Multidimensional Private Information, Market Structure and Insurance Markets

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ONLINE APPENDIX

(Not Intended for Publication)

In this online appendix, we collect the materials omitted from the main text of the paper. The appendices are ordered according to where they are first referenced in the main text. In Appendix A we show that Assumptions 1-5 are satisfied in Examples 1 and 2; in Appendix B, we study imperfect competition and show that our results derived in Section 5 are robust; in Appendix C, we present a continuous-type example based on Example 4; in Appendix D we provide the details of the derivations of the profit function, the demand curve, and the cost curves in Proposition 6; in Appendix E we derive the profit function used in the numerical analysis in Section 7 for $k < 1$.

A Assumptions 1-5 are Satisfied in Examples 1 and 2

As mentioned in the main text, Assumption 3 can be inferred from Assumptions 2 and 4. Therefore, it suffices to show that Assumptions 1, 2, 4 and 5 are satisfied.

Example 1 First, it is clear that

$$\frac{\partial U}{\partial p} = -\left[\frac{m}{\omega}u'\left(y-p-(1-x)\omega;\lambda\right)+\left(1-\frac{m}{\omega}\right)u'\left(y-p;\lambda\right)\right] < 0.$$

Moreover, $U(\theta; x, x\omega) < 0 < U(\theta; x, 0)$ for $x \in (0, 1]$. Therefore, Assumption 1 is satisfied.

Second, we show that Assumption 2 is satisfied. Notice that $v(\theta; x)$ solves $U(\theta; x, v) = 0$. It follows from the implicit function theorem that

$$\frac{\partial v}{\partial x} = -\frac{\partial U/\partial x}{\partial U/\partial v} = \frac{m \cdot u'\left(y-v-(1-x)\omega\right)}{\frac{m}{\omega}u'\left(y-v-(1-x)\omega\right)+\left(1-\frac{m}{\omega}\right)u'(y-v)} > 0.$$

Similarly, we have that

$$\begin{aligned}\frac{\partial v}{\partial m} = -\frac{\partial U/\partial m}{\partial U/\partial v} &= \frac{1}{\omega} \times \frac{u\left(y-v-(1-x)\omega\right)-u(y-v)-u(y-\omega)+u(y)}{\frac{m}{\omega}u'\left(y-v-(1-x)\omega\right)+\left(1-\frac{m}{\omega}\right)\cdot u'(y-v)} \\ &= \frac{1}{m} \times \frac{u(y)-u(y-v)}{\frac{m}{\omega}u'\left(y-v-(1-x)\omega\right)+\left(1-\frac{m}{\omega}\right)\cdot u'(y-v)} > 0,\end{aligned}$$

where the last equality follows from the rearrangement of consumers' indifference condition. To show that $\partial v/\partial \lambda > 0$, suppose to the contrary that $\lambda_1 > \lambda_0$ and $v_1 \equiv v(m, \lambda_1; x) \leq v(m, \lambda_0; x) \equiv v_0$.

By the definition of $v(\cdot)$, we have that:

$$\frac{m}{\omega}u(y - v_1 - (1-x)\omega; \lambda_1) + \left(1 - \frac{m}{\omega}\right) \cdot u(y - v_1; \lambda_1) = \frac{m}{\omega}u(y - \omega; \lambda_1) + \left(1 - \frac{m}{\omega}\right) \cdot u(y; \lambda_1),$$

which is equivalent to,

$$\frac{u(y; \lambda_1) - u(y - v_1; \lambda_1)}{u(y - v_1 - (1-x)\omega; \lambda_1) - u(y - \omega; \lambda_1)} = \frac{\frac{m}{\omega}}{1 - \frac{m}{\omega}} > 0.$$

It is straightforward to see that $u(y - v_1 - (1-x)\omega; \lambda_1) - u(y - \omega; \lambda_1) > 0$, or equivalently, $y - v_1 - (1-x)\omega > y - \omega$. Similarly, we have that

$$\frac{u(y; \lambda_0) - u(y - v_0; \lambda_0)}{u(y - v_0 - (1-x)\omega; \lambda_0) - u(y - \omega; \lambda_0)} = \frac{\frac{m}{\omega}}{1 - \frac{m}{\omega}} > 0.$$

Therefore, we must have that

$$\frac{u(y; \lambda_1) - u(y - v_1; \lambda_1)}{u(y - v_1 - (1-x)\omega; \lambda_1) - u(y - \omega; \lambda_1)} = \frac{u(y; \lambda_0) - u(y - v_0; \lambda_0)}{u(y - v_0 - (1-x)\omega; \lambda_0) - u(y - \omega; \lambda_0)}.$$

However, the above equality cannot hold because

$$\begin{aligned}\frac{u(y; \lambda_1) - u(y - v_1; \lambda_1)}{u(y - v_1 - (1-x)\omega; \lambda_1) - u(y - \omega; \lambda_1)} &< \frac{u(y; \lambda_0) - u(y - v_1; \lambda_0)}{u(y - v_1 - (1-x)\omega; \lambda_0) - u(y - \omega; \lambda_0)} \\ &\leq \frac{u(y; \lambda_0) - u(y - v_0; \lambda_0)}{u(y - v_0 - (1-x)\omega; \lambda_0) - u(y - \omega; \lambda_0)},\end{aligned}$$

where the first inequality follows from the fact that $y > y - v_1 \geq y - v_1 - (1-x)\omega > y - \omega$ and Theorem 1 in Pratt (1964); and the second inequality follows from the postulated $v_1 \leq v_0$ and the fact that $u(\cdot)$ is a strictly increasing function. Therefore, Assumption 2 is satisfied.

Next, we show that Assumption 4 is satisfied. Fixing $x_i > x_j$, $U\big(\theta; x_i, p_j + (x_i - x_j)m\big) \geq U\big(\theta; x_j, p_j\big)$ is equivalent to

$$\begin{aligned}&\frac{m}{\omega}u\big(y - p_j - (x_i - x_j)m - (1 - x_i)\omega; \lambda\big) + \left(1 - \frac{m}{\omega}\right)u(y - p_j - (x_i - x_j)m; \lambda) \\ &\geq \frac{m}{\omega}u\big(y - p_j - (1 - x_j)\omega; \lambda\big) + \left(1 - \frac{m}{\omega}\right)u(y - p_j; \lambda).\end{aligned}$$

The above inequality holds due to the observation that the lottery $m \circ [y - p_j - (1 - x_j)\omega] + (1 - m) \circ [y - p_j]$ is a mean-preserving spread of the lottery $m \circ [y - p_j - (x_i - x_j)m - (1 - x_i)\omega] + (1 - m) \circ [y - p_j - (x_i - x_j)m]$. Therefore, Assumption 4 is satisfied.

It remains to show that Parts (i) and (ii) of Assumption 5 are satisfied. For Part (i), notice that $U(\theta; x_i, p_i) = U(\theta; x_j, p_j)$ is equivalent to

$$\frac{u\left(y-p_j;\lambda\right)-u\left(y-p_i;\lambda\right)}{u\left(y-p_i-(1-x_i)\omega;\lambda\right)-u\left(y-p_j-(1-x_j)\omega;\lambda\right)}=\frac{\frac{m}{\omega}}{1-\frac{m}{\omega}}. \tag{A1}$$

Notice that we must have $p_i > p_j$. Otherwise, contract (x_i, p_i) dominates (x_j, p_j) in terms of both premium and coverage, and consumers strictly prefer contract (x_i, p_i) over (x_j, p_j). Therefore, we have that $u\left(y-p_j;\lambda\right)-u\left(y-p_i;\lambda\right) > 0$ and $u\left(y-p_i-(1-x_i)\omega;\lambda\right)-u\left(y-p_j-(1-x_j)\omega;\lambda\right) > 0$. This implies instantly that

$$y-p_j > y-p_i \geq y-p_i-(1-x_i)\omega > y-p_j-(1-x_j)\omega.$$

It follows from Theorem 1 in Pratt (1964) that the left hand side of (A1) is strictly decreasing in λ. Moreover, the right hand side of (A1) is strictly increasing in m. Therefore, the indifference curve $\lambda = \mathcal{I}_{ij}(m)$ is strictly decreasing in m.

For Part (*ii*), suppose that type-(m, λ) is indifferent between contract (x_i, p_i) and (x_j, p_j) and $\lambda > \dot{\lambda}$ and $\dot{m} > m$. It follows from (A1) that

$$\begin{aligned}&\frac{u\left(y-p_j;\dot{\lambda}\right)-u\left(y-p_i;\dot{\lambda}\right)}{u\left(y-p_i-(1-x_i)\omega;\dot{\lambda}\right)-u\left(y-p_j-(1-x_j)\omega;\dot{\lambda}\right)}\\ &<\frac{u\left(y-p_j;\lambda\right)-u\left(y-p_i;\lambda\right)}{u\left(y-p_i-(1-x_i)\omega;\lambda\right)-u\left(y-p_j-(1-x_j)\omega;\lambda\right)}=\frac{\frac{m}{\omega}}{1-\frac{m}{\omega}}<\frac{\frac{\dot{m}}{\omega}}{1-\frac{\dot{m}}{\omega}},\end{aligned}$$

which is equivalent to $U(\dot{m}, \dot{\lambda}; x_i, p_i) > U(\dot{m}, \dot{\lambda}; x_j, p_j)$. This completes the proof.

Example 2 Recall that

$$U(\theta; x, p) = xm + \frac{x\,(2-x)}{2}\sigma^2\lambda - p,$$

and

$$v(\theta; x) = xm + \frac{x\,(2-x)}{2}\sigma^2\lambda.$$

It follows immediately that

$$\frac{\partial U}{\partial p} = -1 < 0,\ \frac{\partial v}{\partial x} = m > 0,\ \frac{\partial v}{\partial m} = x > 0, \text{ and } \frac{\partial v}{\partial \lambda} = \frac{x\,(2-x)}{2}\sigma^2 > 0.$$

Therefore, Assumptions 1 and 2 are satisfied.

Fix $0 \le x_j < x_i \le 1$. We have that

$$
\begin{aligned}
&U\left(\theta; x_i, p_j + (x_i - x_j)m\right) - U\left(\theta; x_j, p_j\right) \\
&= \left\{ x_i m + \frac{x_i\left(2 - x_i\right)}{2}\sigma^2\lambda - \left[p_j + (x_i - x_j)m\right] \right\} - \left\{ x_j m + \frac{x_j\left(2 - x_j\right)}{2}\sigma^2\lambda - p_j \right\} \\
&= \left[\frac{x_i\left(2 - x_i\right)}{2} - \frac{x_j\left(2 - x_j\right)}{2} \right]\sigma^2\lambda > 0,
\end{aligned}
$$

where the inequality follows from the fact that $\frac{x(2-x)}{2}$ is strictly increasing in x for $x \in [0, 1]$. Therefore, Assumption 4 is satisfied and it remains to show that Assumptions 5 is satisfied. By definition, the indifference curve $\mathcal{I}_{ij}(m)$ solves $U(\theta; x_i, p_i) = U(\theta; x_j, p_j)$, or equivalently,

$$
x_i m + \frac{x_i\left(2 - x_i\right)}{2}\sigma^2\lambda - p_i = x_j m + \frac{x_j\left(2 - x_j\right)}{2}\sigma^2\lambda - p_j.
$$

Solving for $\mathcal{I}_{ij}(m)$ yields,

$$
\lambda = \mathcal{I}_{ij}(m) = \frac{1}{\sigma^2}\left[-\frac{2}{2 - x_i - x_j}m + \frac{2(p_i - p_j)}{(x_i - x_j)(2 - x_i - x_j)} \right]. \tag{A2}
$$

It is clear that $\mathcal{I}_{ij}(m)$ is strictly decreasing in m. For $\left(\dot{m}, \dot{\lambda}\right) > \left(m, \mathcal{I}_{ij}(m)\right)$, we have that

$$
\begin{aligned}
U(\dot{m}, \dot{\lambda}; x_i, p_i) - U(\dot{m}, \dot{\lambda}; x_j, p_j) &= \left(x_i\dot{m} + \frac{x_i\left(2 - x_i\right)}{2}\sigma^2\dot{\lambda} - p_i \right) - \left(x_j\dot{m} + \frac{x_j\left(2 - x_j\right)}{2}\sigma^2\dot{\lambda} - p_j \right) \\
&= \left(x_i - x_j\right)\dot{m} + \left[\frac{x_i\left(2 - x_i\right)}{2} - \frac{x_j\left(2 - x_j\right)}{2} \right]\sigma^2\dot{\lambda} - \left(p_i - p_j\right) \\
&> \left(x_i - x_j\right)m + \left[\frac{x_i\left(2 - x_i\right)}{2} - \frac{x_j\left(2 - x_j\right)}{2} \right]\sigma^2\mathcal{I}_{ij}(m) - \left(p_i - p_j\right) \\
&= U(m, \mathcal{I}_{ij}(m); x_i, p_i) - U(m, \mathcal{I}_{ij}(m); x_j, p_j) = 0,
\end{aligned}
$$

where the inequality follows instantly from $x_i > x_j$ and $\frac{x_i(2-x_i)}{2} > \frac{x_j\left(2-x_j\right)}{2}$. This indicates that type-$\left(\dot{m}, \dot{\lambda}\right)$ consumer strictly prefers contract (x_i, p_i) over (x_j, p_j). Next, it follows from (A2) that

$$
-\mathcal{I}'_{i0}(m) = \frac{1}{\sigma^2} \times \frac{2}{2 - x_i} > \frac{1}{\sigma^2} \times \frac{2}{2 - x_i} = -\mathcal{I}'_{j0}(m).
$$

Therefore, the single crossing condition holds and Assumption 5 is satisfied. This completes the proof.

B Imperfectly Competitive Insurance Market

In this section, we generalize the market structure and study imperfect competition. We model imperfectly competitive insurance market as follows.[1] Suppose that there are two insurance firms on the market where they engage in a modified "Bertrand competition" by setting a price for insurance of quality x. Different from the standard Bertrand model, we assume that consumers cannot compare prices perfectly; instead, a consumer receives a *noisy* signal regarding which of the two firms has a lower price, and then he/she inspects the actual price of the firm indicated by the noisy signal, and finally he/she decides whether to buy the product accordingly.[2, 3] The noisy signal regarding which firm has the lower price creates spurious product differentiation and gives rise to market power to firms and thus induces imperfect competition.

Specifically, after firm $i \in \{1, 2\}$ posts price p_i, each consumer receives a signal $s \in \{1, 2\}$ about which firm has the lower price as follows: given (p_1, p_2),

$$s = \begin{cases} 1 & \text{if } p_1 - p_2 + \epsilon \leq 0 \\ 2 & \text{otherwise,} \end{cases}$$

where $\epsilon \sim \mathcal{N}(0, \sigma_s^2)$.[4] It is clear that a consumer always follows the signal: if $s = i$, she will find out the actual price p_i and decide between purchasing insurance at price p_i and staying uninsured. Hence, conditional on the price vector (p_1, p_2), the probability that a consumer considers purchasing from firm i is $\Phi\left(\frac{p_j - p_i}{\sigma_s}\right)$.[5] Conditional on observing firm i's price p_i, the purchase decision of type-θ consumer remains the same as before: she will purchase insurance at price p_i if and only if $v(\theta; x) \geq p_i$, i.e., if and only if $\theta \in \mathcal{B}(p_i)$ where $\mathcal{B}(\cdot)$ is defined in (2). Note that σ_s^2 represents a measure of the market competitiveness. When $\sigma_s^2 = 0$, consumers always buy from the firm with the lower price, which indicates fierce price competition. When $\sigma_s^2 = \infty$, the signal is completely uninformative and the consumer randomly chooses between the two offers and each firm behaves as if they were a monopoly.

Now we can set up the strategic pricing game between the two firms. Fixing p_j, firm i chooses p_i to maximize the expected profit:

$$\Pi_i(p_i, p_j) = \Phi\left(\frac{p_j - p_i}{\sigma_s}\right) \int_{\theta \in \mathcal{B}(p_i)} (p - xm)\, dH(m, \lambda) \equiv \Phi\left(\frac{p_j - p_i}{\sigma_s}\right) \pi(p_i), \tag{A3}$$

[1] Our approach of modeling imperfect competition is in the same spirit as Fisher and Plan (2015).

[2] It is assumed that the consumer cannot switch to the other firm once the price of the chosen firm fully reveals. This specification of the consumer behavior can be rationalized by a sufficiently high switching cost.

[3] Note that our approach to parameterize the imperfect competition is related to but distinct from Lester et al. (2016). In their paper they assume that a buyer samples one offer with probability $p_1 \in (0, 1)$ and two offers with probability $p_2 = 1 - p_1$. The higher p_2 is, the more fierce the competition is.

[4] The assumption of Gaussian noise can be easily relaxed. All the results obtained in this section can be generalized to a noise Z parameterized by $\alpha \in [\underline{\alpha}, \overline{\alpha}]$ with support $-\infty \leq \underline{z} \leq \overline{z} \leq \infty$, whose CDF $\Psi(z; \alpha)$ and PDF $\psi(z; \alpha)$ satisfy: (i) $\psi(z; \alpha) = \psi(-z; \alpha)$; (ii) $\partial \frac{\psi(z;\alpha)}{\Psi(z;\alpha)} \Big/ \partial z < 0$; (iii) $\partial \psi(0; \alpha) \big/ \partial \alpha > 0$; (iv) $\lim_{\alpha \to \underline{\alpha}} \psi(0; \alpha) = 0$ and $\lim_{\alpha \to \overline{\alpha}} \psi(0; \alpha) = \infty$.

[5] An alternative way of modeling is to assume that each potential consumer receives two signals $s_i = p_i + \epsilon_i$ and examines the price of the firm with the lower signal, where ϵ_i is assumed to be independently identically distributed according to a normal distribution with mean 0 and variance $\sigma_s^2/2$.

where $\pi(\cdot)$ is defined in (5) in the paper. Notice that a nice feature of our formulation of the imperfect competition is that the price competition between the two firms only affects which firm is in the consideration set of the consumer, not the subsequent decision of whether to purchase the insurance.

Assumption A1 *(i) $\pi(p)$ is strictly single-peaked in p for $p \in \left(v(\underline{m}, \underline{\lambda}; x), v(\overline{m}, \overline{\lambda}; x)\right)$; (ii) $\max_{p\geq 0}\{\pi(p)\} > 0$; (iii) $\pi(p)$ is differentiable and log-concave in $p \in (p^*, p^m]$ and $\pi'(p^m) = 0$, where p^* is the competitive equilibrium price and p^m is the monopolist's profit-maximizing price.*

It is straightforward to verify that the profit function of the example in Proposition 6 for $k > 1$ satisfies Assumption A1. The differentiability and single-peakedness of the profit function guarantees that the profit function under monopoly is well-behaved and the first-order condition is sufficient to pin down the optimal price; and $\max_{p\geq 0}\{\pi(p)\} > 0$ ensures that a monopolistic insurer will not exit the market. Finally, as it will be clear later, log-concavity ensures the symmetry and uniqueness of the equilibrium.

Lemma A1 ***(Equilibrium Prices under Imperfect Competition)*** *Suppose Assumption A1 is satisfied. For any $\sigma_s^2 > 0$, there exists a unique equilibrium $\left(\hat{p}_1^*, \hat{p}_2^*\right)$ where $\hat{p}_1^* = \hat{p}_2^* = p^e \in (p^*, p^m)$ and p^e is the solution to*

$$\frac{\pi'(p^e)}{\pi(p^e)} = \frac{1}{\sigma_s}\sqrt{\frac{2}{\pi}}. \tag{A4}$$

Proof. First, notice that fixing p_j, we must have $p_i \in [p^*, p^m]$. For a price below p^*, the corresponding profit is negative, which is strictly dominated by $p_i = p^*$. For a price above p^m, firm $i's$ profit is less than $\Pi_i\left(p^m, p_j\right)$ due to the fact that $\Phi\left(\frac{p_j - p_i}{\sigma_s}\right)$ is strictly decreasing in p_i and $\pi(p_i) \leq \pi(p^m)$. The first order condition with respect to p_i yields,

$$\frac{1}{\sigma_s}\frac{\phi\left(\frac{p_j-p_i}{\sigma_s}\right)}{\Phi\left(\frac{p_j-p_i}{\sigma_s}\right)} = \frac{\pi'(p_i)}{\pi(p_i)}. \tag{A5}$$

Next, we show that if an equilibrium exists, it must be symmetric. Suppose to the contrary that $\hat{p}_1^* > \hat{p}_2^*$ without loss of generality. It follows directly that $(\hat{p}_1^* - \hat{p}_2^*)/\sigma_s > 0$; together with the first order condition (A5), we must have

$$\frac{\pi'(\hat{p}_1^*)}{\pi(\hat{p}_1^*)} = \frac{1}{\sigma_s}\frac{\phi\left(\frac{\hat{p}_2^*-\hat{p}_1^*}{\sigma_s}\right)}{\Phi\left(\frac{\hat{p}_2^*-\hat{p}_1^*}{\sigma_s}\right)} = \frac{1}{\sigma_s}\frac{\phi\left(\frac{\hat{p}_1^*-\hat{p}_2^*}{\sigma_s}\right)}{1-\Phi\left(\frac{\hat{p}_1^*-\hat{p}_2^*}{\sigma_s}\right)} > \frac{1}{\sigma_s}\frac{\phi\left(\frac{\hat{p}_1^*-\hat{p}_2^*}{\sigma_s}\right)}{\Phi\left(\frac{\hat{p}_1^*-\hat{p}_2^*}{\sigma_s}\right)} = \frac{\pi'(\hat{p}_2^*)}{\pi(\hat{p}_2^*)}.$$

Therefore, $\hat{p}_1^* < \hat{p}_2^*$ from Assumption A1, a contradiction. Imposing the symmetry condition $\hat{p}_1^* = \hat{p}_2^* = p^e$ to the above first order condition yields Equation (A4).

Lastly, we prove the existence and uniqueness of the equilibrium. From the definition of p^* and Assumption A1, $\pi'(p^m) = 0$ and $\pi(p^*) = 0$, which implies directly that $\lim_{p^e \to p^*} \pi'(p^e)/\pi(p^e) = \infty$

and $\lim_{p^e \to p^m} \pi'(p^e)/\pi(p^e) = 0$. Therefore, for any $\sigma_s^2 > 0$, there exists a unique solution to equation (A4). This completes the proof. ■

The following lemma derives the comparative statics of the equilibrium price with respect to the market competitiveness.

Lemma A2 *Suppose Assumption A1 is satisfied. The equilibrium price p^e is strictly increasing in σ_s^2. Moreover, $\lim_{\sigma_s^2 \to \infty} p^e = p^m$ and $\lim_{\sigma_s^2 \to 0} p^e = p^*$.*

The proof follows immediately from Assumption A1 that $\pi(p)$ is log-concave. This result is intuitive. When the market is imperfectly competitive, the equilibrium price lies between the competitive equilibrium price and the monopoly price. The equilibrium price becomes lower as the insurance market becomes more competitive.

Proposition A1 *Suppose Assumption A1 is satisfied and suppose $\mathbb{E}\left[M|\mathcal{B}(p^m)\right] < \mathbb{E}[M]$. Then there exist $\underline{\sigma}_s$ and $\overline{\sigma}_s$, with $\underline{\sigma}_s \leq \overline{\sigma}_s$, such that the positive (respectively, negative) correlation property emerges in equilibrium for $\sigma_s^2 < \underline{\sigma}_s^2$ (respectively, $\sigma_s^2 > \overline{\sigma}_s^2$). Furthermore, if there exists a unique solution to $AC(p) = \mathbb{E}[M]$ for $p \in [p^*, p^m]$, then $\overline{\sigma}_s = \underline{\sigma}_s$.*

Proof. From Proposition 1, we must have $\mathbb{E}\left[M|\mathcal{B}(p^*)\right] > \mathbb{E}[M]$. In addition, $\mathbb{E}\left[M|\mathcal{B}(p^m)\right] < \mathbb{E}[M]$ holds by assumption. From Lemma A2 and the continuity of $\mathbb{E}\left[M|\mathcal{B}(p)\right]$, we have

$$\begin{aligned}\lim_{\sigma_s^2 \to 0} \mathbb{E}\left[M|\mathcal{B}(p^e)\right] &= \mathbb{E}\left[M|\mathcal{B}\left(p^*\right)\right] > \mathbb{E}[M];\\ \lim_{\sigma_s^2 \to \infty} \mathbb{E}\left[M|\mathcal{B}(p^e)\right] &= \mathbb{E}\left[M|\mathcal{B}(p^m)\right] < \mathbb{E}[M].\end{aligned}$$

Therefore, the positive (respectively, negative) correlation property emerges in equilibrium when σ_s^2 is sufficiently low (respectively, high).

Denote the unique solution (if it exists) to $AC(p) = \mathbb{E}[M]$ for $p \in [p^*, p^m]$ as $\tilde{p}$. Because $\mathbb{E}\left[M|\mathcal{B}(p^*)\right] > \mathbb{E}[M]$, we must have $\mathbb{E}\left[M|\mathcal{B}(p)\right] > \mathbb{E}[M]$ for $p \in [p^*, \tilde{p})$ and $\mathbb{E}\left[M|\mathcal{B}(p)\right] < \mathbb{E}[M]$ for $p \in (\tilde{p}, p^m]$. Define $\tilde{\sigma}_s^2$ as the solution to $p^e(\sigma_s^2) = \tilde{p}$. From Lemma A2, $\tilde{\sigma}_s^2$ exists and is unique. Moreover, $p^e(\sigma_s^2) \lesseqgtr \tilde{p}$ for $\sigma_s^2 \lesseqgtr \tilde{\sigma}_s^2$. Therefore, the positive (respectively, negative) correlation property emerges in equilibrium for $\sigma_s^2 < \tilde{\sigma}_s^2$ (respectively, $\sigma_s^2 > \tilde{\sigma}_s^2$). This completes the proof. ■

C Continuous-Type Version of Example 4

In this section, we present a continuous-type example based on Example 4. When consumers' type is drawn from a continuous joint CDF, the MC curve is well-defined. Different from Example 4 where there are two competitive equilibrium premiums, there exists a unique equilibrium premium in the following example.

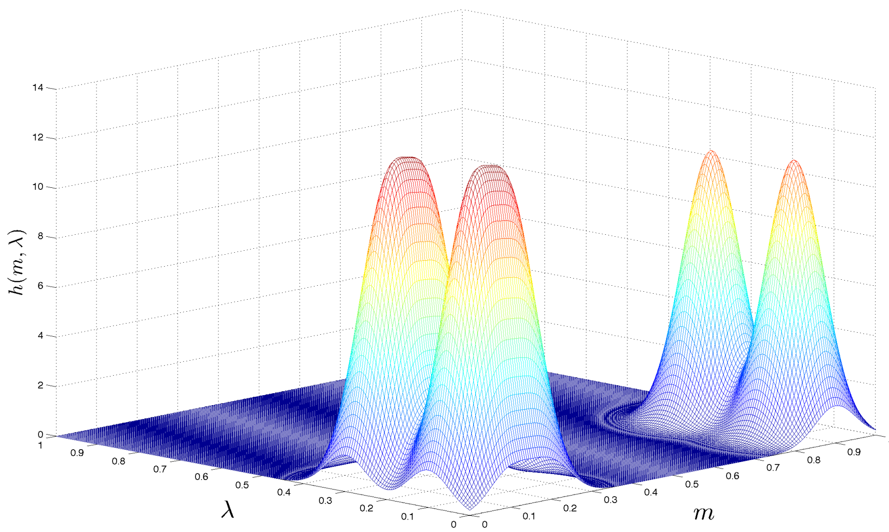

Figure 7: The Joint Density Function $h(m, \lambda)$ in Example A1

Example A1 *Suppose M and λ are independent and are both drawn from a mixture of left-right truncated normal distributions between 0 and 1 with the following marginal density functions,*

$$f(m) = \frac{1}{3\sigma_m} \frac{\phi\left(\frac{m-m_1}{\sigma_m}\right)}{\Phi\left(\frac{1-m_1}{\sigma_m}\right) - \Phi\left(\frac{0-m_1}{\sigma_m}\right)} + \frac{1}{3\sigma_m} \frac{\phi\left(\frac{m-m_2}{\sigma_m}\right)}{\Phi\left(\frac{1-m_2}{\sigma_m}\right) - \Phi\left(\frac{0-m_2}{\sigma_m}\right)} + \frac{1}{3\sigma_m} \frac{\phi\left(\frac{m-m_3}{\sigma_m}\right)}{\Phi\left(\frac{1-m_3}{\sigma_m}\right) - \Phi\left(\frac{0-m_3}{\sigma_m}\right)},$$

and

$$g(\lambda) = \frac{1}{2\sigma_\lambda} \frac{\phi\left(\frac{\lambda-\lambda_1}{\sigma_\lambda}\right)}{\Phi\left(\frac{1-\lambda_1}{\sigma_\lambda}\right) - \Phi\left(\frac{0-\lambda_1}{\sigma_\lambda}\right)} + \frac{1}{2\sigma_\lambda} \frac{\phi\left(\frac{\lambda-\lambda_2}{\sigma_\lambda}\right)}{\Phi\left(\frac{1-\lambda_2}{\sigma_\lambda}\right) - \Phi\left(\frac{0-\lambda_2}{\sigma_\lambda}\right)}.$$

The joint density function is $h(m, \lambda) = f(m)g(\lambda)$. Suppose that $v\left((m, \lambda); x\right) = m+2\lambda$, $(m_1, m_2, m_3) = (0.1, 0.2, 0.9)$, $(\lambda_1, \lambda_2) = (0.1, 0.3)$, and $\sigma_m = \sigma_\lambda = 0.05$. The joint density function is illustrated in Figure 7. The profit curve and the marginal cost curve are illustrated in Figure 8 and Figure 9 respectively. Figure 8 indicates that the equilibrium premium is unique and lies between 0.5 and 0.6. While Figure 9 indicates that the market is subject to local advantageous selection for $p \in [0.5, 0.6]$. From Proposition 1, the market always exhibits positive correlation property in competitive equilibrium. Therefore, from this example we know the market can still be subject to local advantageous selection at the equilibrium premium even though the positive correlation property holds.

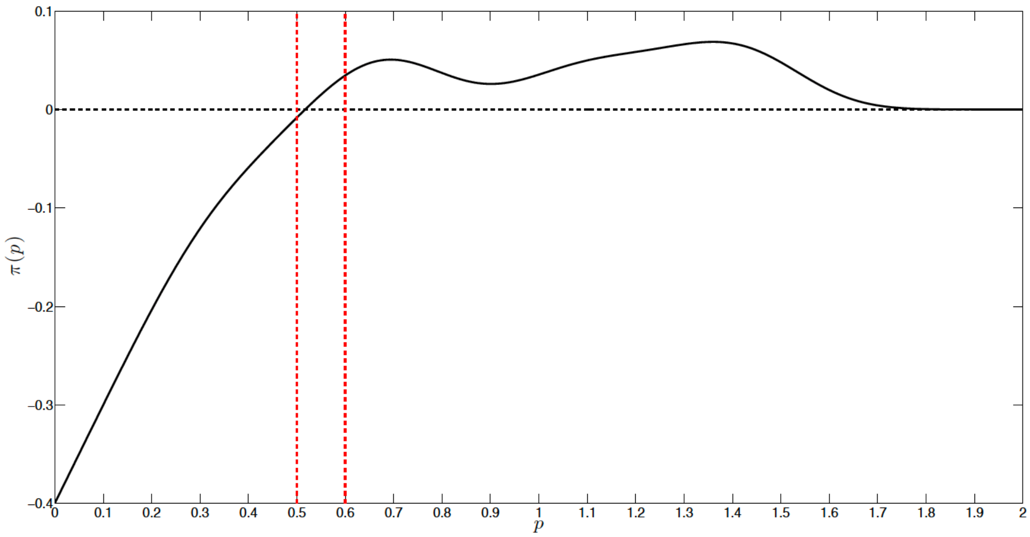

Figure 8: The Profit Curve $\pi(p)$ in Example A1

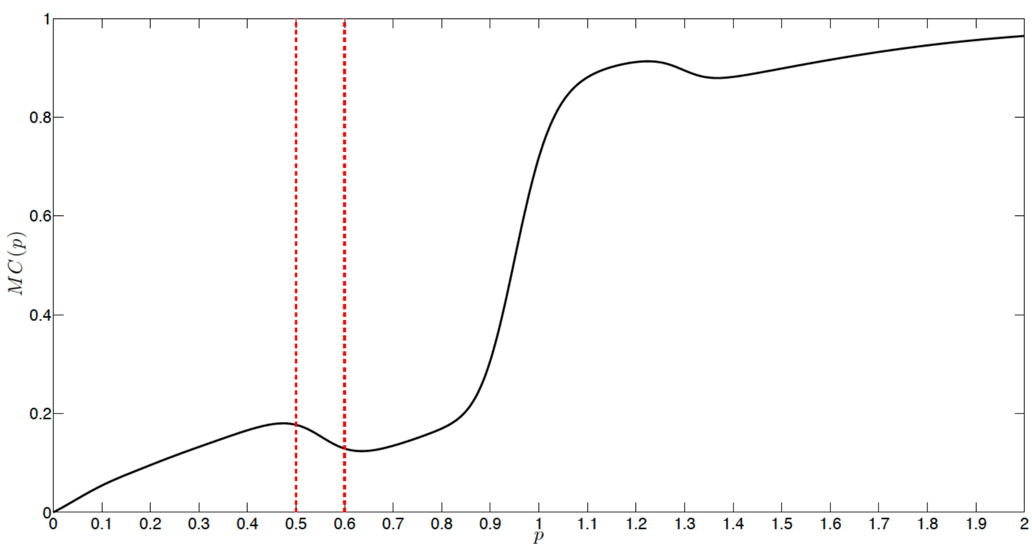

Figure 9: The Marginal Cost Curve $MC(p)$ in Example A1

D Derivation of the Profit Function, the Demand Curve and the Cost Curves in Proposition 6

Profit Function. For $p \leq 0$, all consumers purchase insurance and the firm's expected profit is $\pi(p) = p - \mathbb{E}[M] = p - \frac{1}{2}$. For $p \geq k+1 \equiv \overline{m} + k\overline{\lambda}$, the price exceeds the highest WTP and no consumers purchase insurance. Therefore, $\pi(p) = 0$.

For $p \in [0,1]$, the expected profit is,

$$\begin{aligned}
\pi(p;\mu) &= \int_{\theta \in \mathcal{B}(p) \equiv \left\{\theta : v(\theta;x) \geq p\right\}} (p - xm)\, dH(m,\lambda) \\
&= p - \mathbb{E}[M] - \left[\mu \int_{\theta \in \mathcal{NB}(p)} (p-m)\, d\mathcal{W}(m,\lambda) + (1-\mu) \int_{\theta \in \mathcal{NB}(p)} (p-m)\, d\Pi(m,\lambda)\right] \\
&= (p - \frac{1}{2}) - (1-\mu) \int_0^p \int_0^{\frac{p-m}{k}} (p-m) d\lambda dm \\
&= (p - \frac{1}{2}) - (1-\mu) \frac{1}{k} \int_0^p (p-m)^2 dm = (p - \frac{1}{2}) - (1-\mu) \frac{1}{3k} p^3.
\end{aligned}$$

For $p \in [1,k]$, the expected profit is,

$$\begin{aligned}
\pi(p;\mu) &= \mu \int_{\theta \in \mathcal{B}(p)} (p-m)\, d\mathcal{W}(m,\lambda) + (1-\mu) \int_{\theta \in \mathcal{B}(p)} (p-m)\, d\Pi(m,\lambda) \\
&= (1-\mu) \int_0^1 \int_{\frac{p-m}{k}}^1 (p-m)\, d\lambda dm + \mu \int_0^{\frac{k-p}{k-1}} (p-m)\, dm \\
&= (1-\mu) \left[(p - \frac{1}{2}) + \frac{-3p^2 + 3p - 1}{3k}\right] + \frac{1}{2}\mu \frac{k-p}{k-1} \left[2p - \frac{k-p}{k-1}\right].
\end{aligned}$$

For $p \in [k, k+1]$, the expected profit is,

$$\begin{aligned}
\pi(p;\mu) &= \mu \int_{\theta \in \mathcal{B}(p)} (p-m)\, d\mathcal{W}(m,\lambda) + (1-\mu) \int_{\theta \in \mathcal{B}(p)} (p-m)\, d\Pi(m,\lambda) \\
&= (1-\mu) \int_{p-k}^1 \int_{\frac{p-m}{k}}^1 (p-m)\, d\lambda dm \\
&= (1-\mu) \left[\frac{(p-1)^3}{3k} - \frac{(p-1)^2}{2} + \frac{k^2}{6}\right].
\end{aligned}$$

To summarize,

$$\pi(p;\mu)=\begin{cases} p-\frac{1}{2} & \text{for} \quad p\in(-\infty,0] \\ (p-\frac{1}{2})-(1-\mu)\frac{1}{3k}p^3 & \text{for} \quad p\in[0,1] \\ (1-\mu)\left[(p-\frac{1}{2})+\frac{-3p^2+3p-1}{3k}\right]+\frac{1}{2}\mu\frac{k-p}{k-1}\left[2p-\frac{k-p}{k-1}\right] & \text{for} \quad p\in[1,k] \\ (1-\mu)\left[\frac{(p-1)^3}{3k}-\frac{(p-1)^2}{2}+\frac{k^2}{6}\right] & \text{for} \quad p\in[k,k+1] \\ 0 & \text{for} \quad p\in[k+1,\infty). \end{cases}$$

Demand curve, total cost curve, and average cost curve. The derivation of the demand curve and the total cost curve is similar to the derivation of the profit function and is omitted. The total cost curve is given by,

$$TC(p;\mu)=\begin{cases} \frac{1}{2} & \text{for} \quad p\in(-\infty,0] \\ (1-\mu)(\frac{1}{2}-\frac{1}{6k}p^3)+\frac{1}{2}\mu & \text{for} \quad p\in[0,1] \\ \frac{1}{2}\mu(\frac{k-p}{k-1})^2+(1-\mu)[\frac{1}{2}+\frac{1}{k}(\frac{1}{3}-\frac{1}{2}p)] & \text{for} \quad p\in[1,k] \\ (1-\mu)\left\{\frac{1}{2}(1-\frac{p}{k})[1-(p-k)^2]+\frac{1}{3k}[1-(p-k)^3]\right\} & \text{for} \quad p\in[k,k+1] \\ 0 & \text{for} \quad p\in[k+1,\infty), \end{cases}$$

and the demand curve is given by,

$$D(p;\mu)=\begin{cases} 1 & \text{for} \quad p\in(-\infty,0] \\ \mu+(1-\mu)(1-\frac{1}{2k}p^2) & \text{for} \quad p\in[0,1] \\ \mu(\frac{k-p}{k-1})+(1-\mu)[1+\frac{1}{k}(\frac{1}{2}-p)] & \text{for} \quad p\in[1,k] \\ (1-\mu)\frac{1}{2k}(1+k-p)^2 & \text{for} \quad p\in[k,k+1] \\ 0 & \text{for} \quad p\in[k+1,\infty). \end{cases}$$

Finally, the average cost is given by,

$$AC(p;\mu)=\mathbb{E}\left[M|\mathcal{B}\left(p\right)\right]=\frac{TC(p;\mu)}{D(p;\mu)}.$$

Marginal Cost Curve. The marginal cost is defined as,

$$MC(p;\mu)=\mathbb{E}\left[M|v(\theta)=p\right]=\frac{\int_{\theta\in\left\{\theta:v(\theta;1)=p\right\}}mdH(m,\lambda)}{\int_{\theta\in\left\{\theta:v(\theta;1)=p\right\}}dH(m,\lambda)}\equiv\frac{dTC(p)}{dp}\Big/\frac{dD(p)}{dp}.$$

Notice that, $MC(p)$ is well-defined only when there exists positive demand for insurance at a price. For $\mu=1$, the price that induces positive demand lies between 1 to k. Therefore, $MC(p;\mu)$ is given by,

$$MC(p)=\frac{k-p}{k-1}\text{ for }p\in[1,k].$$

For $\mu\in[0,1)$, the price range associated with positive demand is $[0,k+1]$. Therefore, $MC(p;\mu)$ is given by,

$$MC(p;\mu)=\begin{cases} \frac{1}{2}p & \text{for} \quad p\in[0,1] \\ \left[\frac{\mu}{k-1}\frac{k-p}{k-1}+\frac{1-\mu}{2k}\right]\Big/\left[\frac{\mu}{k-1}+\frac{1-\mu}{k}\right] & \text{for} \quad p\in[1,k] \\ \frac{1}{2}[(p-k)+1] & \text{for} \quad p\in[k,k+1]. \end{cases}$$

E Derivation of the Profit Function in the Numerical Analysis in Section 7 for $k<1$

For $p\leq 0$, all consumers purchase the insurance and firm's expected profit is $\pi(p)=p-\mathbb{E}[M]=p-\frac{1}{2}$. For $p\geq k+1\equiv\overline{m}+k\overline{\lambda}$, the price exceeds the highest WTP and no consumers purchase insurance. Therefore, $\pi(p)=0$.

For $p\in[0,k]$, the expected profit is,

$$\begin{aligned}
\pi(p,1;k) &= \int_{\theta\in\mathcal{B}(p)\equiv\left\{\theta:v(\theta;x)\geq p\right\}}(p-xm)\,dH(m,\lambda) \\
&= p-\mathbb{E}[M]-\left[\mu\int_{\theta\in\mathcal{NB}(p)}(p-m)\,d\mathcal{W}(m,\lambda)+(1-\mu)\int_{\theta\in\mathcal{NB}(p)}(p-m)\,d\Pi(m,\lambda)\right] \\
&= \left(p-\frac{1}{2}\right)-(1-\mu)\int_0^p\int_0^{\frac{p-m}{k}}(p-m)d\lambda dm \\
&= \left(p-\frac{1}{2}\right)-(1-\mu)\frac{1}{k}\int_0^p(p-m)^2dm=\left(p-\frac{1}{2}\right)-(1-\mu)\frac{1}{3k}p^3.
\end{aligned}$$

For $p\in[k,1]$, the expected profit is,

$$\begin{aligned}
\pi(p,1;k) &= \mu\int_{\theta\in\mathcal{B}(p)}(p-m)\,d\mathcal{W}(m,\lambda)+(1-\mu)\int_{\theta\in\mathcal{B}(p)}(p-m)\,d\Pi(m,\lambda) \\
&= (1-\mu)\int_0^1\int_{p-k\lambda}^1(p-m)\,dmd\lambda+\mu\int_{\frac{k-p}{k-1}}^1(p-m)\,dm \\
&= (1-\mu)\left[\frac{1}{6}k^2-\frac{1}{2}(p-1)^2\right]+\frac{1}{2}\mu\left(1-\frac{k-p}{k-1}\right)\left(2p-1-\frac{k-p}{k-1}\right) \\
&= (1-\mu)\left[\frac{1}{6}k^2-\frac{1}{2}(p-1)^2\right]+\mu\left(k-\frac{1}{2}\right)\frac{(p-1)^2}{(k-1)^2}.
\end{aligned}$$

For $p\in[1,k+1]$, the expected profit is,

$$\begin{aligned}
\pi(p,1;k) &= \mu\int_{\theta\in\mathcal{B}(p)}(p-m)\,d\mathcal{W}(m,\lambda)+(1-\mu)\int_{\theta\in\mathcal{B}(p)}(p-m)\,d\Pi(m,\lambda) \\
&= (1-\mu)\int_{p-k}^1\int_{\frac{p-m}{k}}^1(p-m)\,d\lambda dm \\
&= (1-\mu)\left[\frac{(p-1)^3}{3k}-\frac{(p-1)^2}{2}+\frac{k^2}{6}\right].
\end{aligned}$$

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