work while on welfare is an available option, though for a typical welfare recipient it represents relatively little of her time use or income. With respect to the “at home” choice, we interpret this evidence to suggest that our model makes two empirically relevant, implicit assumptions: (1) Those who work while on welfare would receive these same part-time earnings if they instead chose to stay home without welfare (see footnote 8); and (2) part-time unofficial work experience does *not* contribute to the experience valued by the official labor market.

2.2.4. The wage experience profile is deterministic and fertility and marriage decisions are exogenous

These assumptions are clearly unrealistic and they are made here merely for tractability. We believe, however, that the various effects that we highlight in the deterministic setting will persist in an environment with more realistic, stochastic wage offers.

2.3. States, strategies and payoffs

When a period- t self makes her work–welfare–home choice, her relevant *state variable* is denoted by $h_t \equiv (t, \tau, \kappa)$, indicating that her previous selves have worked for a total of τ periods, have received welfare for a total of κ periods, and have been at home with no welfare for $(t - 1) - (\tau + \kappa)$ periods. We refer to the second and third component of state h_t as $\tau(h_t)$ and $\kappa(h_t)$, respectively. The set of possible states for a period- t self, H_t , is:

$$H_t \equiv \{(t, \tau, \kappa) : \tau \in \mathbb{Z}_+, \kappa \in \mathbb{Z}_+, 0 \leq \tau + \kappa \leq t - 1\}.$$

The *action space* for a period- t self is given by $A \equiv \{0, 1, 2\}$ where 0 stands for welfare, 1 for work and 2 for home without work or welfare. The *feasible* action space in state h_t , $A(h_t)$ is given by:

$$A(h_t) = \begin{cases} A & \text{if } \kappa(h_t) < L \\ \{1, 2\} & \text{if } \kappa(h_t) \geq L \end{cases}.$$

A *feasible strategy* for a period- t self is a mapping, $\sigma_t : H_t \rightarrow A$ such that $\sigma_t(h_t) \in A(h_t)$ for all $h_t \in H_t$. Note that we are restricting our attention to Markovian strategies, that is, σ_t is a function of past actions only to the extent that they are summarized in the state variable h_t .

To simplify the analysis, we assume that instantaneous utility u_t is given by period income. That is, if the state variable for the period- t self is h_t , her instantaneous utility $u_t(h_t, \sigma_t)$ from strategy σ_t is:

$$u_t(h_t, \sigma_t) = \frac{[1 - \sigma_t(h_t)][2 - \sigma_t(h_t)]}{2}m + \sigma_t(h_t)[2 - \sigma_t(h_t)]w(\tau(h_t)). \tag{2}$$

Given h_t and σ_t , the state inherited by the period- $(t + 1)$ self is denoted $h_{t+1}(h_t, \sigma_t)$ and is given by:

$$h_{t+1}(h_t, \sigma_t) = \left(t + 1, \tau(h_t) + \sigma_t(h_t)(2 - \sigma_t(h_t)), \kappa(h_t) + \frac{(1 - \sigma_t(h_t))(2 - \sigma_t(h_t))}{2} \right).$$

For any period $s > t$, we denote the period- s state that will be reached from h_t if the strategy profile σ is followed by $h_s(h_t, \sigma)$.

A *strategy profile* is a vector of mappings $\sigma \equiv (\sigma_t)_{t=1}^T$, specifying for each self her action in all possible states. Given a strategy profile σ , we can define the period- t self's *continuation payoff*, $U^t(h_t, \sigma)$, as:

$$U^t(h_t, \sigma) = \delta^t u_t(h_t, \sigma_t) + \beta \sum_{s=t+1}^T \delta^s u_s(h_s(h_t, \sigma), \sigma)$$

We also define the agent's period- t continuation *long-run utility*, $V_t(h_t, \sigma)$, recursively from the terminal period:

$$V_T(h_T) = u_T(h_T, \sigma_T), \quad \text{and}$$

$$V_t(h_t) = u_t(h_t, \sigma_t) + \delta V_{t+1}(h_{t+1}(h_t, \sigma_t)) \quad \text{for } t = T - 1, \dots, 1 \tag{3}$$

In words, an agent's long run utility represents her intertemporal preferences from some prior perspective where her own present bias is irrelevant. An agent's long run preferences are therefore represented by Eq. (1) when $\beta = 1$, and thus TCs and PBs have identical long-run utilities.

The *outcome path* generated by a strategy profile σ , denoted by $a(\sigma) = \{a_t(\sigma)\}_{t=1}^\infty$ is defined recursively as follows:

$$a_1(\sigma) = \sigma_1(h_1) \text{ where } h_1 = (0, 0, 0), \quad \text{and}$$

$$a_t(\sigma) = \sigma_t(h_t(h_{t-1}, \sigma_{t-1})) \text{ for } t = 2, \dots, T. \tag{4}$$

Following O'Donoghue and Rabin (1999b) we define a *perception-perfect strategy profile* as:

Definition 2. A perception-perfect strategy profile is a strategy profile σ^* that satisfies for all h_t , and for all t ,

$$\sigma_t^*(h_t) = \operatorname{argmax}_{\sigma \in A(h_t)} \{u_t(h_t, \sigma) + \beta \delta V_{t+1}(h_{t+1}(h_t, \sigma), \sigma^*)\}.$$

Whenever the agent is indifferent between welfare and work, we will break the tie in favor of work.¹³ The perception-perfect strategy profile for TCs and PBs will be denoted by σ^{TC} and σ^{PB} respectively. When there is a time limit L , we will sometimes write these profiles as $\sigma^{\text{TC}}(L)$ and $\sigma^{\text{PB}}(L)$, respectively. In what follows we will characterize the outcome paths generated by the perception-perfect strategy profiles and their associated utility for welfare programs both without and with time limits.

3. Time-consistent agents

3.1. Without time limits

To establish a benchmark, we begin by characterizing the optimal decisions for time-consistent agents without time limits. Note that in this case, the choice of staying home without welfare will never be chosen in a perception perfect strategy profile.

The following proposition establishes that TCs will either start working immediately from period 1 or never work. The intuition is straightforward: if a TC ever works, her wage in the end must be higher than the welfare benefit m ; but by our assumption on the wage experience profile, we know that the wage once exceeding m will never drop below it. Therefore if a TC ever decides to work, she should work as long as possible. In other words, there will be no churning between work and welfare. This prediction, however, is clearly due to our deterministic setting assumption.¹⁴

Proposition 1. *The outcome path generated by σ^{TC} is $a_t(\sigma^{\text{TC}}) = 1$ for all t if and only if*

$$\sum_{s=1}^T \delta^{s-1} w(s-1) \geq \sum_{s=1}^T \delta^{s-1} m;$$

otherwise, $a_t(\sigma^{\text{TC}}) = 0$ for all t .

3.2. With time limits

Now suppose that a time limit $0 < L < T - 1$ is imposed on welfare eligibility. Note that if an agent would choose to work from period one in the absence of time limits then the time limit will have no effect on her equilibrium behavior. Suppose instead that the agent would always choose welfare in the absence of time limits. To characterize a TC's perception-perfect strategy profile with time limits, we need to know most importantly whether the agent would choose to work once her welfare eligibility is depleted, and if so

¹³ Results are qualitatively unchanged if ties are broken in favor of welfare, or by a random mechanism.

¹⁴ In Fang and Silverman (2001) we find that, for the sample of women we select from the NLSY, choices are quite persistent: the probabilities that a period t choice is the same as that of period $t - 1$ are respectively 84.3, 79.3 and 59.7% for welfare, work and home. We rationalize this fact by introducing choice-specific unobserved shocks.

for how long she would choose to work. Following the argument from above, we know that she will work if and only if

$$\max_{L+1 \leq t \leq T} \left\{ \sum_{s=L+1}^t \delta^{s-1} w(s-L-1) \right\} \geq 0,$$

where the left hand of the inequality is the maximum discounted wage she could receive from working up to some future period, and the right-hand side is her discounted utility from staying home. Denote the maximizer of the left-hand side of the above inequality by T^* . The following proposition, whose proof uses arguments similar to those of Proposition 1 and is thus omitted, characterizes the perception-perfect strategy profile with time limits:

Proposition 2. *Suppose $\delta \in (0, 1)$ and $0 < L < T - 1$. The outcome path generated by $\sigma^{TC}(L)$ is given by :*

$$a_t(\sigma^{TC}(L)) = 1 \text{ for all } t \text{ if } \sum_{s=1}^T \delta^{s-1} w(s-1) \geq \sum_{s=1}^L \delta^{s-1} m + \max \left\{ \sum_{s=L+1}^{T^*} \delta^{s-1} w(s-L-1), 0 \right\},$$

otherwise,

$$a_t(\sigma^{TC}(L)) = \begin{cases} 0 & \text{if } t \leq L \\ 1 & \text{if } L < t \leq T^*, \text{ and } \sum_{s=L+1}^{T^*} \delta^{s-1} w(s-L-1) \geq 0 \\ 2 & \text{if otherwise.} \end{cases}$$

In words, there are two types of outcome for TCs under time limits: In the first type the agent works in all periods; in the second type the agent receives welfare through period L , and then works from period $L + 1$ through period T^* and, if $T^* < T$, she stays home in the remaining periods.

Now we can evaluate both the behavioral and the utility consequences of time limits for TCs. The imposition of time limits will only change the behavior of those agents whose experience wage profile $w(\tau)$ satisfies:

$$\sum_{t=1}^T \delta^{t-1} w(t-1) < \sum_{t=1}^T \delta^{t-1} m.$$

Since the choices made under time limits were also available in the absence of time limits, the agents, if their behavior is changed, are made strictly *worse off*. Thus it is clear that in a classical setting “compassionate time limits” is a contradiction in terms: adding con-

straints to an agent's choice set cannot make her better off. This rejection of compassionate time limits by the standard framework motivates the following exploration of the same model, but where agents have present-biased preferences.

4. Present-biased agents

In this section, we study the behavior of agents with present-biased preferences ($0 < \beta < 1$) both with and without time limits on welfare benefits. We find that the behavior of these sophisticated, present-biased agents is considerably more intricate than that of their time-consistent counterparts.

4.1. Solving σ^{PB} recursively

When $T < \infty$, the perception-perfect strategy profile for PBs can be solved recursively as follows:

- Solve $\sigma_T^{\text{PB}}(h_T)$ for all h_T :

$$\sigma_T^{\text{PB}}(h_T) = \operatorname{argmax}_{\sigma \in A(h_T)} u_T(h_T, \sigma);$$

and define

$$V_T(h_T) = u_T(h_T, \sigma_T^{\text{PB}}(h_T)).$$

- For every $t = T - 1, \dots, 1$, for every h_t :

$$\sigma_t^{\text{PB}}(h_t) = \operatorname{argmax}_{\sigma \in A(h_t)} u_t(h_t, \sigma) + \beta \delta V_{t+1}(h_{t+1}(h_t, \sigma))$$

and

$$V_t(h_t) = u_t(h_t, \sigma_t^{\text{PB}}(h_t)) + \delta V_{t+1}(h_{t+1}(h_t, \sigma_t^{\text{PB}}(h_t))).$$

The outcome path generated by σ^{PB} will be $a(\sigma^{\text{PB}})$ as defined in (4).

4.2. Effects of present bias vs. sophistication¹⁵

Before we describe the behavior of PBs, here we give a brief overview of three general and potentially competing effects that influence the decisions of a sophisticated, present-biased agent (see O'Donoghue and Rabin, 1999b). The first effect, called the *present-bias* effect, describes the basic and direct influence of the agent's preference for immediate gratification. In our model this present-bias effect simply makes an agent more likely to forgo work in favor of welfare—even when, from a long-term perspective, she would be

¹⁵ We thank Ted O'Donoghue for clarifying these various effects for us.

better off employed. It is important to note that this effect has force regardless of the agent’s degree of sophistication about her own future preferences. The present-bias effect influences naifs and sophisticates alike.

The second effect, called the *pessimism effect*, describes how the preference of future selves for immediate gratification leads earlier selves to abandon an otherwise optimal plan that exchanges short term costs for long-term rewards. In our model, the pessimism effect makes earlier selves more likely to take welfare than work. Recognizing that they cannot commit their future selves to an optimal plan of work, these earlier selves become more likely to forgo work themselves and choose welfare. The third effect, called the *incentive effect*, describes how the preference of future selves for immediate gratification leads earlier selves to abandon an otherwise optimal plan that takes short term rewards in exchange for long term costs. In our model, the incentive effect makes earlier selves more likely to work than take welfare. While they cannot commit their future selves to an optimal plan of work, earlier selves may be able to induce their future selves to work by choosing work themselves. Thus the incentive effect makes earlier selves, who would have preferred to take welfare and postpone work were they able to commit their future selves to work, more likely to enter the workforce. We emphasize that while the present-bias effect is a direct consequence of present-biased preferences, the pessimism and the incentive effects arise from the agent’s sophisticated anticipation of her self-control problem. Naifs are not motivated by either the pessimism or the incentive effect.

In what follows, we illustrate how these three effects influence the decisions of sophisticated, present-biased agents with a series of examples. Each example represents one of four possible outcomes: the *carpe diem* outcome, the lack of commitment outcome, the free ride outcome, and the now or never outcome. We demonstrate how which of the four outcomes emerges depends on relative magnitudes of the three effects. Later, in Section 4.5, we illustrate how the behavioral and utility consequences of time limits will also depend, in turn, on the relative magnitudes of the three effects in the absence of time limits.

4.3. No time limits: examples

In each of the following examples we illustrate a basic outcome as the result of a combination of the present-bias, pessimism and incentive effects described above. Unless stated otherwise, in all the examples below we set $T = 3, m = 1, \beta = 1/2$ and $\delta = 1$. The examples are constructed so that all the incentive conditions are strict. Therefore by continuity, the outcomes will not change for δ close to, but strictly less than 1. Because there is no time limit, the cumulative number of periods on welfare, κ , is not a payoff relevant state variable; so for simplicity we will in this section write $h_t = (t, \tau)$.

Example 1. (Carpe diem outcome) The wage profile is:

τ	0	1	2
w	− 2	1.4	5

In this example, using the recursive method described in Section 4.1 we can show that σ^{PB} is:

$$\sigma_3^{\text{PB}}(h_3) = 1 \text{ iff } \tau(h_3) \geq 1;$$

$$\sigma_2^{\text{PB}}(h_2) = 1 \text{ iff } \tau(h_2) = 1;$$

$$\sigma_1^{\text{PB}}(h_1) = 0.$$

This example has two important features: First, if she were to work, the period-1 self would induce all her future selves to work as well (note that for $t = 2, 3$, $\sigma_t^{\text{PB}}(h_t) = 1$ if $\tau(h_t) \geq 1$). In other words, the incentive effect is strong. Second, there is no commitment problem, that is, even if the period-1 self had the power to commit her future selves to work, she would not choose to do so because $1 + \beta \times (-2 + 1.4) < 1 + \beta \times (1 + 1)$. These two features together imply that the only reason the period-1 self chooses welfare is that, because of a preference for immediate gratification, the cost of working today ($w(0) - m = -3$) looms too large relative to the future rewards of having worked. In other words, the present bias effect dominates the incentive effect and is the sole reason why the agent chooses welfare. We call this type of outcome where the present-bias effect dominates the incentive effect a *carpe diem outcome*. Note also that if the agent were a TC, then from Proposition 1 we know that she would start working from period 1.

Example 2. (Lack of commitment outcome) The wage profile is:

τ	0	1	2
w	-2	-8	18.5

One can show that in this example σ^{PB} is:

$$\sigma_3^{\text{PB}}(h_3) = 1 \text{ iff } \tau(h_3) = 2$$

$$\sigma_2^{\text{PB}}(h_2) = 0 \text{ for all } h_2$$

$$\sigma_1^{\text{PB}}(h_1) = 0.$$

This example has two salient features: First, if the period-1 self could commit her future selves to work, she would prefer to start work right away because $-2 + \beta \times (-8 + 18.5) = 3.25 > 1 + \beta \times (1 + 1) = 2$; Second, no matter what choice the period-1 self makes, the period-2 self will never choose to work because of her taste for immediate gratification. The incentive effect is absent, and the pessimism and present-bias effects dominate. Even though the equilibrium outcome is identical to that in Example 1, the period-1 self here chooses not work because she is (rightly) pessimistic about the willingness of her future selves' to work. She chooses welfare because she cannot commit her future selves to follow her preferred plan of working in all periods. For this reason, we call this type of outcome the *lack of commitment outcome*.

The role of sophistication in Example 2 is clarified when we consider the behavior of a naif who is uninfluenced by the pessimism effect. In this example, the naif believes

(wrongly) that if she works in the first period she will work from then on. She therefore finds it optimal to work in period one, relying on her future selves to do the same. In fact, her second-period self’s taste for immediate gratification unravels this plan. Both the second and third period selves find it optimal to choose welfare. If she were sophisticated, the agent would have anticipated this unraveling and chosen welfare in period one. Instead, naively optimistic, she works early only to be disappointed by her lack of later self control.

Example 3. (Free ride outcome) The wage profile is:

τ	0	1	2
w	-2	8	3

One can show that σ^{PB} for this example is:

$$\sigma_3^{\text{PB}}(h_3) = 1 \text{ iff } \tau(h_3) \in \{1, 2\}$$

$$\sigma_2^{\text{PB}}(h_2) = 1 \text{ for all } h_2$$

$$\sigma_1^{\text{PB}}(h_1) = 0.$$

This example has two important features: First, no matter what the period-1 self does, the period-2 self will always choose to work (note $\sigma_2^{\text{PB}}(h_2) = 1$ for all h_2) and thus the period-3 self will also work regardless of what the period-1 self chooses. Both the pessimism and incentive effects are absent. Second, with a preference for immediate gratification and knowing that her future selves will work no matter what she chooses, the period-1 self decides to free ride on her future selves, receive welfare, and avoid the immediate cost of work ($w(0) - m = -3$) since $1 + \beta \times (-2 + 8) = 4 > -2 + \beta \times (8 + 3) = 7/2$. Here the period-1 self’s present bias is in effect and leads the first period self to free ride on her later selves. We therefore call this type of outcome a *free ride outcome*.

Example 4. (Now or never outcome) The wage profile is:

τ	0	1	2
w	-2	5.6	3

One can show that σ^{PB} for this example is:

$$\sigma_3^{\text{PB}}(h_3) = 1 \text{ iff } \tau(h_3) \in \{1, 2\}$$

$$\sigma_2^{\text{PB}}(h_2) = 1 \text{ iff } \tau(h_2) = 1$$

$$\sigma_1^{\text{PB}}(h_1) = 1.$$

The relevant features of this example are as follows: First, whether the period-2 self finds it optimal to work depends on whether the period-1 self has worked (note $\sigma_2^{\text{PB}}(h_2) = 1$ iff $\tau(h_2) = 1$); second, if the period-1 self could commit her future selves to work, she would prefer not to work, leaving it for her period-2 self to start working, since $-2 + \beta \times (5.6 + 3) = 2.3 < 1 + \beta \times (-2 + 5.6) = 2.8$. But the incentive effect dominates both the present-bias and the pessimism effects and the period-1 self chooses to work because if she does not work now, she never will. For this reason we call this type of outcome a *now or never outcome*.

Again the role of sophistication in this example is clarified by consideration of a naïf's choices. Here, in period 1, the naïf (wrongly) believes her period-2 self will work regardless of her choice in period 1. Uninfluenced by the incentive effect, the naïf therefore chooses welfare in period 1 and thereby induces all her future selves to do the same. The naïf, therefore, never works.

4.4. No time limits: general analysis

When analyzing the behavior of an agent with present-biased preferences, we need to solve a dynamic game with complete information played by the agent's different selves in different periods. For a specific environment it is straightforward to determine the unique (given our tie-breaking rule) equilibrium of this game. As the examples in Section 4.3 demonstrate, however, the effects determining the equilibrium outcome are quite sensitive to the specific curvature of the wage experience profile. Nonetheless, there are still general insights to be gained without making additional assumptions about the parameters of the model. First we consider whether, in the absence of time limits, a present-biased agent will ever return to welfare after having worked in some prior period. In other words, we ask whether $a_t(\sigma^{\text{PB}}) = 1$ can hold while $a_{t'}(\sigma^{\text{PB}}) = 0$ for some $t' > t$. The answer is no.

Proposition 3. *If $a_t(\sigma^{\text{PB}}) = 1$ then $a_{t'}(\sigma^{\text{PB}}) = 1$ for all $t' > t$.*

The proof of Proposition 3 is more involved than that of Proposition 1 because with present-biased agents the different selves have truly different preferences. What Proposition 3 tells us is that the outcome path generated by σ^{PB} in the absence of time limits is, like a TC's, characterized by an entry date. We will denote the entry date by t^* . However, as the examples in Section 4.3 show, with present-biased agents the entry date is no longer restricted to the set $\{1, T + 1\}$, that is, agents may receive welfare for some periods before they begin their working careers.

As we demonstrate in section 4.5, understanding the various effects underlying the present-biased agent's choices is more important than the particulars of the outcome path itself for studying the effect of time limits. For this purpose, we define:

Definition 3. (Willing-to-work set) $\Gamma^{\text{PB}}(t) \equiv \{\tau(h_t) : \sigma_t^{\text{PB}}(h_t) = 1\}$.

In words, the set $\Gamma^{\text{PB}}(t)$ is the set of experience levels at which a period t self will choose to work under σ^{PB} . We denote $\Gamma^{\text{PB}} \equiv \{\Gamma^{\text{PB}}(t)\}_{t=1}^T$. Observe that Γ^{PB} provides the same information as σ^{PB} : for every t , and every $h_t \in H_t$, $\sigma_t^{\text{PB}}(h_t) = 1$ if and only if $\tau(h_t) \in \Gamma^{\text{PB}}(t)$.

With our existing assumptions on the wage experience profile, it is immediate that $\{\tau^*, \dots, t - 1\} \subseteq \Gamma^{\text{PB}}(t)$ if $t \geq \tau^* + 1$. The following proposition proves useful:

Proposition 4. *If $\tilde{\tau} \in \Gamma^{\text{PB}}(s)$, then $\tilde{\tau} + 1 \in \Gamma^{\text{PB}}(s + 1)$ for every s .*

The entry date t^* established by Proposition 3 is exactly

$$t^* = \min\{t : 0 \in \Gamma^s(t)\} \cup \{T + 1\}$$

where $t^* = T + 1$ means that on the path generated by σ^{PB} the agent never works. With Γ^{PB} we can now formalize the various effects introduced in Section 4.3:

Definition 4. (Carpe diem outcome) We say that a *carpe diem* outcome occurs when $0 \notin \Gamma^{\text{PB}}(t)$ for all t , but $1 \in \Gamma^{\text{PB}}(t')$ for some t' .

In a *Carpe Diem* outcome, the period- $(t' - 1)$ self could start working, and we know from Proposition 4 that since $1 \in \Gamma^{\text{PB}}(t')$, all her future selves will work too. That is, by working the period- $(t' - 1)$ self could have induced all her future selves to work, but she chooses not to because of her bias towards immediate gratification.

Definition 5. (Lack of commitment outcome) We say that a lack of commitment outcome occurs when $0 \notin \Gamma^{\text{PB}}(t)$, and $1 \notin \Gamma^{\text{PB}}(t + 1)$ for all t , but,

$$w(0) + \beta \sum_{s=2}^T \delta^{s-1} w(s - 1) \geq m + \beta \sum_{s=2}^T \delta^{s-1} m. \tag{5}$$

In the lack of commitment outcome the agent will never work because $0 \notin \Gamma^{\text{PB}}(t)$ for all t . However, inequality (5) implies that if the period-1 self were able to, by herself working, commit her future selves to work, she would work too. In other words, the reason $0 \notin \Gamma^{\text{PB}}(1)$ is that $1 \notin \Gamma^{\text{PB}}(2)$.

Definition 6. (Free ride outcome) We say that a free ride outcome occurs if there exists some date $1 \leq t^{\text{FR}} < T$ such that $0 \notin \Gamma^{\text{PB}}(t)$ for all $t \leq t^{\text{FR}}$ and $0 \in \Gamma^{\text{PB}}(t^{\text{FR}} + 1)$.

We call the selves at periods $1, \dots, t^{\text{FR}}$ the free riders. This definition formalizes the idea that the selves at periods $1, \dots, t^{\text{FR}}$ choose not to work because they anticipate that the period- $(t^{\text{FR}} + 1)$ self will work no matter what. In Example 3, $\Gamma^{\text{PB}}(1) = \emptyset$, $\Gamma^{\text{PB}}(2) = \{0, 1\}$ and $\Gamma^{\text{PB}}(3) = \{1, 2\}$, hence $t^{\text{FR}} = 1$.

Definition 7. (Now or never outcome) We say that a now or never outcome occurs if there exists a unique $1 \leq t^{\text{NN}} < T$ such that $0 \in \Gamma^{\text{PB}}(t^{\text{NN}})$, and

$$w(0) + \beta \sum_{s=t^{\text{NN}}+1}^T \delta^{s-t^{\text{NN}}} w(s - t^{\text{NN}}) < m + \beta \sum_{s=t^{\text{NN}}+1}^T \delta^{s-t^{\text{NN}}} w(s - t^{\text{NN}} - 1). \tag{6}$$

We call the period- t^{NN} self the pioneer. Inequality (6) implies that the pioneer would prefer not to work if she could guarantee that her future selves would work. She anticipates, however, that unless she starts working, her future selves never will. Hence, she is compelled to work in order to lead her future selves into the labor market. In Example 4, $\Gamma^{PB}(1) = \{0\}$, $\Gamma^{PB}(2) = \{1\}$ and $\Gamma^{PB}(2) = \{1, 2\}$, hence $t^{NN} = 1$.

The now or never outcome provides an alternative instrumental explanation for the low welfare take-up rate: some of those eligible for welfare may choose low paying work, not because of the psychic costs associated with public aid, but rather because they fear that once they start receiving welfare they will not be able to stop. This differs from Moffitt (1983) who explains the welfare non-participation by welfare stigma, which is the disutility arising from participation in a welfare program per se.

Now we consider the relationship between the above four outcomes. First, in the *carpe diem* and lack of commitment outcomes, the agent will never work because $0 \notin \Gamma^{PB}(t)$ for all t , hence they are not compatible with either the free ride or the now or never outcomes. Second, in both the *carpe diem* and the lack of commitment outcomes the agent will always be on welfare, the difference is whether some period- t self would prefer to work if she were able to commit her future selves to work. Third, the now or never and the free ride outcomes are not exclusive of each other, as demonstrated by the following example:

Example 5. Suppose $T = 4, m = 1, \beta = 1/2, \delta = 1$. The wage profile is:

τ	0	1	2	3
w	-2	5.6	3	1.5

It can be shown that $\Gamma^{PB}(4) = \{1, 2, 3\}$, $\Gamma^{PB}(3) = \{1, 2\}$, $\Gamma^{PB}(2) = \{0, 1\}$ and $\Gamma^{PB}(1) = \emptyset$. Hence $t^{FR} = 1$ and $t^{NN} = 2$. Here a free rider takes advantage of a pioneer. For future reference, we will call the case where $t^{NN} = 1$ a *pure* now or never outcome.

4.5. Time limits on welfare eligibility

Here we analyze the effects of time limits on the choices of present-biased agents. We find that the consequences of time limits may be benign or perverse depending on the effects underlying the agent’s choices in the absence of time limits. We will start by revisiting some examples in section 4.3.

Recall that in Example 2 $a_t(\sigma^{PB}) = 0$ for all t . However, the period-1 self would work if she were able to commit her future selves to work. Lacking that ability, the pessimism and present-bias effects dominate; she chooses welfare anticipating that her period-2 self will not work no matter what. Now suppose that we impose a time limit $L = 1$ in this example. Then the period-1 self has, in effect, a means to commit her period-2 self to work since $-8 + \beta \times 18.5 = 1.25 > 1 + \beta \times 0$. Hence the outcome path generated by the unique $\sigma^{PB}(1)$ is $a_t(\sigma^{PB}(1)) = 1$ for all t . Importantly, both the period-1 self and the long run time-consistent self are made strictly better off with a time limit on welfare. The time limit diffuses the pessimism and present-bias effects and thus allows her to choose work confident that her later selves will do the same. We note that for naifs the gains are even

greater since with the time limit the period-1 self is no longer disappointed by her lack of later self control. The time limit induces her later selves to follow her lead into work. Now suppose instead that there is another woman with a time horizon of two periods, i.e., $T = 1$. Then, facing the one-period time limit, she will choose welfare in the first period and to stay at home in the second period. That is, the lack of commitment outcome is consistent with Grogger and Michalopoulos' (1999, 2002) empirical finding that women with younger children are more likely to exit welfare in response to time limits.

In contrast, in Example 4, $a_t(\sigma^{\text{PB}}) = 1$ for all t . In this example the incentive effect dominates and the period-1 self is a pioneer. With commitment, she would have preferred welfare for herself while the rest of her future selves worked. Without commitment, she works knowing that her future selves will work only if she does. Now suppose that a time limit $L = 1$ is imposed. Then the period-1 self knows that the period-2 self will work if the welfare eligibility has been exhausted since $-2 + \beta \times 5.6 = 0.8 > 0$. Hence the outcome path generated by the unique $\sigma^{\text{PB}}(1)$ is: $a_1(\sigma^{\text{PB}}(1)) = 0$, and $a_2(\sigma^{\text{PB}}(1)) = a_3(\sigma^{\text{PB}}(1)) = 1$. The time limit diffuses the incentive effect and the period-1 self, who was working in the absence of time limits, now takes welfare. By doing so, she, but not the long run self, is made strictly better off. Here again, though, the gains are greater for naifs whose equilibrium choices are, in this example, identical to those of sophisticates. Since with time limits the naive agent now works rather than receiving welfare in the last two periods, both the period one self *and* the long run self are made strictly better off.

Now let us consider imposing a time limit $L = 2$ in Example 4. Now it can be shown that in this example $\sigma^{\text{PB}}(2)$ is the same as when there is no time limits, in other words, the agent will work in all periods. This example has an interesting alternative interpretation. Suppose that one woman's time horizon is three periods, while another's is two periods. Suppose that a time limit of two periods is imposed. Then the woman with a longer time horizon will start working right away and continue working afterwards, while the one with a shorter time horizon will choose welfare in both periods. In this example, the now or never outcome is again consistent with Grogger and Michalopoulos' (1999, 2002) empirical findings.

These examples not only highlight the importance of analyzing the role of time-inconsistent preferences in people's choices of work or welfare, but also show the usefulness of understanding why some people never receive welfare, and why some people never work in states with no time limits on eligibility. If individual preferences are present-biased as considerable evidence suggests, then some may be working out of fear of never having the will power to work if they do not work now. Alternatively, some people may rely on welfare because they lack confidence that their future selves will continue to work through hard times until the rewards of working are finally realized. Finally, some people might postpone the beginning of their careers, relying on a deadline to force their future selves to work. The imposition of time limits on welfare eligibility gives early selves a device with which to commit the behavior of future selves. Whether this commitment power will lead an individual to leave or to join welfare depends on the reason behind this person's work-welfare-home choices in the absence of the time limit.

To make these insights somewhat more general, it is useful to denote $I_{\ell}^{\text{PB}}(t)$ as date t self's *willing-to-work* set when there are ℓ periods of welfare eligibility remaining. It is

difficult to characterize fully the equilibrium of the general game with time limits. In what follows we will therefore consider some special cases.

Proposition 5. *If the equilibrium outcome is a lack of commitment outcome in the absence of a time limit, and $1 \in \Gamma_1^{\text{PB}}(2)$, then $a_t(\sigma^{\text{PB}}(1)) = 1$ for all $t = 1, \dots, T$.*

Proposition 5 states that if $1 \in \Gamma_1^{\text{PB}}(2)$ the imposition of a time limit of $L = 1$ on welfare eligibility can effectively overcome the lack of commitment problem faced by the period 1 self in the absence of time limits.

Proposition 6. *If the equilibrium outcome is a pure now or never outcome (i.e. $t^{\text{NN}} = 1$) in the absence of time limits, and $0 \in \Gamma_0^{\text{PB}}(2)$, then $a_1(\sigma^{\text{PB}}(1)) = 0$.*

Proposition 6 establishes the theoretical possibility that the imposition of a time limit on welfare will trigger some agents who were working in the absence of time limits to take welfare temporarily, knowing that their future selves will be forced to work when welfare eligibility is exhausted.

4.6. The compassion of time limits

Last, we will address the utility consequences of time limits and consider how time limits could make the welfare eligible better off. In the literature, two criteria have been proposed to serve as a basis for comparing an agent's well being with and without time limits: the *Pareto* criterion (Laibson, 1994) and the *long-run utility* criterion (O'Donoghue and Rabin, 1999a). The Pareto criterion asks if all the selves in each period are made better off; while the long-run utility criterion envisions an imaginary agent with time-consistent preferences and asks if she is made better off. In our paper the imposition of time limits is not always Pareto optimal, as can be seen quite clearly from the examples above. If we use the long run utility criterion, then the utility consequences of time limits depend on the relevant outcome in the absence of time limits: in the pure now or never scenario of Proposition 4.5, the long-run utility is reduced by the imposition of a time limit. To see this, note first that by the definition of a now or never outcome we know that $a_t(\sigma^{\text{PB}}) = 1$ for all t in the absence of time limits. When a time limit of one period is imposed, the period one self will receive welfare and all subsequent selves will work. It can be shown that $m + \sum_{s=2}^T \delta^{s-1} w(s-2) < \sum_{s=1}^T \delta^{s-1} w(s-1)$,¹⁶ where the left- and the right-hand sides of the inequality are respectively the long run utility with and without the time limits. In contrast, in the lack of commitment scenario described in Proposition 5, the long-run utility is made higher by a time limit. The reason is as follows: by the definition of lack of commitment outcome, we have inequality (5). Since $w(0) < m$, it must be that $\sum_{s=2}^T \delta^{s-1} w(s-1) - \sum_{s=2}^T \delta^{s-1} m > 0$, hence $w(0) + \sum_{s=2}^T \delta^{s-1} w(s-1) > m + \sum_{s=2}^T \delta^{s-1} m$, where the left- and right-hand sides are respectively the long-run utility with and without the time limits.

¹⁶ By the definition of pure now or never effect, we have the inequality $w(0) + \beta \sum_{t=2}^T \delta^{t-1} w(t-1) \geq m + \beta \sum_{t=2}^T \delta^{t-1} m$. Since $w(0) < m$, the above inequality implies $\sum_{s=1}^T \delta^{s-1} w(s-1) > \sum_{s=1}^T \delta^{s-1} m$. Proposition 1 tells us that a TC would have worked from period one. TC's revealed preference then implies the desired inequality.

A third criterion which we propose is the *period-one self's utility* criterion. That is, we can ask whether the time limits have made the period one self, according to her present-biased preference, better off. This criterion will be of particular interest if we imagine an at least partially sophisticated period-one self has the opportunity to vote on a time limits policy. Although the influence of time limits on behavior depends on which effects are in force in the absence of the time limits, the effects in terms of period-one self's utility are the same in the scenarios described in Propositions 4.5 and 4.5: the period-one self is made strictly better off by the imposition of time limits.

5. Conclusion

In this paper we investigated a theoretical case for compassionate time limits—time limits that benefit the welfare eligible—in a simple model with present-biased preferences. We identified four types of outcome that describe the behavior of a present-biased agent in the absence of time limits, and then showed that the behavioral consequences of time limits are contingent on which outcome(s) characterizes the agent's behavior in the absence of time limits. The evaluation of the utility consequences of time limits depends on the criterion. We show that under some conditions the imposition of time limits could improve the well-being of welfare recipients evaluated both in terms of the long-run utility and the period-one self's utility. We show, in particular, that time limits could serve low-wage parents by providing a commitment to work in the future: in some cases the limits benefit the welfare eligible by allowing them to join the workforce earlier than they would have otherwise (the lack of commitment outcome) and in other cases the limits benefit the welfare-eligible by allowing them to postpone work for longer than they otherwise would (the now or never outcome).

Alternative rationalizations of compassionate time-limited welfare can be formulated via externalities. For example, a parent's productivity could be higher when more of her neighbors work. This production externality may generate multiple equilibria. If the low-income parents of a community ignore the effect that their own work–welfare choices have on others, and choose welfare over work, all may be trapped in a low-utility equilibrium with a large welfare roll. By imposing time limits on welfare eligibility, it is possible that the community switches to a high-utility equilibrium with a large working population. An advantage of our model, with no externalities, is that it is better suited to empirical implementation. In Fang and Silverman (2001), we examine whether the particular features of present-biased preferences highlighted in this paper are empirically important.¹⁷ The model in this paper is clearly too stylized for an estimation of present-bias in the decision-making of the welfare eligible: for example, data on welfare participation indicate that low-income women with children often move on and off of welfare, a pattern that cannot emerge in the deterministic model in this paper. Due to these

¹⁷ Most closely related to our empirical study, Della Vigna and Paserman (2000) and Paserman (2000) have found evidence for present-biased preferences in the context of job search.

considerations, we introduce in Fang and Silverman (2001) stochastic elements in job offer arrival, child birth, wage offer realization etc. into our model, and then empirically implement it to estimate the degree of present-bias reflected in the choices of low-wage women with children in the National Longitudinal Survey of Youth–1979 Cohort. Preliminary estimates suggest some degree of present bias among single mothers. Simulations then suggest that time limits and work requirements do have quite large behavioral consequences in terms of encouraging more agents to work. The simulations also indicate that the utility consequences of time limits and work requirements depend on the agent's schooling, labor market experience at the birth of her first child, and on the generosity of welfare benefits in her state of residence, and some agents can indeed be made slightly better off by the imposition of time limits or work requirements.

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Appendix A. Proofs

Proof of Proposition 1.

The following lemma establishes that the perception-perfect behavior of a TC is characterized by a critical entry date, after which the agent always works.

Lemma 1. *There exists some $t^* \in \{1, \dots, T + 1\}$ such that $a_t(\sigma^{\text{TC}}) = 0$ if $t < t^*$ and $a_t(\sigma^{\text{TC}}) = 1$ if $t \geq t^*$.*

Proof. Suppose not. Then there are two possibilities. The first possibility is that the agent works in one period, then receives welfare for a few periods, and then goes back to work again. Formally, let t' and t'' with $t' + 1 < t''$ be the first pair of dates such that $a_{t'}(\sigma^{\text{TC}}) = a_{t''}(\sigma^{\text{TC}}) = 1$ and for all $\hat{t} \in (t', t'')$, $a_{\hat{t}}(\sigma^{\text{TC}}) = 0$. If $\tau(h_{t'}) \geq \tau^*$, then the agent would be better off working in every period \hat{t} since $w(\tau(h_{\hat{t}})) \geq m$. If $\tau(h_{t'}) < \tau^*$, then in period t' , the agent can make a profitable deviation by choosing $\sigma_{t'} = 0$ and $\sigma_{\hat{t}} = 1$ for an arbitrary $\hat{t} \in (t', t'')$. The change in discounted utility is given by

$$\left[\delta^{t'} m + \delta^{\hat{t}} w(\tau(h_{t'})) \right] - \left[\delta^{t'} w(\tau(h_{t'})) + \delta^{\hat{t}} m \right] = \left(\delta^{t'} - \delta^{\hat{t}} \right) (m - w(\tau(h_{t'}))) > 0.$$

The existence of this profitable deviation contradicts the requirement that σ^{TC} maximize the period-1 self's discounted utility. The second possibility is that after working for some periods, the agent returns to welfare for ever. This is not optimal by the same logic. \square

Proof of Proposition 1. Suppose to the contrary that $1 < t^* \leq T$. If $t^* > T - \tau^*$, then the wages at every period $t^*, t^* + 1, \dots, T$ would be strictly less than m , thus a policy of not working at all would have been strictly preferred by the period-1 self, a contradiction. If $t^* \leq T - \tau^*$ then two inequalities must hold: first, at period t^* working must generate at least as much discounted utility as staying on welfare through period T :

$$w(0) + \delta w(1) + \dots + \delta^{T-t^*} w(T - t^*) \geq m \left(1 + \delta + \dots + \delta^{T-t^*} \right). \tag{A.1}$$

Second, at period $t^* - 1$ starting working must generate strictly less discounted utility than starting to work from period t^* (recall our tie-breaking rule):

$$\begin{aligned} & w(0) + \delta w(1) + \dots + \delta^{T-t^*} w(T - t^*) + \delta^{T-t^*+1} w(T - t^* + 1) \\ & < m + \delta \left[w(0) + \delta w(1) + \dots + \delta^{T-t^*} w(T - t^*) \right] \end{aligned} \tag{A.2}$$

Inequality (A.2) implies that

$$\begin{aligned} w(T - t^* + 1) & < \frac{m - (1 - \delta) \left[w(0) + \delta w(1) + \dots + \delta^{T-t^*} w(T - t^*) \right]}{\delta^{T-t^*+1}} \\ & < \frac{m - (1 - \delta) \left(1 + \delta + \dots + \delta^{T-t^*} \right) m}{\delta^{T-t^*+1}} = m, \end{aligned}$$

where the second inequality follows from inequality (A.1). A contradiction to the supposition that $t^* \leq T - \tau^*$. \square

Proof of Proposition 3. Suppose the contrary. Note that if the agent ever works, she will work in period T . Let $\hat{t} \equiv \max\{t : a_{t-1}(\sigma^{PB}) = 0\}$. Suppose that for some $n \geq 1$, the selves in periods $\{\hat{t} - n, \dots, \hat{t} - 1\}$ choose not to work on the equilibrium path, but $a_{\hat{t}-n-1}(\sigma^{PB}) = 1$. For notational simplicity, denote $\tau_t = \tau(h_t(h_1, \sigma^{PB}))$ as the level of experience in period t on the equilibrium path. It must be the case that $\tau_{\hat{t}-n-1} < \tau^*$, otherwise the date- $(\hat{t} - n)$ self would have chosen to work. Suppose that if period- $(\hat{t} - n - 1)$ self deviates from work to welfare on the path $h_{\hat{t}-n-1}(h_1, \sigma^{PB})$, then the subsequent selves will start working from period t^\dagger , i.e.,

$$t^\dagger = \min\{t \geq \hat{t} - n : \sigma_t^{PB}(h_t((h_{\hat{t}-n}(h_{\hat{t}-n-1}, 0), \sigma^{PB}))) = 1\}.$$

It must be that $t^\dagger \gg \hat{t}$ since otherwise the period- $(\hat{t} - n - 1)$ self would have deviated.. It must be true that the period- $(\hat{t} - n)$ self would prefer not to work if the period- $(\hat{t} - n - 1)$

self deviated from work to welfare. Otherwise, the subsequent plays will be identical to that on the equilibrium path, and the period- $(\hat{t} - n - 1)$ self would have deviated. For this to be true, it must be:

$$\begin{aligned}
 & w(\tau_{\hat{t}-n-1}) + \beta \left[\delta m + \dots + \delta^{n-1} m + \delta^n w(\tau_{\hat{t}}) + \dots + \delta^{T-\hat{t}+n} w(\tau_{\hat{t}} + (T - \hat{t})) \right] \\
 & < m + \beta \left[\delta m + \dots + \delta^{n-1} m + \delta^n m + \dots + \delta^{\hat{t}-\hat{t}+n-1} m \right. \\
 & \quad \left. + \delta^{\hat{t}-\hat{t}+n} V_{\hat{t}}(h_{\hat{t}}(h_{\hat{t}-n}(h_{\hat{t}-n-1}, 0), \sigma^{PB})) \right],
 \end{aligned}$$

where the left-hand side is the period- $(\hat{t} - n)$ self’s utility if she chose to work in the event that the period- $(\hat{t} - n - 1)$ deviated from work to welfare, and the right-hand side is the period- $(\hat{t} - n)$ self’s utility if she chose to stay on welfare in the same event. The above inequality is equivalent to:

$$\begin{aligned}
 & m - w(\tau_{\hat{t}-n-1}) \\
 & > \beta \delta^n \left[\begin{aligned} & w(\tau_{\hat{t}}) + \dots + \delta^{T-\hat{t}} w(\tau_{\hat{t}} + (T - \hat{t})) \\ & - \delta^{\hat{t}-\hat{t}} V_{\hat{t}}(h_{\hat{t}}(h_{\hat{t}-n}(h_{\hat{t}-n-1}, 0), \sigma^{PB})) - (m + \dots + \delta^{\hat{t}-\hat{t}-1} m) \end{aligned} \right]. \tag{A.3}
 \end{aligned}$$

However, knowing this, for the period- $(\hat{t} - n - 1)$ self to still choose to work, it must be:

$$\begin{aligned}
 & w(\tau_{\hat{t}-n-1}) + \beta \left[\delta m + \dots + \delta^n m + \delta^{n+1} w(\tau_{\hat{t}}) + \dots + \delta^{T-\hat{t}+n+1} w(\tau_{\hat{t}} + (T - \hat{t})) \right] \\
 & \geq m + \beta \left[\delta m + \dots + \delta^n m + \delta^{n+1} m + \dots + \delta^{\hat{t}-\hat{t}+n} m \right. \\
 & \quad \left. + \delta^{\hat{t}-\hat{t}+n+1} V_{\hat{t}}(h_{\hat{t}}(h_{\hat{t}-n}(h_{\hat{t}-n-1}, 0), \sigma^{PB})) \right],
 \end{aligned}$$

where the left- and the right-hand sides are respectively the period- $(\hat{t} - n - 1)$ self’s utility if she chooses to work and if she chooses to receive welfare. The above inequality is equivalent to:

$$\begin{aligned}
 & m - w(\tau_{\hat{t}-n-1}) \\
 & \leq \beta \delta^{n+1} \left[\begin{aligned} & w(\tau_{\hat{t}}) + \dots + \delta^{T-\hat{t}} w(\tau_{\hat{t}} + (T - \hat{t})) \\ & - \delta^{\hat{t}-\hat{t}} V_{\hat{t}}(h_{\hat{t}}(h_{\hat{t}-n}(h_{\hat{t}-n-1}, 0), \sigma^{PB})) - (m + \dots + \delta^{\hat{t}-\hat{t}-1} m) \end{aligned} \right]. \tag{A.4}
 \end{aligned}$$

Since $\tau_{\hat{t}-n-1} < \tau^*$, the right-hand side of inequality (A.4) must be positive. But then inequalities (A.3) and (A.4) cannot hold simultaneously. A contradiction. \square

Proof of Proposition 4. The statement is clearly true if $\tilde{\tau} \geq \tau^*$. Suppose $\tilde{\tau} < \tau^*$. Define a new game for which the wage experience profile is given by $\hat{w}(\tau) = w(\tau + \tilde{\tau})$, and the initial state variable is given by $\hat{h}_1 = h_s$. Since σ^{PB} is a perception-perfect strategy for

present-biased agents in the original game, the strategy profile defined by $\hat{\sigma}_t^{\text{PB}} \equiv \sigma_{t+s-1}^{\text{PB}}$ for every $t = 1, \dots, T - s + 1$ is a perception-perfect strategy profile of the new game. By Proposition 3, we know that if the period-1 self starts working on the equilibrium outcome path of the new game, the period-2 self must be working on the equilibrium path too. This implies the desired statement about the original game. \square

Proof of Proposition 5. Recall inequality (5) in the definition of lack of commitment outcome

$$w(0) + \beta \sum_{t=2}^T \delta^{t-1} w(t-1) \geq m + \beta \sum_{t=2}^T \delta^{t-1} m.$$

Since $1 \in \Gamma_1^{\text{PB}}(2)$, the period-1 self is better off if she chooses to work. The conclusion follows. \square

Proof of Proposition 6. If the period-1 self chooses welfare, then the period-2 self will have no welfare eligibility. Since $0 \in \Gamma_0^{\text{PB}}(2)$, we know that the period-2 self will start working. From inequality (6) in the definition of pure now or never outcome (noting that $t^{\text{NN}} = 1$), we have:

$$w(0) + \sum_{t=2}^T \delta^{t-1} w(t-1) < m + \sum_{t=2}^T \delta^{t-1} w(t-2).$$

hence $a_1(\sigma^{\text{PB}}(1)) = 0$. \square

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