Life Insurance and Life Settlement Markets
with Overconfident Policyholders[*]

Hanming Fang[†]   Zenan Wu[‡]

March 31, 2020

Abstract

We analyze how the life settlement market—the secondary market for life insurance—may affect consumer welfare in a dynamic equilibrium model of life insurance with one-sided commitment and overconfident policyholders. In our model, policyholders may lapse their life insurance policies when they lose their bequest motives; however, they are overconfident in the sense that they may underestimate the probability of losing their bequest motives. We show that in the competitive equilibrium without life settlement, overconfident consumers will buy life insurance contracts with "too much" reclassification risk insurance for later periods. The life settlement market can impose a limit on the extent to which primary insurers can exploit overconfident consumers. We show that the life settlement market may increase the equilibrium consumer welfare of overconfident consumers when they are sufficiently "vulnerable" in the sense that they have a sufficiently large intertemporal elasticity of substitution of consumption. Our result is robust to alternative specifications where (i) insurers cannot observe the subjective or objective probability that policyholders will lose their bequest motives; (ii) insurers can include health-contingent cash surrender values (CSVs) in the life insurance contract; and (iii) policyholders underestimate their future mortality risk.

Keywords: Life Insurance; Secondary Market; Overconfidence; Intertemporal Elasticity of Substitution.
JEL Classification Codes: D03, D86, G22, L11.

1 Introduction

Life insurance is a prevalent long-term contract for policyholders who want to prevent economic disaster for their dependents when they die. Life insurance is a large and growing industry. According to Life Insurance Marketing and Research Association International (LIMRA international), 70% of U.S. households owned some type of life insurance in 2010. U.S. families purchased $2.8 trillion of insurance coverage in 2013 and the total life insurance coverage in the U.S. was $19.7 trillion by the end of 2013. The average face amount of the individual life insurance policies purchased increased from $81,000 in 1993 to 165,000 in 2013 at an average annual growth rate of 3.56%.[1]

An important feature of the life insurance market is that policyholders often allow their policies to lapse (i.e., they do not renew them) before the end of the intended coverage period and receive a payment—commonly referred to as the *cash surrender value* (CSV, henceforth)—from the insurer that is a small fraction (typically 3-5%) of the policy's face value. The life insurance market is subject to substantial lapsing. Consider, for instance, universal life insurance, for which 75% of policyholders allow their policies to lapse (Deloitte, 2005).[2] Policyholders may let the contract lapse if they lose their bequest motives and thus no longer need life insurance (due, for example, to the death of a spouse, a divorce, or changes in circumstances of the intended beneficiaries of the insurance policy) or if they are pressed for liquidity (due, for example, to a negative income shock or to a large unexpected medical expenditure). Fang and Kung (2012) show that income shocks are relatively more important than shocks to bequest motives in explaining lapsation when policyholders are young; however, as policyholders age, shocks to bequest motives become the more important factor in lapsation.

Recently, the secondary market for life insurance (also known as the life settlement market) has emerged, offering policyholders the option of selling their unwanted policies for more than the CSV. More than 20% of all policyholders above age 65 considered selling their policy on the secondary market to be an attractive alternative to surrendering it or allowing it to lapse (Doherty and Singer, 2003). If a policyholder decides to sell his/her insurance contract to the settlement firm, the settlement firm continues to pay the premium for the policyholders; in return, the life settlement firm becomes the beneficiary of the policy and collects the death benefits if the insured dies. Since the candidates for life settlements tend to be relatively old,[3] we will assume in this paper that the driver of the lapsation is the loss of bequest motives (Fang and Kung, 2012).

Although the life settlement market is in its infancy, it draws attention from life insurance firms, who lobby intensively to prohibit the securitization of life settlement contracts.[4] They argue that the life insurance contract is designed to take into consideration the fact that a fraction of policyholders let their contracts lapse without receiving the death benefits; the existence of the settlement market forces the insurance firms to pay death benefits on more policies than expected,

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[1]See American Council of Life Insurers (2014).

[2]According to Gottlieb and Smetters (2019), the annualized lapsation rates of all life insurance policies are about 4.2%. Similarly, Gatzert (2010) documents that lapsation rates are around 7% per year.

[3]See https://www.lisa.org/consumer-advisors/life-settlement-basics/defining-life-settlements.

[4]See Martin (2010) for detailed discussions of life insurance and the life settlement market.

which will lead to higher premiums for policyholders in the long run and thus will eventually hurt consumers. The life settlement industry, on the other hand, has been working hard to justify its existence, emphasizing its role of enhancing liquidity to policyholders.[5] It is interesting to note that the life settlement industry has gained some success recently. For instance, in 2010, the General Assembly in Kentucky passed a bill requiring insurers to inform policyholders who are considering surrendering their policy that the settlement is a potential alternative.[6]

Should the life settlement industry be banned? To resolve this theoretically and practically important question, it would be useful to first understand the role of the life settlement market and its impact on policyholders' welfare. In this paper we extend the models of Daily, Hendel, and Lizzeri (2008) and Fang and Kung (2020) to study the welfare implications of the life settlement market in a setting where consumers are *biased*. Specifically, consumers may be *overconfident* about the probability of losing their bequest motives at the time they purchase the contract, or about their future mortality risk.[7] Daily et al. (2008) are aware of the importance of the investigation of the consequences of consumers' behavioral bias in the life insurance market, as they stated: "In our model, all agents are forward-looking and standard-expected-utility maximizers. We thus abstract from potential violations of 'rational' behavior. Our view is that considerations such as regret or misperceptions of probabilities may very well be relevant in this [life insurance] market."

We show in this paper that, when life insurance policyholders exhibit overconfidence with regard to either their bequest motives or their future mortality risk, the life insurers will exploit policyholders' biased beliefs in their contract design. As a result, the competitive equilibrium contract deviates from the rational benchmark and leads to dynamic inefficiency in consumption. Specifically, the equilibrium contracts under overconfidence in the persistence of bequest motives are overly front-loaded relative to the rational benchmark. Furthermore, we show that the magnitude of the dynamic inefficiency caused by policyholders' overconfidence depends crucially on the curvature—the intertemporal elasticity of substitution (IES)—of their utility functions, and policyholders become more vulnerable due to their overconfidence as the degree of their IES increases. The presence of the settlement market provides policyholders a channel to correct their earlier mistakes in later periods, thus mitigating the loss due to their misperception. This new role of the settlement market generates a potential source of welfare gain that is absent when consumers are fully rational. We show that the potential welfare gain is substantial when policyholders are

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[5]As mentioned in Martin (2010): "In 2008, the executive of the life settlement industry's national trade organization testifies to the Florida Office of Insurance Regulation that the 'secondary market for life insurance has brought great benefits to consumers, unlocking the value of life insurance policies.'"

[6]Similar requirements exist in Maine, Oregon, Washington (see Martin, 2010) and the U.K. (see Januário and Naik, 2014).

[7]For overconfidence in marriage quality—which may lead to overconfidence in the persistence of bequest motives— Baker and Emery (1993) find that individuals dramatically underestimate the likelihood that their marriage will end in divorce: despite recognizing that the overall divorce rate in the population was 50%, the respondents' median estimate of their own chance of divorce was 0%. Mahar (2003) also provides evidence that false optimism that marriages will last plays an important role in explaining why only about 5% of married couples have prenuptial agreements (see Marston, 1997). Further, many studies document that people are unrealistically optimistic about future life events and their future mortality risk. For instance, Robb, Miles, and Wardle (2004) detect underestimation of risk among patients who participated in cancer examinations. They find that the self-perceived risk is lower than the actual risk of colorectal cancer determined by flexible sigmoidoscopy screening.

sufficiently overconfident and have a large IES value. As a result, life settlement can potentially increase consumer welfare in equilibrium. This finding stands in contrast to the findings of Daily, Hendel, and Lizzeri (2008) and Fang and Kung (2020), which demonstrate that introducing the life settlement market always reduces consumer welfare in equilibrium when policyholders have rational beliefs and when lapsation is due to loss of bequest motives. Our results thus contribute to the debate over the potential welfare consequences of the life settlement market.

Related Literature   This paper is naturally linked to the growing literature on life insurance. In a seminal paper, Hendel and Lizzeri (2003) use a two-period model to analyze the role of commitment on the long-term life insurance contract. In their model, risk-neutral life insurance firms compete to offer contracts to risk-averse consumers who are subject to mortality risk. Consumers' health states may change over time and thus they may face *reclassification risk* in the spot market. Insurance firms are able to commit to contractual terms while consumers lack commitment power because they can allow the contract to lapse in the second period. In this environment with one-sided commitment, they prove that the equilibrium contract is *front-loaded*: consumers are offered a contract with a first-period premium that is higher than an actuarially fair one in exchange for reclassification risk insurance in the second period. Daily, Hendel, and Lizzeri (2008) and Fang and Kung (2020) investigate this problem further by introducing a settlement market and analyzing its effect on the equilibrium contract and consumer welfare. In their models, policyholders may lose their bequest motives in the second period, resulting in lapsation and a potential demand for the settlement market. Using a model similar to Hendel and Lizzeri (2003), Fang and Kung (2020) show that, when consumers are fully rational, the presence of the settlement market reduces consumer welfare. In addition, they show that the equilibrium life insurance contract would have zero cash surrender value in the absence of the life settlement market, while in contrast, in the presence of the life settlement market, health-contingent cash surrender values in the life insurance contract can improve consumer welfare. Relatedly, Polborn, Hoy, and Sadanand (2006) develop a three-period model of a life insurance market and investigate the effects of regulations prohibiting the use of information to risk-rate premiums. They conclude that it may be welfare-enhancing to prohibit the use of genetic tests for rate-making. We extend the theoretical literature on life insurance by explicitly pointing out the disciplinary role of a secondary market in limiting primary insurers' exploitation of consumers' biases through contract design.[8]

This paper also belongs to the extensive literature on behavioral contract theory.[9] Most papers assume consumers exhibit some type of behavioral bias, and investigate how firms respond to biased consumers and design contracts accordingly. For instance, de la Rosa (2011) and Santos-Pinto (2008) study the incentive contract in a principal-agent model of moral hazard when the agents are

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[8]Gao, He, and He (2018) consider a model in which competing financial intermediaries offer contracts to investors with time-inconsistent preferences who may liquidate their investment prematurely when the liquidation cost is low. They show that financial intermediaries are compelled to offer more options for early withdrawals on a linear scheme if a secondary market for long-term contracts opens for trading, and the welfare of naïve investors can improve as a result.

[9]See Kőszegi (2014) for a comprehensive survey on this topic. See also Heidhues and Kőszegi (2018) and Grubb (2015) for surveys on behavioral industrial organization and overconfident consumers in the marketplace.

overconfident. Fang and Moscarini (2005) show that firms have incentives to compress wages so as not to reveal their private information about workers' productivity in an environment where workers are overconfident about their ability, and where ability and effort are complements in the production function. Grubb (2009) proposes a model of optimal contracting with overconfident consumers in the cellular phone services market. In the context of the insurance market, Sandroni and Squintani (2007) modify the textbook Rothschild-Stiglitz model to study the equilibrium contract by assuming that a fraction of the insurees are overconfident about their risk types. They find that when a significant fraction of the consumers are overconfident, compulsory insurance may serve as a transfer of income between different types of agents. Their results have different implications than those of Rothschild and Stiglitz (1976) on government intervention in the insurance market. Spinnewijn (2015) studies the optimal unemployment insurance contract under perfect competition where the insuree has a misperception about the probability of finding a job. Gottlieb (2008) considers the impact of non-exclusivity in a competitive market when firms offer contracts to compete for the business of present-biased consumers. He shows that non-exclusive contracts would invalidate the type of profit-maximizing contracts proposed in DellaVigna and Malmendier (2006) for leisure goods, i.e., goods with immediate rewards and deferred costs, such as tobacco, alcohol, and unhealthy food, where present-biased consumers would receive a lump-sum transfer and pay a usage price higher than the marginal cost. In some sense, the non-exclusivity of the contracts in Gottlieb (2008) plays a role similar to the secondary market in our setting. Heidhues and Kőszegi (2017) investigate the welfare consequences of discrimination based on consumers' naïveté, and show that they differ starkly from those obtained under discrimination based on consumers' preference.[10] Our paper advances the exploitative contracting literature by identifying the intertemporal elasticity of substitution (IES) of consumption as a measure of consumers' vulnerability to their behavioral bias, a new insight which we believe is not restricted to the life insurance and life settlement markets.[11]

Our paper is most closely related to Schumacher (2016) and Gottlieb and Smetters (2019). Both studies introduce consumer behavioral biases into a dynamic insurance model. Schumacher (2016) models an insurance market in which consumers exhibit time-inconsistent preferences and suffer from self-control problems. Consumers, who are either sophisticated or naive, can exert costly effort to remain in the low-risk category. Sophisticated consumers engage in healthy lifestyles and end up being low-risk consumers. Naive consumers believe that they will behave the same as the sophisticates, but may shirk and become high-risk consumers. When firms compete to offer long-term contracts, all consumers initially select the same contract and the sophisticates may subsidize or receive a transfer from the naifs, depending on the fraction of the naifs in the population.[12] In

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[10] We will discuss the connection between our results and theirs in Section 3.2.

[11] See Fang and Wu (2020) for further investigations in this direction.

[12] To see this, note that naive consumers would generate higher costs to the insurer due to their present bias. Therefore, they exert a negative externality on the sophisticates as in Sandroni and Squintani (2007) if the insurance contract is inflexible. When firms can offer long-term flexible contracts—which allow consumers to switch among insurance options offered by the same insurer—naive consumers would alter their insurance option in equilibrium after realizing that they will shirk. As such, the transfer from the sophisticates to the naifs will be lowered and even turn negative, because the naive consumers have a higher willingness to pay than the sophisticates and are locked in

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contrast, cross-subsidy will not occur if insurers compete through short-term contracts as in the Rothschild-Stiglitz model. Schumacher (2016) obtains results similar to ours and shows that spot contracting may protect naive consumers from being exploited by the sophisticates under long-term contracting. It is noteworthy that Schumacher (2016) compares consumer welfare in a competitive market—where only short-term contracts are provided—to the case where only long-term contracts are provided, whereas in our model long-term contracts and short-term contracts can coexist. Further, consumers are heterogeneous in Schumacher (2016), while we assume homogeneous agents. Therefore, the welfare comparison in our model is driven by the threat of the spot market on the long-term insurance market, rather than the degree of externality that sophisticated consumers impose on the naifs.

Gottlieb and Smetters (2019) investigate equilibrium life insurance contracts where consumers may (i) forget to pay premiums, or (ii) fail to sufficiently account for uncorrelated income shocks. Unlike our setup, in which lapsation is driven by policyholders' loss of bequest motives, lapsation in their model is due to policyholders' forgetfulness or negative income shocks. They show that policyholders endogenously allow their policies to lapse after they miss a payment or suffer a large negative income shock, and lapsers cross-subsidize non-lapsers in the competitive equilibrium. In an early version of Gottlieb and Smetters (2012), the authors show that the presence of the settlement market can help smooth consumption and increase consumer welfare if lapsation is driven by negative income shocks. Our study differs from Gottlieb and Smetters (2012, 2019) in terms of both focus and insight. Gottlieb and Smetters (2019) aim to explain the pattern of primary life insurance contracts observed in practice (e.g., being lapse-based, allowing for policy loans). Therefore, it is important that lapsation emerges endogenously in equilibrium.[13] In contrast, our paper focuses on the welfare consequences of introducing a secondary life settlement market, and lapsation in our model can be considered exogenous.[14] Further, consumers in Gottlieb and Smetters (2012) are unambiguously better off when the settlement market is introduced. Our setup allows us to identify the curvature of consumers' utility function as a crucial factor of welfare comparison.

The remainder of the paper is organized as follows. In Section 2, we first present the baseline model of dynamic life insurance when policyholders underestimate the probability of losing their bequest motives and characterize the set of equilibrium contracts without the life settlement market; we then introduce the settlement market and describe its impact on the equilibrium contracts. In Section 3, we present our main result (i.e., Theorem 1), which characterizes the welfare consequences of introducing the settlement market. In Section 4, we show that our analysis can be extended to the situation where policyholders possess private information regarding the actual or perceived probability of losing their bequest motives, and Theorem 1 is robust when insurers are allowed to include health-contingent cash surrender values into a life insurance contract, and when policyholders exhibit overconfidence regarding their future mortality risk. In Section 5, we

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ex post.

[13]For instance, insurers can allow consumers to keep their policy after missing a payment. Similarly, insurers can avoid lapsation through providing policy loans to consumers after an income shock.

[14]To see this, note that policyholders' demand for life insurance dissolves once they lose their bequest motives, and that lapsation always occurs in the absence of the settlement market.

discuss the assumptions of our model and the empirical implications of our results. In Section 6, we summarize our main findings and suggest directions for future research. All proofs are relegated to the Appendix.

2   The Baseline Model

In this section, we propose a model of dynamic life insurance that is slightly modified from that of Hendel and Lizzeri (2003) and Fang and Kung (2020), and introduce consumer bias on the prediction of the probability of losing their bequest motives. We will discuss the case of overconfidence regarding future mortality risk in Section 4.3.

2.1   Preliminaries

Consider an environment with a continuum of consumers (potential policyholders) who may live up to two periods. The life insurance market is perfectly competitive.

Income, Health, and Preference.   The policyholder receives an income of y_1 in the first period and y_2 in the second period. In the first period, the policyholder has a mortality risk (i.e., death probability) of $p_1 \in (0, 1)$. The death probability is interpreted as the health state of the policyholder. In the second period, the mortality risk $p_2 \in [0, 1]$ is randomly drawn from a distribution with continuous density $\phi(\cdot)$ and corresponding CDF $\Phi(\cdot)$. We follow the literature (e.g., Hendel and Lizzeri, 2003; Daily, Hendel, and Lizzeri, 2008; and Fang and Kung, 2020) and assume that (i) p_1 is common knowledge between policyholders and insurance firms; and (ii) health state p_2 is not known in the first period when the policyholder purchases the insurance and is symmetrically learned by the insurance firms and the policyholder at the beginning of the second period.[15]

A policyholder has two potential sources of utility. If the policyholder is alive and consumes $c \geq 0$, he receives utility $u(c)$ from his own consumption; if the policyholder dies, then he receives utility $v(c)$ from his dependent's consumption c, provided that the policyholder retains his bequest motives at the time of his death. We assume that both $u(\cdot)$ and $v(\cdot)$ are strictly increasing, twice differentiable and strictly concave. Furthermore, we assume that $u(\cdot)$ and $v(\cdot)$ satisfy the Inada conditions: $\lim_{c \searrow 0} u'(c) = \infty$, $\lim_{c \searrow 0} v'(c) = \infty$, and $\lim_{c \nearrow \infty} v'(c) = 0$.

The following assumption is made throughout the paper:

Assumption 1 $y_2 > y_1 - \delta$, where $\delta \in (0, y_1)$ is the unique solution to $u'(y_1 - \delta) = v'\left(\delta/p_1\right)$.

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[15]Cawley and Philipson (1999) find no evidence of adverse selection in term life insurance, which may justify our assumption on the period-1 mortality risk. As discussed in Hendel and Lizzeri (2003), the absence of period-1 adverse selection can be potentially explained by the facts that (i) buyers have to pass a medical examination and answer a detailed questionnaire when they buy insurance; (ii) misrepresenting or hiding material information would invalidate the policy; and (iii) insurers share information about policyholders. Moreover, Hendel and Lizzeri (2003) argue that more front-loading would be associated with a higher coverage cost if policyholders possess superior information about future death probabilities and find the opposite. Their empirical finding indicates that period-2 information asymmetry is not crucial in shaping the equilibrium contracts, and supports the assumption of symmetric learning about period-2 mortality risk.

Assumption 1 allows for different patterns of the policyholders' income dynamics between the two periods: income can either increase or decrease as policyholders age. Note that Assumption 1 requires that the second-period income y_2 be larger than $y_1 - \delta$. To better understand the assumption, note that the variable δ specifies the maximum amount of income that an individual is willing to sacrifice in exchange for his dependent's consumption upon his death, given the constraint that the expected first-period consumption level of the family is y_1. Therefore, $y_1 - \delta$ refers to the minimum first-period consumption level required by an individual policyholder. If y_2 falls below $y_1 - \delta$, then policyholders have strong incentives to smooth consumption across periods through saving, and as a result the equilibrium contract may contain an investment component, which significantly complicates the analysis.

Bequest Motives and Overconfidence. A policyholder does not lose his bequest motives in the first period. However, the policyholder may lose his bequest motives with probability $q \in (0, 1)$ at the beginning of period 2. If the policyholder loses his bequest motives, then he no longer derives utility from his dependent's consumption, in which case he receives some constant utility normalized to zero if he dies.

The policyholder *believes* that the probability of losing his bequest motives is $\tilde{q} \in [0, q]$. When $\tilde{q} = q$, the policyholder is rational. When $\tilde{q} < q$, the policyholder exhibits *overconfidence* in the sense that he *underestimates* the probability of losing his bequest motives. For the ease of exposition, let us denote

$$\Delta \equiv \frac{q - \tilde{q}}{q} \tag{1}$$

as the degree of the consumer's overconfidence. When there is a continuum of consumers, the variable $\Delta \in [0, 1]$ also indicates the fraction of policyholders who lose their bequest motives unexpectedly in period 2. Both $\tilde{q}$ and q are assumed to be observable to firms. We discuss in Section 4.1 the issue of observability and show that our results would remain intact when these restrictions are relaxed.

Timing, Commitment, and Contracts At the beginning of the first period, the consumer learns his first-period health state p_1 and then chooses to purchase a long-term life insurance contract. As in Hendel and Lizzeri (2003), a long-term insurance contract is in the form of:[16]

$$\langle (Q_1, F_1), (Q_2(p_2), F_2(p_2)) : p_2 \in [0, 1] \rangle, \tag{2}$$

where (Q_1, F_1) specifies a premium and face value for the first period, and $(Q_2(p_2), F_2(p_2))$ specifies the corresponding premium and face value for each health state $p_2 \in [0, 1]$ in the second period.

Two remarks are in order. First, we implicitly exclude cash surrender values (CSVs) from the contract, or equivalently, we assume that CSVs are restricted to be zero in the baseline model. We will enrich the set of insurers' admissible contracts to allow for health-contingent CSVs in

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[16]The contract terms should all be indexed by $(q, \tilde{q})$ or (q, Δ). We ignore this for expositional ease.

Section 4.2. Second, the menu of the second-period premiums and face values specified in (2) is state-dependent. Contingent contracts are common in the life insurance market. For instance, a "select and ultimate annual renewable term product" (S&U ART) allows for reclassification and offers state-contingent prices. Specifically, an S&U ART will reward an existing policyholder with a premium discount if the policyholder shows that he/she is still in good health; and thus premiums vary by issue age and duration since underwriting.[17]

At the end of the first period, with probability p_1, the policyholder dies and his dependent receives the face value F_1. With the remaining probability, the policyholder continues to period 2 and observes, as does the insurance company, his period-2 health state p_2. We assume *one-sided commitment* by the insurance firm, i.e., the insurance firm can commit to future premiums and face values specified in the long-term contract. However, the policyholder can choose to continue with the long-term contract purchased in the first period, but he is also free to terminate the long-term contract purchased in period 1 and purchase a spot contract from the perfectly competitive spot market if he desires.[18] When we later introduce the life settlement market, he can also sell the long-term contract in the secondary market.

2.2 Equilibrium Contracts without the Settlement Market

We first characterize the equilibrium contract without the settlement market. The key here is to understand how competitive insurers will design their dynamic long-term contracts so as to most appeal to the overconfident consumers. The equilibrium long-term contract $\langle (Q_1, F_1), (Q_2(p_2), F_2(p_2)) : p_2 \in [0, 1] \rangle$ solves:[19]

$$\max \left[u(y_1 - Q_1) + p_1 v(F_1) \right] \tag{3}$$
$$+ (1 - p_1) \int_0^1 \left\{ (1 - \tilde{q}) \left[u \left(y_2 - Q_2(p_2) \right) + p_2 v \left(F_2(p_2) \right) \right] + \tilde{q} u(y_2) \right\} d\Phi(p_2)$$

$$\text{s.t. } (Q_1 - p_1 F_1) + (1 - p_1)(1 - q) \int_0^1 \left[Q_2(p_2) - p_2 F_2(p_2) \right] d\Phi(p_2) = 0, \tag{4}$$

$$Q_2(p_2) - p_2 F_2(p_2) \leq 0, \text{ for all } p_2 \in [0, 1], \tag{5}$$

$$Q_2(p_2) \geq 0, \text{ for all } p_2 \in [0, 1]. \tag{6}$$

Note that the objective function (3) in the above maximization problem is the policyholders' expected *perceived* utility using the subjective belief $\tilde{q}$ about losing their bequest motives, instead of the utility based on the *objective* probability q of losing their bequest motives.[20] Constraint (4)

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[17]See Table 1 in Hendel and Lizzeri (2003) for a description of an S&U ART contract.

[18]A spot contract in our model can be interpreted as a life insurance policy with a shorter term.

[19]The maximization problem (3) is similar to that of Fang and Kung (2020) with two main differences: first, the perceived probability $\tilde{q}$ of losing bequest motives is used in the objective function of the consumers; second, the non-negative constraints for second-period premiums (6) may be binding in this environment, in contrast to the case when $\tilde{q} = q$, as we will show below in Lemma 4.

[20]Thus, the perceived expected utility in (3) is the *decision utility* in the terminology of Kahneman, Wakker, and Sarin (1997). As we will explain later, policyholders' expected utility according to the *correct*, or *objective*, probability of losing their bequest motives, q, is used when we evaluate consumer welfare, corresponding to the

is the zero-profit condition that captures the competition in the primary insurance market. It is important to note that in (4), the insurance company uses the objective probability q of the policyholder losing his bequest motives in evaluating the second-period expected profit. Constraint (5) is the no-lapsation condition for policyholders whose bequest motives remain in period 2. Constraint (5) is important because the consumer is not committed to continuing with the long-term contract, and he will opt not to terminate the contract only if staying with the long-term contract is preferable to purchasing a spot contract. The intuition for why (5) ensures no lapsation for those with bequest motives is as follows.[21] For any long-term contract as specified by (2), $p_2 F_2(p_2) - Q_2(p_2)$ is the actuarial value of the second-period contract in health state p_2. Due to one-sided commitment, the policyholder can opt for a spot contract in the second period. Since the spot market is perfectly competitive, the actuarial value of the spot contract must be zero. Thus in order to prevent the policyholder from letting the long-term contract lapse and purchasing a spot contract instead, the actuarial value of the period-2 contract must be non-negative for all p_2, i.e., $p_2 F_2(p_2) - Q_2(p_2) \geq 0$. Finally, constraint (6) simply states that the second-period premium for any health state cannot be negative.[22]

Note that the objective function (3) is concave and that constraints (4), (5), and (6) are linear. Therefore, the Kuhn-Tucker conditions are necessary and sufficient for the global maximum. The first-order conditions with respect to Q_1 and F_1 imply immediately that:

$$u'\left(y_1 - Q_1\right) = v'\left(F_1\right). \tag{7}$$

That is, in equilibrium in period 1, the marginal utility of policyholder's consumption must be equal to the marginal utility of his dependent's consumption; this is referred to as *full-event first-period insurance*. Analogously, it would also be useful to define the *fair premium and face value* of the *full-event second-period insurance* in health state $p_2 \in [0, 1]$, which we denote by $Q_2^{FI}(p_2)$ and $F_2^{FI}(p_2)$ respectively, as the solution to the following pair of equations:

$$u'\left(y_2 - Q_2^{FI}(p_2)\right) = v'\left(F_2^{FI}(p_2)\right), \tag{8a}$$

$$Q_2^{FI}(p_2) - p_2 F_2^{FI}(p_2) = 0. \tag{8b}$$

This is indeed the equilibrium *spot* contract given health state p_2 in period 2.[23] It is straightforward to verify that $Q_2^{FI}(p_2)$ is strictly increasing in p_2, and $F_2^{FI}(p_2)$ is strictly decreasing in p_2.

To characterize the equilibrium contracts, we divide the support of the second-period health states p_2 into two subsets $\mathcal{B}$ and $\mathcal{NB}$: for $p_2 \in \mathcal{B}$, the no-lapsation constraint (5) binds; for $p_2 \in \mathcal{NB}$, the no-lapsation constraint (5) does not bind. In other words, an insurance firm breaks even at

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notion of *experienced utility* in Kahneman et al. (1997).

[21] For a formal argument of constraint (5), see Hendel and Lizzeri (2003).

[22] The non-negativity constraints for $F_2(p_2)$ and F_1 are ignored because they will never bind due to the Inada condition on $v(\cdot)$. Moreover, Q_1 will be strictly positive and hence the non-negativity constraint for Q_1 can be dropped from the maximization problem because constraints (4) and (5) imply that $Q_1 - p_1 F_1$ must be non-negative.

[23] The second-period spot contract $\langle Q_2(p_2), F_2(p_2) \rangle$ solves $\max u\left(y_2 - Q_2(p_2)\right) + p_2 v\left(F_2(p_2)\right)$ subject to $Q_2(p_2) - p_2 F_2(p_2) = 0$, which leads to the same first-order conditions as in (8).

health state p_2 if $p_2 \in \mathcal{B}$, and suffers a loss if $p_2 \in \mathcal{NB}$.

Lemma 1 *If $p_2 \in \mathcal{B}$ and $p'_2 \in \mathcal{NB}$, then $p_2 < p'_2$, $Q_2(p_2) \le Q_2(p'_2)$, and $F_2(p_2) \ge F_2(p'_2)$.*

Lemma 1 indicates that there exists a threshold p_2^* such that $p_2 \in \mathcal{B}$ if $p_2 < p_2^*$ and $p_2 \in \mathcal{NB}$ if $p_2 > p_2^*$. The following lemma fully characterizes the shape of the period-2 contracts for the case where $p_2^* \in (0, 1)$:

Lemma 2 *Suppose that $p_2^* \in (0, 1)$. Then the policyholder receives full-event second-period insurance for all health states $p_2 \in [0, 1]$, i.e.,*

$$u' \left(y_2 - Q_2(p_2) \right) = v' \left(F_2(p_2) \right). \tag{9}$$

Moreover, the equilibrium period-2 premiums $Q_2(p_2)$ must satisfy

$$Q_2(p_2) = \begin{cases} Q_2^{FI}(p_2) & if \quad p_2 \le p_2^*, \\ Q_2^{FI}(p_2^*) & if \quad p_2 > p_2^*, \end{cases} \tag{10}$$

where $Q_2^{FI}(p_2)$ is uniquely determined by (8).

By Equation (10), reclassification risk insurance is provided for the states above the mortality risk threshold p_2^*. Figure 1 illustrates Lemma 2 and shows that more period-2 health states are provided an actuarially *favorable* contract in equilibrium as p_2^* decreases. Therefore, the endogenous variable p_2^* measures the extent of reclassification risk insurance in the competitive equilibrium.[24]

Lemma 3 *If there exists a health state $\tilde{p}_2 \ne 0$ such that constraint (6) is binding, i.e., $Q_2(\tilde{p}_2) = 0$, then it must be the case that (6) is binding, namely, $Q_2(p_2) = 0$, for all $p_2 \in [0, 1]$.*

Lemma 3 indicates that $p_2^* = 0$ if the equilibrium period-2 premium is zero for some non-zero health state. Our next result provides a sufficient and necessary condition under which these zero period-2 premiums delineated in Lemma 3 arise:

Lemma 4 *There exist $\bar{q} \in (0, 1)$ and $\bar{\Delta}(q) \in (0, 1)$ such that $p_2^* = 0$ if and only if $q \ge \bar{q}$ and $\Delta \ge \bar{\Delta}(q)$.*

The intuition for Lemma 4 is as follows. Since the insurance company only needs to pay out the death benefit in period 2 if the policyholder retains his bequest motives, which occurs with probability $1 - q$, the higher q is, the lower the cost for the insurance company to provide the policyholder with reclassification risk insurance in period 2. Indeed, the competitive market ensures that in equilibrium, firms will push p_2^* lower so that more states have the level premiums. Importantly, if consumers are overconfident, then they *subjectively value* the period-2 reclassification risk insurance more than the *objective cost* for the firms to offer such insurance, thus creating a "wedge."

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[24]See the proof of Proposition 1 in the Appendix for more details on the derivation of p_2^*.

Competitive pressure among the insurance firms to attract the consumers in the first period will force the insurance firms to exploit this wedge, thus pushing firms to offer more reclassification insurance in period 2, pushing p_2^* even lower. In order to break even, the firms will of course raise the period-1 premium. When these two effects are strong enough (i.e., for sufficiently high q and Δ), contracts with zero period-2 premiums can emerge in equilibrium.

Note that our model assumes non-negative premiums from constraint (6), which imposes a lower bound on the equilibrium period-2 premiums. If negative premiums are allowed, then Lemma 4 predicts that zero or negative future premiums will be observed if consumers are sufficiently over-confident. To see this, suppose that the firms provide contracts with zero period-2 premiums in equilibrium for some (q, Δ) and policyholders become more overconfident. Because policyholders subjectively value the period-2 contracts more than before, firms have incentives to further take advantage of policyholders' overconfidence by offering better period-2 contracts. With the restriction of non-negative premiums, the only way a firm can improve the second-period contracts is to increase the face values. If this restriction is dropped, then firms have more tools available to them and can improve the second-period contracts by further reducing the period-2 premium and increasing the period-2 face value simultaneously. This leads to the possibility of negative premiums.

Combining Lemmas 1 through 4, the following proposition can be obtained:

Proposition 1 *(Equilibrium Contracts without Life Settlement Market)* *In the absence of the life settlement market, the equilibrium contracts satisfy the following properties:*

1. *Policyholders receive full-event insurance in period 1.*

2. *There is a period-2 threshold health state $p_2^* \in [0, 1]$ such that $p_2 \in \mathcal{B}$ if $p_2 < p_2^*$ and $p_2 \in \mathcal{NB}$ if $p_2 > p_2^*$.*

3. (a) *If $p_2^* = 0$, policyholders lose full-event insurance in period 2. Moreover, they are fully insured against reclassification risk and receive zero premiums in period 2 for all health states $p_2 \in [0, 1]$.*

   (b) *If $0 < p_2^* < 1$, policyholders receive full-event insurance in period 2. Moreover, the period-2 premiums are actuarially fair for all $p_2 \leq p_2^*$; the premiums are constant across all health states such that $p_2 > p_2^*$, and thus are actuarially favorable to the policyholder.*

   (c) *If $p_2^* = 1$, the equilibrium contract coincides with spot contracts.*

4. *When q and Δ are sufficiently large, $p_2^* = 0$.*

The wedge between the policyholder's *subjective valuation* of the period-2 reclassification risk insurance and firms' *objective cost* of offering such insurance is the key for the welfare effects of life settlement that we will describe in Section 3. Note that this wedge is amplified as consumers become more overconfident.
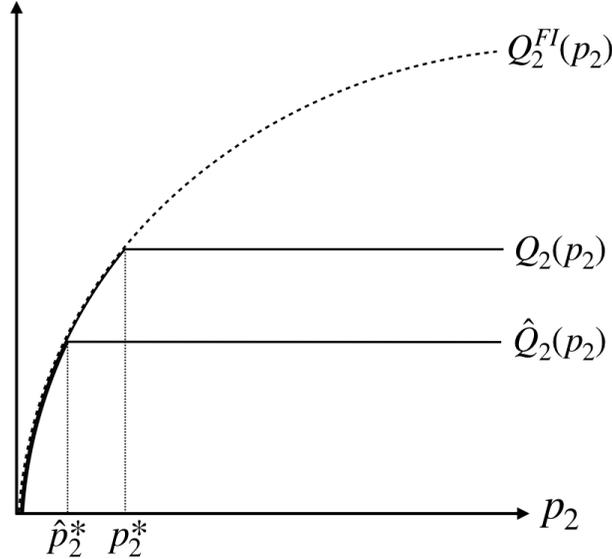
Figure 1: Equilibrium Period-2 Premium Profiles without the Settlement Market under Different Levels of Consumer Overconfidence: $\hat{\Delta} > \Delta$.

The next result illustrates how the equilibrium contracts are affected by consumers' increasing overconfidence. Let us fix the objective probability of losing bequest motives $q \in (0, 1)$. Consider two levels of subjective beliefs $\hat{\tilde{q}}$ and $\tilde{q}$, with $\hat{\tilde{q}} < \tilde{q} < q$. Define $\hat{\Delta} := \left(q - \hat{\tilde{q}}\right)/q$ and $\Delta := (q - \tilde{q})/q$. It follows immediately that $\hat{\Delta} > \Delta$. In words, the hat symbol denotes cases where consumers have a higher degree of overconfidence. Further, let $\langle \hat{Q}_1, \hat{F}_1, \hat{Q}_2(p_2), \hat{F}_2(p_2) \rangle$ and $\langle Q_1, F_1, Q_2(p_2), F_2(p_2) \rangle$ be the equilibrium long-term contracts under $\hat{\Delta}$ and Δ respectively, and let $\hat{p}_2^*$ and p_2^* be the threshold probabilities characterized in Proposition 1 respectively under $\hat{\Delta}$ and Δ. We have the following result:

Proposition 2 *(Higher Overconfidence Exacerbates Front-loading in the Absence of the* ***Life Settlement Market****) Suppose that* $\hat{\Delta} > \Delta$ *and* $0 < p_2^* < 1$. *Then* $\hat{Q}_1 > Q_1$, $\hat{F}_1 < F_1$, *and* $\hat{p}_2^* < p_2^*$.

Proposition 2 shows that, when policyholders' overconfidence level increases from Δ to $\hat{\Delta}$, a higher degree of reclassification risk insurance (i.e., $\hat{p}_2^* < p_2^*$) is offered in the second period, as Figure 1 depicts. The intuition is as follows. As policyholders become more overconfident, they place more weight on the second-period expected utility from the case that their bequest motives remain. As a result, they prefer more actuarially favorable period-2 contract terms. To maximize the policyholders' *perceived* expected utility, the life insurance firms respond by lowering the period-2 premiums and providing a higher degree of reclassification risk insurance in the second period, which implies that the insurance firms will suffer a greater loss in period 2. This loss is compensated by a more front-loaded contract in the first period in equilibrium: the first-period premium is higher, and the first-period death benefit is lower under $\hat{\Delta}$ than under Δ, namely, $\hat{Q}_1 > Q_1$ and $\hat{F}_1 < F_1$.

The issue, however, is that the policyholders' *experienced utility* from the equilibrium contract is not the same as the *perceived expected utility* when they make their life insurance purchase decisions. Following the literature, we use the consumer's *experienced utility* with the *objective probability* of losing his bequest motives to evaluate consumer welfare; that is, if a consumer who has an objective probability q of losing his bequest motives in the second period purchases a *generic* long-term contract $\mathcal{C} \equiv \langle (Q_1, F_1), (Q_2(p_2), F_2(p_2)) : p_2 \in [0, 1] \rangle$, then his welfare from the contract is:

$$W := \Big[u(y_1 - Q_1) + p_1 v(F_1) \Big] \tag{11}$$
$$+ (1 - p_1) \int_0^1 \Big\{ (1 - q) \Big[u\big(y_2 - Q_2(p_2)\big) + p_2 v\big(F_2(p_2)\big) \Big] + q u(y_2) \Big\} d\Phi(p_2).$$

Note that in expression (11), the objective probability q enters the calculation, while in the objective function (3) in the optimization problem of the competitive insurers, it is $\tilde{q}$ that enters the calculations. Now we can define the consumer's equilibrium welfare from market environment (q, Δ) simply as W evaluated at the equilibrium contract $\langle (Q_1(q, \Delta), F_1(q, \Delta)), (Q_2(p_2; q, \Delta), F_2(p_2; q, \Delta)) : p_2 \in [0, 1] \rangle$ that solves Problem (3):[25]

Definition 1 *The consumer equilibrium welfare in the absence of the life settlement market, $W(q, \Delta)$, is defined by:*

$$W(q, \Delta) := \Big[u\big(y_1 - Q_1(q, \Delta)\big) + p_1 v\big(F_1(q, \Delta)\big) \Big] \tag{12}$$
$$+ (1 - p_1) \int_0^1 \Big\{ (1 - q) \Big[u\big(y_2 - Q_2(p_2; q, \Delta)\big) + p_2 v\big(F_2(p_2; q, \Delta)\big) \Big] + q u(y_2) \Big\} d\Phi(p_2).$$

Given that expression (11) coincides with the objective function (3) in the optimization problem of the competitive insurers, and the set of feasible contracts defined by the constraints (4)-(6) does not depend on consumer's subjective belief $\tilde{q}$ or Δ when consumers are unbiased, i.e., when $\Delta = 0$, it immediately follows that the contract that maximizes (11) and the equilibrium contracts proposed by the life insurance firms coincide. More generally, the following result shows that higher overconfidence leads to lower equilibrium consumer welfare.

Proposition 3 *(Higher Overconfidence Reduces Equilibrium Consumer Welfare in the Absence of the Life Settlement Market) Fix $q \in (0, 1)$. $W(q, \Delta)$ is weakly decreasing in Δ.*

When consumers are more overconfident, they are more biased in their beliefs about the probability of losing their bequest motives. As stated in Proposition 2, competitive insurers will cater to the consumers' more biased beliefs by offering more front-loaded contracts (higher first-period premium, lower first-period coverage) in exchange for more risk reclassification insurance in period 2, which consumers value more highly when they have more biased beliefs. Such catering is effective in attracting the consumers in the first period, but they will come to *regret, albeit too late*, that the

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[25]Here we highlight the fact that the equilibrium contract depends on (q, Δ) in our notation.

risk reclassification insurance offered in the long-term contract is not as valuable as they initially thought, once they observe that the objective probability of losing their bequest motives q is higher than they initially thought. A greater behavioral bias leads to more deviation from the socially optimal contract. Thus, consumer welfare is decreasing in the level of policyholders' overconfidence.

Notice that in the baseline model, when the consumers realize that they have purchased too much reclassification insurance in the second period, it is too late. They realize that they have paid too high a first-period premium for the second-period risk reclassification risk insurance they no longer value as highly, but they have no recourses. In the next subsection, we argue that the life settlement market can serve as just such a recourse for the consumers in the second period when they realize the mistake they made due to overconfidence, and the very presence of this recourse will provide a *discipline* on the competitive insurers in their equilibrium contracts.

2.3 Equilibrium Contracts in the Presence of the Settlement Market

We now introduce a life settlement market to the model at the beginning of period 2. After the policyholder learns about his second-period health state p_2, and learns whether his bequest motives remain, he has the option to sell the contract to a settlement firm prior to the resolution of his mortality risk. If the policyholder loses his bequest motives in period 2, he now has a better option than just allowing his contract to lapse; he can sell his contract on the settlement market, and receive a fraction $\beta \in [0,1]$ of the actuarial value of the contract. The actuarial value of a life insurance contract is the difference between the expected death benefit from the contract and the premium; specifically, in health state p_2, the actuarial value of the contract is simply $p_2 F_2(p_2) - Q_2(p_2)$. If a policyholder decides to sell his insurance to the settlement firm, the settlement firm will continue to pay the second-period premium $Q_2(p_2)$, and in return, the life settlement firm becomes the beneficiary of the policy and collects the corresponding death benefits if the policyholder dies at the end of period 2. Intuitively, the parameter β measures the market competitiveness of the life settlement market. In addition, β can be understood and interpreted more broadly as other frictions associated with the secondary market for life insurance.

The presence of the settlement market introduces two main changes to the primary insurers' problem. The first change relates to the consumer's expected utility function: the consumer now expects to receive a fraction β of the actuarial value of the contract by selling it to the settlement firm in the event that he loses his bequest motives. The second change relates to the insurer's zero-profit condition: because a policyholder without bequest motives in the second period will always sell the policy to the settlement firm instead of letting it lapse, the life insurance firm will *always* have to pay the death benefits. We will show below that these changes fundamentally alter the way insurance firms provide long-term insurance contracts in equilibrium.

The equilibrium long-term contract in the presence of the life settlement market, which we denote by $\langle (Q_{1s}, F_{1s}), (Q_{2s}(p_2), F_{2s}(p_2)) : p_2 \in [0,1] \rangle$, where we use subscript s to indicate "settle-

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ment," solves:

$$\max \left[u(y_1 - Q_{1s}) + p_1 v(F_{1s}) \right] \tag{13}$$

$$+ (1 - p_1) \int_0^1 \left\{ (1 - \tilde{q}) \left[\begin{array}{c} u\left(y_2 - Q_{2s}(p_2)\right) \\ + p_2 v\left(F_{2s}(p_2)\right) \end{array} \right] + \tilde{q} u\left(y_2 + \beta V_{2s}(p_2)\right) \right\} d\Phi(p_2)$$

$$\text{s.t. } (Q_{1s} - p_1 F_{1s}) + (1 - p_1) \int_0^1 \left[Q_{2s}(p_2) - p_2 F_{2s}(p_2) \right] d\Phi(p_2) = 0, \tag{14}$$

$$Q_{2s}(p_2) - p_2 F_{2s}(p_2) \leq 0 \text{ for all } p_2 \in [0, 1], \tag{15}$$

$$Q_{2s}(p_2) \geq 0 \text{ for all } p_2 \in [0, 1], \tag{16}$$

where

$$V_{2s}(p_2) \equiv p_2 F_{2s}(p_2) - Q_{2s}(p_2) \tag{17}$$

is the actuarial value of the period-2 contract with health state p_2. From the no-lapsation condition (15), $V_{2s}(p_2)$ is always non-negative.

Note that the objective probability of losing bequest motives, q, enters neither the zero-profit condition (14) nor the objective function (13). Therefore, fixing $\tilde{q}$, the set of equilibrium contracts is *independent* of q. This is because life insurance firms have to pay the face value when policyholders die in period 2 regardless of whether they lose their bequest motives. Therefore, the life insurance firms do not take into account the actual probability of losing bequest motives when they maximize policyholders' perceived utility.[26]

The following result demonstrates that the presence of a life settlement market has a potential disciplinary effect on the extent to which the life insurers can exploit policyholders' overconfidence:

Lemma 5 *For all $(q, \Delta) \in (0, 1) \times [0, 1]$, the solution to Problem (13) must satisfy that $Q_{2s}(p_2) > 0$ for all $p_2 \in (0, 1]$.*

Lemma 5 states that the non-negativity condition for $Q_{2s}(p_2)$ never binds. Recall from Lemma 4 that, when there is no settlement market, it is possible for the life insurers to offer contracts with zero period-2 premiums in equilibrium in order to maximally exploit overconfident consumers, given the wedge between their subjective valuation of period-2 reclassification insurance and the objective cost to the insurers to offer such insurance when q and Δ are sufficiently large. However, such a pricing strategy will not emerge in equilibrium in the presence of the settlement market. This is explained by (14): when there is a settlement market, the objective cost of providing zero-premium period-2 contracts is independent of q; hence, the wedge does not increase with q. Thus, the presence of the settlement market can be seen to provide a *disciplinary effect* on the extent to which the life insurers can exploit overconfident consumers. If they are too aggressive in offering risk reclassification insurance and charging highly front-loaded premiums, they will lose money in the second period by paying the death benefits to the settlement firms.

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[26]Interestingly, this property no longer holds when we allow the insurance firms to compete with the settlement companies by offering health-contingent CSVs, as we will see in Section 4.2.

Given the fact that q does not appear in the optimization problem (13), Lemma 5 immediately implies that the characterization of the equilibrium contracts in our setting is identical to that in Fang and Kung (2020) once we replace the variable q in Fang and Kung (2020) with $\tilde{q}$.[27] Although the equilibrium contract depends only on $\tilde{q}$, we will see below that the equilibrium consumer welfare depends on both q and $\tilde{q}$.

Similar to Proposition 1, we can characterize the set of equilibrium contracts as follows:

Proposition 4 *(Equilibrium Contracts with Settlement Market) In the presence of the life settlement market, the equilibrium contracts satisfy the following properties:*

1. *Policyholders receive full-event insurance in both periods.*

2. *There exists a threshold $p_{2s}^* \in (0, 1]$ such that the period-2 premiums are actuarially fair for all $p_2 < p_{2s}^*$, and the period-2 premiums are actuarially favorable to the policyholder for all $p_2 > p_{2s}^*$.*

3. (a) *If $p_{2s}^* < 1$, then the equilibrium period-2 premium $Q_{2s}(p_2)$ and actuarial value $V_{2s}(p_2)$ are both strictly increasing in policyholders' mortality risk p_2 when $\beta > 0$.*

   (b) *If $p_{2s}^* = 1$, then the equilibrium period-2 contracts coincide with the spot contracts.*

Note that in the presence of the settlement market, life insurance firms no longer provide flat premiums in period 2. Instead, they provide partial insurance against reclassification risk in equilibrium. The set of period-2 equilibrium contracts take the form of *premium discounts* relative to the spot market contracts. Policyholders with higher mortality risks are charged higher premiums, but the equilibrium contract still favors a higher p_2 in the sense that policyholders with higher p_2 are offered contracts with higher actuarial values. These insights are identical to those in Fang and Kung (2020).

Next, we provide comparative statics on the equilibrium contracts with respect to consumer overconfidence in the presence of the settlement market. Recall that in Proposition 2 we use the hat symbol to indicate a higher degree of consumer overconfidence ($\hat{\Delta} > \Delta$). In what follows, we use the hat symbol to refer to the variables for which the degree of consumer overconfidence is $\hat{\Delta}$. More formally, let $\langle \hat{Q}_{1s}, \hat{F}_{1s}, \hat{Q}_{2s}(p_{2s}), \hat{F}_{2s}(p_{2s}) \rangle$ and $\langle Q_{1s}, F_{1s}, Q_{2s}(p_{2s}), F_{2s}(p_{2s}) \rangle$ be the equilibrium long-term contracts under $\hat{\Delta}$ and Δ respectively, and let $\hat{p}_{2s}^*$ and p_{2s}^* be the threshold probabilities established in Proposition 4 under $\hat{\Delta}$ and Δ, respectively.

Proposition 5 *(Greater Overconfidence Exacerbates Front-loading in the Presence of the Settlement Market) Suppose that $\hat{\Delta} > \Delta$ and $p_{2s}^* < 1$. Then $\hat{Q}_{1s} > Q_{1s}$ and $\hat{F}_{1s} < F_{1s}$.*

When consumers become more overconfident, life insurance firms respond by offering a set of contracts with a higher degree of front-loading (i.e., a higher premium and a lower face value in the

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[27]Recall that Fang and Kung (2020) analyze the case where consumers have rational beliefs regarding the probability that they will lose their bequest motives, i.e., $\tilde{q} = q$.
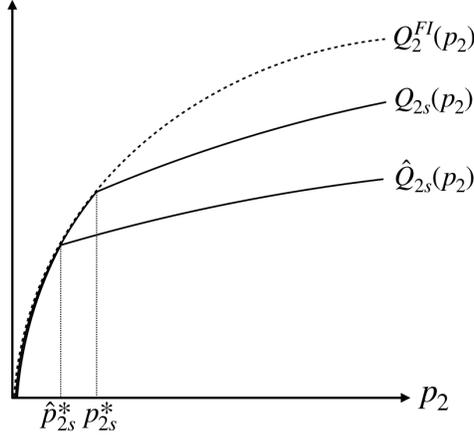
(a) Case I: $\hat{p}_{2s}^* < p_{2s}^*$
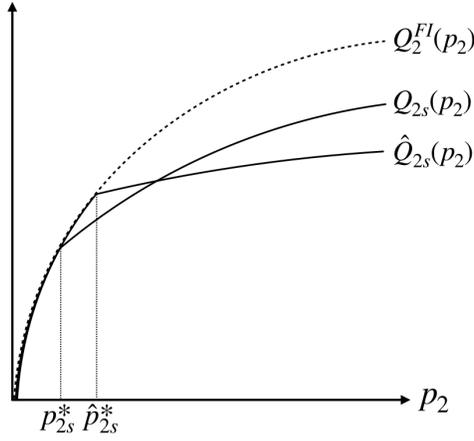
(b) Case II: $\hat{p}_{2s}^* > p_{2s}^*$

Figure 2: Equilibrium Period-2 Premium Profiles in the Presence of the Settlement Market under Different Levels of Consumer Overconfidence: $\hat{\Delta} > \Delta$.

first period). The intuition is similar to that of Proposition 2. When policyholders become more overconfident, they demand contracts that are more actuarially favorable in the second period. As a result, insurance firms suffer greater losses than before. Therefore, the first-period premium increases in equilibrium so as to satisfy the zero-profit condition.

Unlike Proposition 2, we can no longer obtain clean comparative statics on the degree of reclassification risk (i.e., p_{2s}^*) with respect to the degree of consumer overconfidence. Figure 2 depicts two possibilities for the change in the equilibrium second-period premiums as the policyholders' overconfidence increases from Δ to $\hat{\Delta}$, where $\hat{p}_{2s}^* < p_{2s}^*$ holds in Figure 2(a) and $\hat{p}_{2s}^* > p_{2s}^*$ holds in Figure 2(b).[28] When the life settlement market is present, insurance firms provide contracts with premium discounts rather than flat premiums, whose shape depends not only on p_{2s}^* but also on

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[28]Assuming that $u(c) = v(c) = c^{0.9}$, $(y_1, y_2, q, p_1) = (0.998, 1, 0.8, 0.01)$, and $p_2 \sim \mathcal{U}[0, 1]$, the shapes of the equilibrium period-2 premiums in Figure 2(a) can be obtained with $\Delta = 0.05$ and $\hat{\Delta} = 0.5$; and the shapes of the equilibrium period-2 premiums in Figure 2(b) can be obtained with $\Delta = 0.65$ and $\hat{\Delta} = 0.9$.

$u(\cdot)$ and $v(\cdot)$.[29] Intuitively, the possibility to cash out a positive fraction of the actuarial value of the period-2 contract changes the policyholders' marginal utility when they lose their bequest motives, and hence the shape of the equilibrium premiums changes accordingly.

Analogous to the experienced utility W defined in (11) for the case where there is no settlement market, we can define the consumer's *experienced utility* with the *objective probability* q of losing bequest motives in the second period from a *generic* long-term contract $\mathcal{C}_s \equiv \langle (Q_{1s}, F_{1s}), (Q_{2s}(p_2), F_{2s}(p_2)) : p_2 \in [0,1] \rangle$ as:

$$W_s := \left[u(y_1 - Q_{1s}) + p_1 v(F_{1s}) \right] \tag{18}$$
$$+ (1 - p_1) \int_0^1 \left\{ \begin{array}{c} (1-q) \left[u\left(y_2 - Q_{2s}(p_2) \right) + p_2 v\left(F_{2s}(p_2) \right) \right] \\ + q u\left(y_2 + \beta V_{2s}(p_2) \right) \end{array} \right\} d\Phi(p_2).$$

Note that the above expression is very similar to (11) except for the term $\beta V_{2s}(p_2)$, which is the cash amount a policyholder obtains from the settlement firm. The consumer's equilibrium welfare in the market environment (q, Δ) with the settlement market is simply W_s evaluated at the equilibrium contract $\langle (Q_{1s}(q,\Delta), F_{1s}(q,\Delta)), (Q_{2s}(p_2;q,\Delta), F_{2s}(p_2;q,\Delta)) : p_2 \in [0,1] \rangle$, which solves Problem (13):[30]

Definition 2 *The consumer equilibrium welfare in the presence of the settlement market, $W_s(q, \Delta)$, is defined by:*

$$W_s(q, \Delta) := \left[u\left(y_1 - Q_{1s}(q, \Delta) \right) + p_1 v\left(F_{1s}(q, \Delta) \right) \right] \tag{19}$$
$$+ (1 - p_1) \int_0^1 \left\{ \begin{array}{c} (1-q) \left[u\left(y_2 - Q_{2s}(p_2; q, \Delta) \right) + p_2 v\left(F_{2s}(p_2; q, \Delta) \right) \right] \\ + q u\left(y_2 + \beta V_{2s}(p_2; q, \Delta) \right) \end{array} \right\} d\Phi(p_2).$$

Proposition 6 *(Greater Overconfidence Lowers Equilibrium Consumer Welfare in the Presence of the Settlement Market) Fix $q \in (0, 1)$. $W_s(q, \Delta)$ is weakly decreasing in Δ.*

Proposition 6 establishes similar comparative statics of consumer welfare with respect to the level of policyholders' overconfidence. Although the proof is more complicated due to the fact that the equilibrium period-2 premiums are no longer flat, as they are in the case without the settlement market, the intuition is reminiscent of that for Proposition 3. In general, overconfidence reduces consumer welfare regardless of the presence of the settlement market.

3 Welfare Comparison and Consumer Vulnerability

So far, Propositions 3 and 6 deliver comparative statics results on consumer welfare with respect to *overconfidence level* in the absence and in the presence of the life settlement market,

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[29]See Equation (A22) in the Appendix.

[30]Here we again highlight the fact that the equilibrium contract depends on (q, Δ) in our notation.

respectively. In this section, we present our main result regarding the equilibrium effect of the settlement market on consumer welfare when consumers are overconfident about the persistence of their bequest motives. It turns out that the welfare comparison hinges on the curvature—captured by the intertemporal elasticity of substitution (IES), or the inverse of the relative risk aversion—of the function $v(\cdot)$. For ease of exposition, we denote the IES of $v(\cdot)$ at c by $\eta(c)$:

$$\eta(c) := -\frac{v'(c)}{cv''(c)}. \tag{20}$$

Theorem 1 *(Welfare Effect of the Life Settlement Market with Overconfidence in the Persistence of Bequest Motives)* *The following statements hold:*

1. *$W(q, \Delta) \geq W_s(q, \Delta)$ if Δ is sufficiently small.*

2. *Suppose that $\eta(\cdot)$ is positively bounded away from one, i.e., there exists $\alpha > 1$ such that $\eta(c) \geq \alpha$ for all $c > 0$. Then there exists a threshold $\check{q}$ such that for $q \geq \check{q}$, $W_s(q, \Delta) > W(q, \Delta)$ if Δ is sufficiently large.*

Theorem 1 provides sufficient conditions under which life settlement improves or reduces consumer welfare in equilibrium. The first part of Theorem 1 shows that introducing the life settlement market reduces consumer welfare in equilibrium when policyholders' overconfidence about the probability of losing their bequest motives is sufficiently small. The intuition can be better explained in the extreme case when consumers have correct beliefs regarding the probability of losing their bequest motives, i.e., when $\Delta = 0$.[31] Recall that policyholders' perceived expected utility and experienced expected utility coincide under such a scenario. On one hand, the settlement market allows policyholders to access the actuarial value in their policies, contributes to market completeness, and thus leads to potential welfare improvement. Suppose that the primary insurance firms do not modify the long-term insurance contract upon the opening of the settlement market. Then consumer welfare will increase, as shown by comparing Equations (11) and (18). On the other hand, the presence of the settlement market increases the primary insurers' cost of providing long-term insurance against reclassification risk. In other words, the settlement market weakens consumers' ability to commit to not asking for a return of their front-loaded premiums in the event that they lose their bequest motives. Expecting this, the primary life insurance firms will provide contracts with premium discounts instead of flat premiums in the second period, as shown in Proposition 4. This weakening of consumers' commitment power contributes further to market incompleteness and hence leads to potential welfare losses. The first part of Theorem 1 indicates that the latter welfare loss effect dominates the former welfare boost effect when consumers are fully rational or exhibit only slight overconfidence.

Next, let us consider the second part of Theorem 1. As we emphasized in Section 2.3, when consumers are overconfident about the persistence of their bequest motives, the life settlement

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[31]Proposition 4 in Fang and Kung (2020) proves the same welfare comparison assuming that policyholders' income increases over time, i.e., $y_1 \leq y_2$ [see also Claim 3 of Proposition 2 in Daily, Hendel, and Lizzeri (2008)]. The first part of our Theorem 1 shows that their result can be generalized to allow for income dynamics that satisfy Assumption 1.

market has an additional disciplinary effect on the extent to which the primary life insurers can exploit consumers' irrational beliefs in the design of their contracts. Whether the unambiguous result stated in the first part of Theorem 1 for the case of consumers with rational or slightly biased beliefs will be overturned crucially depends on the strength of the additional disciplinary effect of the life settlement market.

The second part of Theorem 1 requires that q and Δ be sufficiently large. When policyholders are overconfident, the settlement market has a new role: it allows policyholders to take actions in the second period to correct the *a posteriori* mistaken contractual choices in the first period resulting from their biased beliefs. In particular, a fraction $q\Delta$ of the policyholders no longer retain their bequest motives in the second period even though they expected to do so when they purchased their insurance policies. When there is no settlement market, they have no option but to allow the contract to lapse and suffer the utility loss caused by their biased beliefs. However, in the presence of the settlement market, policyholders can recover part of the actuarial value of their contracts. More importantly, the threat of those consumers who unexpectedly find themselves without bequest motives selling their contracts to life settlement firms in the second period curbs the life insurers' ability to design contracts that aggressively exploit overconfident consumers. In order for the settlement market to be welfare-increasing, q and Δ must be large enough that the welfare gain from the disciplinary effect can outweigh the welfare loss due to the lower degree of market completeness as highlighted for the cases where consumers have rational or slightly biased beliefs.

Importantly, the second part of Theorem 1 also requires the IES of $v(\cdot)$ to be positively bounded away from one. This turns out to be related to the general idea that the curvature of $v(\cdot)$ is related to the degree to which consumers with biased beliefs are *vulnerable* to exploitation (in terms of welfare) by firms.[32] Bisin, Lizzeri, and Yariv (2015) consider a political economy model of public debt where voters have self-control problems and attempt to commit using illiquid assets, and investigate the welfare consequence of banning illiquid assets. They show that in equilibrium, the government accumulates an inefficiently high level of debt to respond to individuals' desire to undo their commitments using illiquid assets. Similarly to our Theorem 1, their welfare comparison depends on the shape of individuals' utility function. Specifically, they find that banning illiquid assets is welfare-improving if the coefficient of relative risk aversion is greater than one.

To explain the intuition most cleanly, let us consider a utility function with constant IES $\rho > 0$. Specifically, suppose that $v(\cdot)$ takes the form

$$v(c) = \begin{cases} \frac{c^{1-\frac{1}{\rho}}-1}{1-\frac{1}{\rho}} & \text{if} \quad \rho > 0 \text{ and } \rho \neq 1, \\ \ln(c) & \text{if} \quad \rho = 1. \end{cases} \tag{21}$$

It follows immediately that $\eta(c) = \rho$ for all $c > 0$, where $\eta(\cdot)$ is defined in (20).[33] From Lemma 4,

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[32]See Fang and Wu (2020) for a more general exploration of consumer vulnerability and behavioral biases.

[33]We assume constant IES for expositional convenience. There is no strong consensus in the life-cycle literature about the appropriate functional form of utility from leaving a bequest (e.g., Gourinchas and Parker, 2002).

when q and Δ are sufficiently large, $Q_2(p_2) = 0$ for all $p_2 \in [0,1]$ in the absence of the settlement market. That is, the life insurance company will cater to the overconfidence of the policyholders by offering zero-premium contracts in the second period in exchange for a high first-period premium. Moreover, the equilibrium face values $F_2(p_2)$ and F_1 must satisfy:

$$\frac{v'\left(F_2(p_2)\right)}{v'(F_1)} = \frac{1-q}{1-\tilde{q}} \text{ for all } p_2 \in [0,1].^{34} \tag{22}$$

Exploiting the constant IES functional form of $v(\cdot)$ as described in (21), we obtain:

$$\frac{F_2(p_2)}{F_1} = \left(\frac{1-\tilde{q}}{1-q}\right)^{\rho} = \left(1 + \frac{q}{1-q}\Delta\right)^{\rho}. \tag{23}$$

Equation (23) shows that, when q and Δ are sufficiently large, the life insurers will offer long-term life insurance policies with higher face values in the second period than in the first period. There are two things to note. First, the value of the death benefits are enjoyed by the policyholder's dependent according to $v(\cdot)$; this explains why the curvature of $v(\cdot)$ plays an important role in Theorem 1. Second, a higher death benefit in the second period is valuable only if policyholder's bequest motives remain in effect. Since the policyholder overestimates his period-2 insurance demand due to his overconfidence in the persistence of his bequest motives, a higher ratio of $F_2(p_2)/F_1$ is valuable to the policyholder ex ante, but not as valuable ex post. Once the policyholder loses his bequest motives in the second period, the promised high death benefit $F_2(p_2)$ is not actually costly to the insurer. Of course, the insurers will charge a higher upfront premium payment Q_1. This is precisely how the insurer exploits the consumer's biased belief.

Equation (23) highlights that the higher the IES parameter ρ is, the larger the distortion resulting from consumer overconfidence will be in the equilibrium contract. Specifically, when $\rho > 1$, the equilibrium consumption growth for the dependent is sensitive to changes in the level of policyholders' overconfidence Δ. In the absence of the settlement market, policyholders will obtain an equilibrium contract that specifies a very low face value and a high premium in the first period in exchange for period-2 contracts of high actuarial values as they become sufficiently overconfident. Such a contract harms policyholders' experienced utility as defined by (12) because the high actuarial values of the period-2 contracts are not objectively valuable to the policyholder when q and Δ are sufficiently large. In equilibrium, a large portion (i.e., $q\Delta$) of the expected utility promised by the set of equilibrium contracts in the second period is not realized due to policyholder's misperception of the probability of losing bequest motives. To summarize, policyholders with a high value of IES are more vulnerable due to their overconfidence and can potentially benefit more from the presence of the settlement market than those with a low value of IES.

With the presence of the settlement market, the set of equilibrium contracts will not deviate as much from the socially optimal one in terms of the degree of front-loading as policyholders become more overconfident. In fact, we can establish a lower bound on the expected utility for

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[34]See the first-order conditions (A1b) and (A1d) in the Appendix.

policyholders in the first period. To see this, recall from Lemma 5 that, in the presence of the settlement market, contracts with zero period-2 premiums cannot be sustained in equilibrium for *all* $(q, \Delta) \in (0, 1) \times [0, 1]$. From the zero-profit condition (14), this in turn implies that there is an upper bound on the amount of front-loading. Thus, the presence of the settlement market protects policyholders from obtaining contracts with excessive front-loading in the first period as they become more overconfident. Such protection is more valuable to vulnerable policyholders with high values of IES than to those with low values of IES, and leads to consumer welfare being greater when there is a settlement market than when there is not.

3.1 Role of IES and Consumer Overconfidence

Theorem 1 is a limiting result and the welfare comparison established in the second part depends only on the shape of $v(\cdot)$. It does not depend on $u(\cdot)$. To understand this, note that policyholders lose full-event insurance and obtain zero premiums in the second period in the absence of the settlement market by Lemma 4 when they become sufficiently overconfident (i.e., $\Delta \nearrow 1$) and the objective probability of losing their bequest motives is sufficiently large (i.e., $q \nearrow 1$). In other words, they always consume y_2 in the second period if they remain alive, regardless of whether or not they lose their bequest motives. Therefore, the equilibrium trade-off between the expected utility of the first-period contract and that derived from the second-period contracts is mainly determined by the face value rather than by the premium in the limit as Equation (22) illustrates. Because the face value and the premium enter consumer welfare through $v(\cdot)$ and $u(\cdot)$ respectively, the theoretical prediction of Theorem 1 hinges only on the curvature of $v(\cdot)$.

It should be noted that the property of zero period-2 premiums that arises under extreme values of q and Δ greatly simplifies the welfare comparison in Theorem 1, but is not necessary for the introduction of the settlement market to be welfare-enhancing. Indeed, it is possible that the welfare comparison in the second part of Theorem 1 remains for moderate values of q and Δ, under which the equilibrium period-2 premiums are positive. In such a scenario, consumers obtain full-event insurance in both periods—i.e., $u'(y_1 - Q_1) = v'(F_1)$ and $u'(y_2 - Q_2(p_2)) = v'(F_2(p_2))$ for all $p_2 \in [0, 1]$—and Equation (22) becomes

$$\frac{u'(y_2 - Q_2(p_2))}{u'(y_1 - Q_1)} = \frac{v'(F_2(p_2))}{v'(F_1)} = \frac{1-q}{1-\tilde{q}} \text{ for all } p_2 \in [p_2^*, 1].^{35}$$ (24)

By Equation (24), the dynamic distortion from consumer overconfidence in the absence of the settlement market is governed by both $u(\cdot)$ and $v(\cdot)$. Therefore, the curvatures of the two utility functions, as well as consumers' degree of overconfidence, should all play a part in determining whether the life settlement market can improve consumer welfare in equilibrium.

Next, we provide some numerical results in order to elaborate on the role of IES and consumer overconfidence. Throughout this subsection, consumers are assumed to exhibit constant IES value

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[35]The condition can be obtained from the first-order conditions (A1a)-(A1d) in the Appendix.

of $\rho_1 > 0$ and $\rho_2 > 0$ for $u(\cdot)$ and $v(\cdot)$ respectively:

$$u(c) = \begin{cases} \frac{c^{1-\frac{1}{\rho_1}}-1}{1-\frac{1}{\rho_1}} & \text{if} \quad \rho_1 > 0 \text{ and } \rho_1 \neq 1, \\ \ln(c) & \text{if} \qquad \rho_1 = 0; \end{cases} \qquad v(c) = \begin{cases} \frac{c^{1-\frac{1}{\rho_2}}-1}{1-\frac{1}{\rho_2}} & \text{if} \quad \rho_2 > 0 \text{ and } \rho_2 \neq 1, \\ \ln(c) & \text{if} \qquad \rho_2 = 0. \end{cases} \tag{25}$$

Let $(y_1, y_2) = (1, 1)$, $p_1 = 0.1$, and $p_2 \sim \mathcal{U}[0,1]$. In addition, we set $\beta = 0$ to highlight the *indirect* equilibrium impact of the settlement market on the primary insurance market. Recall that the primary life insurance firms always pay out the actuarial value of the period-2 contract when the life settlement market is in place, regardless of whether policyholders lose or retain their bequest motives. If we assume that $\beta = 0$, the settlement market offers no *direct* value to policyholders. That is, consumers receive no actuarial value from the contract if they lose their bequest motives. Therefore, if consumer welfare increases in the presence of the settlement market, the welfare gain must stem from the disciplinary role of the settlement market on competitive life insurance contracts in equilibrium.

Role of IES of $u(\cdot)$ and $v(\cdot)$ Figure 3 graphically illustrates how policyholders' IES affects the welfare consequences of the introduction of the settlement market. The dashed curve is the combination of (ρ_1, ρ_2) for which the consumer welfare in the presence of the settlement market is equal to that in the absence of the settlement market for $q = 0.4$ and $\Delta = 0.3$.[36] The region of (ρ_1, ρ_2) to the right (to the left, respectively) of the dashed curve depicts the combination of (ρ_1, ρ_2) for which introducing the settlement market is welfare-enhancing (welfare-reducing, respectively) under the equilibrium contract. Similarly, the solid curve represents the contour plot for $q = 0.4$ and $\Delta = 0.5$; and the dotted curve represents the contour plot for $q = 0.9$ and $\Delta = 0.9$.

The first pattern to notice is that the contour plot is downward-sloping. In words, fixing ρ_1 (ρ_2, respectively), the presence of the settlement market increases consumer welfare as ρ_2 (ρ_1, respectively) becomes sufficiently large. As previously mentioned, the IES of $u(\cdot)$ and $v(\cdot)$ each will have an impact on the IES of policyholder's expected utility. This result confirms the intuition that consumers with a higher IES of either $u(\cdot)$ or $v(\cdot)$ are more vulnerable due to their overconfidence, and hence can benefit more from the presence of the settlement market. Second, the solid curve lies below the dashed curve. Therefore, fixing q, consumer welfare is more likely to be greater in the presence of the settlement market than without it as Δ increases. This is because the beneficial effect of the settlement market increases as Δ increases. Last, the dotted curve is flatter than the dashed curve in the (ρ_1, ρ_2) space, suggesting that the welfare comparison relies more on the IES of $v(\cdot)$ relative to that of $u(\cdot)$ as q and Δ become large. This confirms the result established in Theorem 1.

Role of Consumer Overconfidence Δ Another key determinant in our welfare comparison is consumer overconfidence. Next, we place reasonable parameters on consumer preferences and

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[36]The contour plots are shown only for $(\rho_1, \rho_2) \in [0.1, 5.1] \times [0.1, 5.1]$.
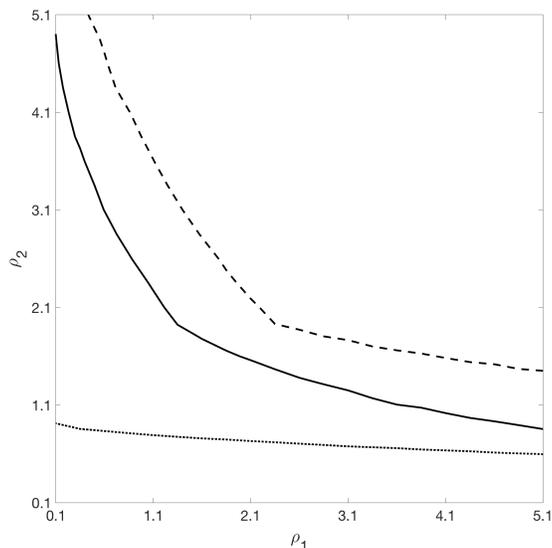
Figure 3: Welfare Comparison with Overconfidence in Persistence of Bequest Motives.
Each curve illustrates combinations of (ρ_1, ρ_2) for which the consumer welfare in the presence of the settlement market is equal to that when it is absent, for different combinations of q and Δ. To the right of the curve, consumer welfare is higher with the life settlement market than without. For the dashed curve, $q = 0.4$ and $\Delta = 0.3$; for the solid curve, $q = 0.4$ and $\Delta = 0.5$; for the dotted curve, $q = 0.9$ and $\Delta = 0.9$.

explore how overconfident an individual would have to be for the introduction of the settlement market to be welfare-enhancing. To proceed, we set $q = 0.4$ and $\rho_1 = \rho_2$. IES is a parameter of central importance in macroeconomics and finance and estimates of its magnitude vary.[37] The estimates of IES range widely from an elasticity of around zero (e.g., Jappelli and Pistaferri, 2007; Best, Cloyne, Ilzetzki, and Kleven, 2019) to an elasticity of about one (e.g., Dunsky and Follain, 2000) and 1.5-3.5 (e.g., Follain and Dunsky, 1997; Bansal and Yaron, 2004), to as high as 3.8 estimated in an influential article by Susumu and Keane (2004).

Figures 4(a) through 4(c) depict how policyholders' overconfidence level affects consumer welfare when the IES parameter equals 0.5, 1.5, and 2.5, respectively. The dashed curve and the solid curve in each figure represent consumer equilibrium welfare when the settlement market is present and when it is not, respectively.

Figures 4(a) and 4(b) indicate that the first part of Theorem 1 extends to all levels of Δ for small values of IES. Figure 4(c) shows that introducing the settlement market is welfare-enhancing (welfare-reducing, respectively) when $\Delta > 0.25$ ($\Delta < 0.25$, respectively). This confirms the intuition for the second part of Theorem 1 that the disciplinary effect of the settlement market is stronger as consumers become more overconfident regarding the persistence of their bequest motives.

3.2 Relation to Heidhues and Kőszegi (2017)

Heidhues and Kőszegi (2017) (HK, hereafter) develop a general framework to analyze the welfare

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[37]See Havránek (2015) for a meta-analysis of the existing literature.

(a) $\rho_1 = \rho_2 = 0.5$

(b) $\rho_1 = \rho_2 = 1.5$
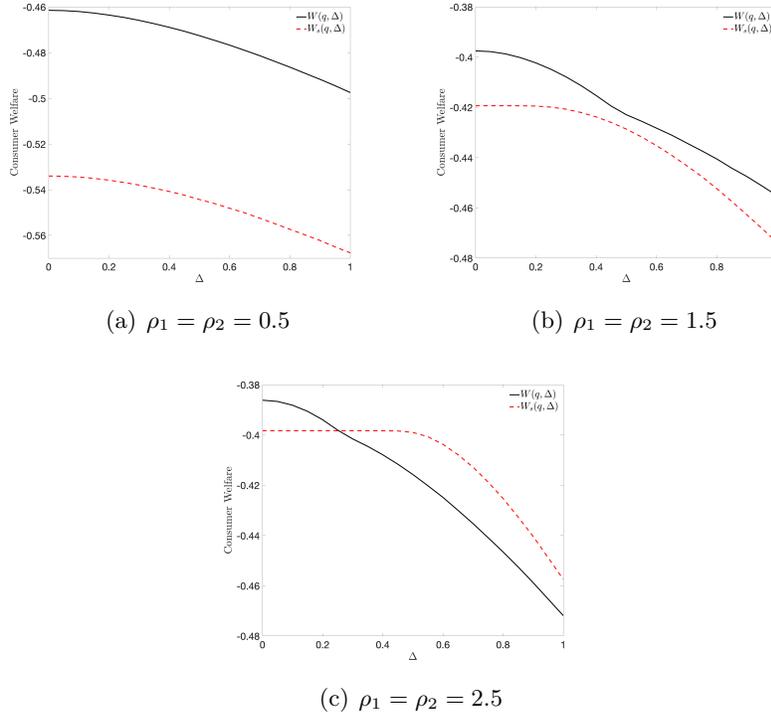
(c) $\rho_1 = \rho_2 = 2.5$

Figure 4: Welfare Comparison with Overconfidence in the Persistence of Bequest Motives Given Different Levels of IES.

effects of naïveté-based discrimination, under which firms are able to acquire and use information about consumers' naïveté to discriminate. In contrast to the classical conclusion that perfect discrimination based on consumer preference always leads to an increase in social welfare, HK show that the welfare effect of naïveté-based price discrimination is ambiguous in general. Despite the differences in model specifications, our study is conceptually connected with HK, and our results share some similarities with theirs. For instance, they show in their credit-card application that the shape of consumer's utility function allows them to sign the exploitation distortion. Specifically, naïveté-based discrimination strictly lowers social welfare if consumers exhibit prudence, or equivalently, if $u'''(c) \geq 0$ for all $c \geq 0$. In a similar vein, our Theorem 3 points to the importance of the IES of consumers' utility function in determining the welfare consequences of introducing the settlement market. Next, we elaborate on the linkage in greater detail.

A Snapshot of HK We first present a model that is slightly different from that in HK. Consider a perfectly competitive market. Firms have identical marginal costs c and simultaneously choose anticipated prices $f_l \in \mathbb{R}$ and additional prices $a_l \in [0, a_{max}]$, with $a_{max} > 0$. Consumers' willingness to pay for the product is v. We assume that v is large enough that consumers' participation constraint never binds, i.e., they always buy the product in equilibrium. There exists a fraction $\theta \in (0, 1)$ of naive consumers and a fraction $1 - \theta$ of sophisticated (or equivalently, rational) consumers. A naive consumer does not take into account the additional price a_l but ends up paying

it once he/she makes a purchase. A sophisticated consumer can successfully avoid paying a_l. The additional price a_l creates a social cost of trades of $k(a_l)$, where $k(\cdot)$ satisfies $k(0) = k'(0) = 0$ and $k''(a_l) > 0$, and $k'(a_{max}) \geq 1$. We assume that the social cost falls on firms.[38]

When firms are not able to distinguish between naive consumers and sophisticated consumers, the equilibrium price vector (f_l, a_l) solves:

$$\max v - f_l,$$

subject to the zero-profit condition

$$f_l + \theta a_l - c - k(a_l) = 0.$$

Note that the anticipated price f_l is simply a transfer from the consumers to firms and thus does not generate any social cost given that both sides are risk-neutral, and it suffices to focus on the additional price a_l. Simple algebra yields that

$$k'(a_l) = \theta. \tag{26}$$

Therefore, $a_l = k'^{-1}(\theta)$ and the associated deadweight loss relative to the first best, where $a_l = 0$, is given by

$$DWL(\theta) = k\left(k'^{-1}(\theta)\right).$$

The analysis is similar when firms are able to price discriminate, except that the zero-profit condition is $f_l + a_l - c - k(a_l) = 0$ for naive consumers and is $f_l - c - k(a_l) = 0$ for sophisticated consumers. Therefore, naïveté-based discrimination strictly lowers welfare if

$$DWL(\theta) < \theta DWL(1) + (1 - \theta)DWL(0).$$

The inequality indicates that the welfare comparison hinges on the concavity/convexity of $DWL(\cdot)$, which in turn depends on $k(\cdot)$. Formally, HK demonstrate that the above inequality holds when $k'(a)/k''(a)$ strictly increases with a.

Analogue to HK   The connection between HK and our paper lies in the comparison of the above first-order condition (26) without discrimination and Equation (24), derived in our model without the settlement market. Let us first take a closer look at Equation (26). The exogenous variable θ on the the right-hand side can be alternatively interpreted as the probability of a representative consumer being naive, and thus measures the degree of consumer naïveté. The endogenous additional price a_l on the left-hand side refers to the deviation from the rational benchmark, which results from the wedge that exists between the utility that firms promise to the representative consumer

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[38]The analysis remains intact if we instead assume that the social cost is borne by both types of consumers. HK also consider the case where the cost $k(a_l)$ only impacts trades with sophisticated consumers and the case where it only impacts trades with naive consumers.

ex ante (i.e., $v - f_l$) and the expected utility that they deliver ex post (i.e., $v - f_l - \theta a_l$).[39] Note that the wedge θa_l is supermodular in the bias θ and the additional price a_l. When consumers are fully sophisticated—i.e., $\theta = 0$—zero additional price will be charged in equilibrium by (26). As consumers' degree of naïveté θ increases, firms have more incentive to exploit consumers' misperception due to the aforementioned supermodularity. A higher additional price results. This higher price will in turn be translated into a higher degree of exploitation distortion $k\left(k'^{-1}(\theta)\right)$, generating inefficiency.

Next, let us consider Equation (24) in our model, which can be rewritten as

$$\frac{u'(y_1 - Q_1)}{u'\left(y_2 - Q_2(p_2)\right)} = \frac{v'(F_1)}{v'\left(F_2(p_2)\right)} = 1 + \frac{q}{1-q}\Delta \text{ for all } p_2 \in [p_2^*, 1]. \tag{27}$$

Note that the right-hand side, $1 + \frac{q}{1-q}\Delta$, is greater than one and increases with the degree of consumer overconfidence Δ. Equation (27) governs how firms exploit consumers' overconfidence through contract design via the premiums and face values. Fixing a generic long-term contract $\mathcal{C} \equiv \langle (Q_1, F_1), (Q_2(p_2), F_2(p_2)) : p_2 \in [0,1] \rangle$, the wedge between policyholders' perceived utility (3) and their experienced utility (11) amounts to

$$\Delta \times \underbrace{(1 - p_1)q \int_0^1 \left[u\left(y_2 - Q_2(p_2)\right) + p_2 v\left(F_2(p_2)\right) - u(y_2) \right] d\Phi(p_2)}_{\mathcal{T}\left(Q_2(p_2), F_2(p_2)\right)}.$$

Again, the wedge is supermodular in the degree of consumer overconfidence Δ and the term $\mathcal{T}\left(Q_2(p_2), F_2(p_2)\right)$, which depends on the second-period contracts. When policyholders are fully rational, i.e., $\Delta = 0$, their perceived utility coincides with their experienced utility. As a result, the right-hand side of (27) is equal to one, implying that firms would provide a set of contracts that equalize the marginal utility of consumption to consumers and their dependents across the two periods in equilibrium. As consumers become more overconfident, firms have more incentive to exploit consumers' misperception due to the aforementioned supermodularity and increase the term $\mathcal{T}\left(Q_2(p_2), F_2(p_2)\right)$. Because $\mathcal{T}$ depends only on the second-period contractual terms, firms will undercut the second-period premiums and increase the face values. By firms' zero-profit condition (4), an overly front-loaded contract relative to the rational benchmark would arise in equilibrium. This leads to the dynamic inefficiency in consumption as (27) depicts. Evidently, the contractual distortion influences consumer welfare through policyholders' utility functions $u(\cdot)$ and $v(\cdot)$.

The above comparison and analogy explain why the welfare effects of naïveté-based discrimination in HK hinge on the shape of $k(\cdot)$, and why the welfare consequences of introducing the settlement market in our model are based on the curvature of consumers' utility functions $u(\cdot)$ and $v(\cdot)$, despite the differences in model setup and focus between the two papers. Indeed, we can raise

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[39]To see this, note that with probability θ, consumers pay the additional price and derive utility $v - f_l - a_l$. With probability $1 - \theta$, consumers can avoid the additional price and receive a payoff of $v - f_l$. Therefore, the expected experienced utility of the representative consumer is $\theta(v - f_l - a_l) + (1 - \theta)(v - f_l)$, which can be simplified to $v - f_l - \theta a_l$.

a question similar to HK in our framework: What is the welfare effect of naïveté-based discrimination in a life insurance market where consumers differ in their actual probability of losing bequest motives q but have the same belief $\tilde{q}$? Based on the previous discussion, a natural conjecture is that the answer depends crucially on consumers' attitudes toward risk, or equivalently, on the curvature of the utility function. We leave the exploration of this question for future research.

4   Extensions

In this section, we consider three extensions and show that our main result regarding the welfare comparison derived in Theorem 1 is robust. Section 4.1 extends the baseline model to a situation where the insurance company cannot observe either the subjective or the objective probability that policyholders may lose their bequest motives. Section 4.2 enlarges the set of admissible contracts and allows the primary insurance firms to include CSVs in long-term insurance policies. Section 4.3 analyzes the case in which the policyholders exhibit overconfidence in their mortality risk instead of in the persistence of their bequest motives.

4.1   Observability of Objective/Subjective Probability of Losing Bequest Motives

In the baseline model, we assume that both the objective probability q and the subjective probability $\tilde{q}$ of losing bequest motives are observable to the firms. Relaxing this assumption would lead to screening issues.[40] Next, we discuss this modeling nuance and show that our main results can be readily adapted to the situation where consumers have private information regarding q or $\tilde{q}$.[41]

To proceed, we slightly abuse the notations and denote the equilibrium contract without the life settlement market—i.e., that established in Proposition 1—by $\mathcal{C}(q, \tilde{q})$. Similarly, denote the equilibrium contract with the presence of the settlement market characterized in Proposition 4 by $\mathcal{C}_s(q, \tilde{q})$. Note that these are the equilibrium contracts firms provide when they are able to observe both q and $\tilde{q}$.

Observability of q   Our results in the baseline model do not depend on the assumption that the insurance firms perfectly observe q. To see this, suppose that the policyholders' objective probability of losing their bequest motives $q \in \{q^1, \ldots, q^n\}$, and denote the mass of type-q^i consumers by $\omega^i > 0$, with $i \in \{1, \ldots, n\}$. Without loss of generality, we normalize the total mass of consumers to unity, i.e., $\sum_{i=1}^n \omega^i = 1$. Furthermore, we assume that q is an individual policyholder's private information and is not observed by the insurance firms.[42]

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[40]A sizable theoretical literature explores the screening of potentially naive consumers based on their beliefs. See, for instance, Eliaz and Spiegler (2006, 2008), Heidhues and Kőszegi (2010), Spinnewijn (2013), and Galperti (2015).

[41]We thank an anonymous referee for suggesting this extension.

[42]The model degenerates to the baseline if policyholders do not know q.

We first consider the scenario where the settlement market is absent. Evidently, the life insurance firms have incentives to screen consumers because q varies the cost of providing insurance as Equation (4) illustrates: consumers with a higher q allow their contracts to lapse more often and are less costly to serve. However, screening is impossible because consumers have the same *beliefs* regarding the probability of losing their bequest motives and thus behave the same way at the time of purchase.

Next, suppose that the settlement market is in place. Recall that firms have to pay out the actuarial value of the second-period contract regardless of whether the policyholder loses or retains his/her bequest motives in such a scenario. As a result, policyholders with different q's would generate the same expected costs to the insurance firms, and the firms' incentive to screen would fade away.

Let $q^\star := \sum_{i=1}^{n} \omega^i q^i$. The above discussion leads to the following result.

Proposition 7 *(Equilibrium Contracts when q is Unobservable) Suppose that (i) all consumers share the same beliefs $\tilde{q}$, and (ii) $q \in \{q^1, \ldots, q^n\}$ is observed by each policyholder but not by firms. Then all policyholders receive the same contract $\mathcal{C}(q^\star, \tilde{q})$ in equilibrium without the settlement market, and the same contract $\mathcal{C}_s(q^\star, \tilde{q})$ in the presence of the settlement market.*

By Proposition 7, all results established in Sections 2 and 3 continue to hold once we interpret q in the baseline model as the average objective probability of losing bequest motives among policyholders who share the same beliefs $\tilde{q}$.

Observability of $\tilde{q}$ Next, we discuss the situation where $\tilde{q}$ is each policyholder's private information but is not observed by the insurance firms. Formally, suppose that the possible $\tilde{q}$'s of policyholders in the market are $\tilde{q}^1 < \tilde{q}^2 < \ldots < \tilde{q}^{\tilde{n}}$. Denote $\tilde{\omega}^i > 0$ as the mass of type-$\tilde{q}^i$ policyholders, with $i \in \{1, \ldots, \tilde{n}\}$. Again, we normalize the total mass of consumers to unity, i.e., $\sum_{i=1}^{\tilde{n}} \tilde{\omega}^i = 1$.

We restrict our attention to the case where there is no settlement market and derive the set of equilibrium contracts. The analysis for the case where the settlement market is present is similar. It is clear that each insurance firm must earn zero expected profits in equilibrium due to competitive pressure. In addition, two observations ensue. First, cross-subsidy will not arise in equilibrium and all equilibrium contracts with positive demand must yield zero expected profits. Otherwise, a firm that cross-subsidizes contracts has incentives to offer only contracts with positive profits.

Second, consumers of different types would opt for different long-term contracts in equilibrium. To see this, suppose to the contrary that type-$\tilde{q}^i$ and type-$\tilde{q}^j$ consumers, with $i, j \in \{1, \ldots, \tilde{n}\}$ and $i \neq j$, purchase the same contract. As mentioned above, this contract yields zero expected profits. Moreover, because both types have the same objective probability of losing bequest motives q, they generate the same expected costs to firms. Therefore, it must be the case that firms earn zero profits from each type. Next, notice that this contract cannot maximize the perceived utility of both types simultaneously because $\tilde{q}^i \neq \tilde{q}^j$. Suppose that the perceived utility of type-$\tilde{q}^i$ consumers is not maximized. Then the insurance firm can craft a long-term contract that coincides with

$\mathcal{C}(q, \tilde{q}^i)$ (except that it has a slightly higher period-1 premium) to attract type-$\tilde{q}^i$ consumers and earn positive profits,[43] which is a contradiction to the first observation.

The above discussion implies that the equilibrium insurance policy to each consumer type must maximize his perceived utility subject to the zero-profit condition. Evidently, consumers have no incentive to switch to any other long-term insurance policy that breaks even on the market. The following proposition can then be obtained.

Proposition 8 *(Equilibrium Contracts when $\tilde{q}$ is Unobservable) Suppose that (i) all consumers have the same objective probability q, and (ii) $\tilde{q} \in \{\tilde{q}^1, \ldots, \tilde{q}^{\tilde{n}}\}$ is observed by each policyholder but not by firms. Then type-$\tilde{q}^i$ policyholders receive a contract $\mathcal{C}(q, \tilde{q}^i)$ in equilibrium without the settlement market, and a contract $\mathcal{C}_s(q, \tilde{q}^i)$ in the presence of the settlement market.*

4.2 Endogenous Cash Surrender Values

Thus far, we have assumed that the CSVs are zero in the baseline model, i.e., the primary insurance company pays nothing to a policyholder when he/she voluntarily terminates the life insurance contract before its maturity. In this subsection, we enrich the primary insurers' contract space by allowing them to include CSVs into the long-term contract, and show that our welfare comparison remains unchanged, provided that the settlement market is sufficiently competitive (i.e., β is sufficiently large).[44]

Analysis without Life Settlement Market We first consider the case without the settlement market. The long-term contract is now characterized by $\langle (Q_1^s, F_1^s), (Q_2^s(p_2), F_2^s(p_2), S_2^s(p_2)) : p_2 \in [0,1] \rangle$, where the new term $S_2^s(p_2) \geq 0$ is the CSV specified in health state $p_2 \in [0,1]$. We use superscript s to indicate "cash surrender value." The following result, similar to Proposition 8 in Fang and Kung (2020), can then be obtained:

Lemma 6 $S_2^s(p_2) = 0$ for all $p_2 \in [0,1]$.

By Lemma 6, the equilibrium life insurance contract in the absence of the life settlement market will not include a positive CSV. This implies immediately that the equilibrium premiums [i.e., Q_1^s and $Q_2^s(p_2)$] and face values [i.e., F_1^s and $F_2^s(p_2)$] coincide with those derived in Proposition 1.

Analysis with Life Settlement Market Denote the equilibrium long-term contract in the presence of the life settlement market by $\langle (Q_{1s}^s, F_{1s}^s), (Q_{2s}^s(p_2), F_{2s}^s(p_2), S_{2s}^s(p_2)) : p_2 \in [0,1] \rangle$. Again, we use subscript s to indicate "settlement" and the superscript s to indicate "cash surrender value."

Lemma 7 $S_{2s}^s(p_2) = \beta V_{2s}^s(p_2)$ for all $p_2 \in [0,1]$.

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[43]The crafted contract can sustain positive profits even if it attracts other types of consumers due to the fact that all types behave identically upon purchase and thus generate the same expected costs to firms.

[44]We thank an anonymous referee for suggesting this extension.

Lemma 7 is intuitive: the insurance firms will set the CSV at the precise level where consumers become indifferent between surrendering the contract and selling it on the secondary markets. This is clearly the least costly way for the primary insurance firms to undercut the settlement firms.

Note that in the case of extreme consumer overconfidence, i.e., $\Delta = 1$, policyholders never expect to lose their bequest motives and thus do not value CSVs specified in the contract at the time they make their purchase decisions. In other words, the insurance firms include second-period cash surrender options to the biased consumers, who believed in the first period that they would not exercise those options.[45] Still, the primary insurers will set positive CSVs by Lemma 7 if $\beta > 0$, so as to provide the policyholders incentives to not sell their policies when they lose their bequest motives in the second period.

Replacing $S_{2s}^s(p_2)$ by $\beta V_{2s}^s(p_2)$ in the objective function, we can see that the objective function with endogenous CSVs corresponds to that without [i.e., Equation (13)]. However, the equilibrium contracts under the two scenarios still differ. To see this, note that the zero-profit condition with endogenous CSVs can be rewritten as

$$(Q_{1s}^s - p_1 F_{1s}^s) + (1 - p_1)(1 - q + \beta q) \int_0^1 \left[Q_{2s}^s(p_2) - p_2 F_{2s}^s(p_2) \right] d\Phi(p_2) = 0.$$

Recall that when the CSVs are exogenously set at zero, the zero-profit condition (14) is independent of β and q because the life insurance firms have to pay out the actuarial value of the second-period contract $V_{2s}(p_2)$ regardless of whether the policyholder loses or retains his/her bequest motives. However, when CSVs are allowed in the design of the long-term contract, the primary life insurance firms can respond to the threat from the settlement market and undercut the cost by offering a contract that contains health-contingent CSVs that are equal to the cash benefit a policyholder can receive from the settlement firm by selling his/her policy, which is $\beta V_{2s}^s(p_2)$. Consequently, a new term $(1 - q + \beta q)$ appears in the above zero-profit condition.

Lemma 8 *There exists a threshold $\overline{\beta} < 1$ such that for all $(q, \Delta) \in (0, 1) \times [0, 1]$, $Q_{2s}^s(p_2) > 0$ for all $p_2 \in (0, 1]$ if $\beta > \overline{\beta}$.*

Lemma 8 states that the equilibrium period-2 premiums would be positive if β is above a certain threshold. In other words, when CSVs can be included in the contract, the life settlement markets can discipline the primary insurance market if the secondary market is sufficiently competitive (e.g., commission amounts required by the settlement firms are low, or other market frictions on the secondary market are small). To see why the condition is required, let us consider the extreme case in which $\beta = 0$. Lemma 7 implies that the CSVs are zero for all health states, implying that the face values and premiums of the equilibrium contract correspond to those when the settlement market is absent. In such a scenario, the primary insurance firms are again able to exploit the overconfident policyholders by setting zero period-2 premiums and promising high period-2 face values, as predicted in Lemma 3, when the settlement market is absent.

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[45]This feature is standard in the behavioral contract theory literature. See, for example, Eliaz and Spiegler (2006) and Heidhues and Kőszegi (2010).

Welfare Comparison   The following result, analogous to Theorem 1, can then be obtained:

Theorem 2 *(Welfare Comparison with Endogenous CSVs)* *Suppose that the life insurance firms can offer health-contingent CSVs. The following statements hold:*

1. *Consumer welfare is weakly reduced by the presence of the life settlement market if Δ is sufficiently small.*

2. *Suppose that $\beta > \overline{\beta}$,[46] and $\eta(\cdot)$ is positively bounded away from one, i.e., there exists $\alpha > 1$ such that $\eta(c) \geq \alpha$ for all $c > 0$. Then consumer welfare is higher when there is a settlement market than when there is no settlement market if q and Δ are sufficiently large.*

Similar to the intuition for Theorem 1, the life settlement market can improve consumer welfare when consumers are vulnerable due to their overconfidence, i.e., when the IES of the utility function $\eta(\cdot)$ is greater than one. In addition, because insurance firms can react to the threat of the settlement market by designing CSVs, the welfare-improving result also requires that the settlement market be sufficiently competitive, i.e., that β is above a certain threshold.

4.3   Overconfidence Regarding Future Mortality Risk

In this subsection, we investigate the influences of policyholder's bias about the *probability of his future mortality* on the equilibrium contract and consumer welfare. In order to simplify the modeling of overconfidence with regard to mortality risk, we assume that the second-period mortality risk p_2 follows a Bernoulli distribution, $p_2 \in \{p_L, p_H\}$, with $p_H > p_L > p_1$, and that the *objective* distribution is such that

$$\Pr(p_2 = p_L) = \phi_L \in (0, 1),$$
$$\Pr(p_2 = p_H) = \phi_H \equiv 1 - \phi_L.$$

However, we assume that in period 1, policyholders are overconfident about their future mortality risk; specifically, they *believe* that p_2 is drawn from a "better" distribution with

$$\tilde{\phi}_L = \phi_L + \Delta_m(1 - \phi_L),$$
$$\tilde{\phi}_H = 1 - \tilde{\phi}_L = (1 - \Delta_m)(1 - \phi_L),$$

where $\Delta_m \in [0, 1]$ can be interpreted as the degree of overconfidence regarding the second-period mortality risk.[47]

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[46]Note that $\overline{\beta}$ is independent of $v(\cdot)$ from the proof of Lemma 8.

[47]Relatedly, Schumacher (2016) considers a model in which competitive insurers offer long-term contracts to present-biased consumers. Naive consumers are unable to manage their self-control problem and may form incorrect beliefs about their future risks when they purchase insurance. Future risks in Schumacher (2016) are consumers' private information. In contrast, we assume risks are symmetrically learned by the insurers and the policyholders.

Equilibrium in the Absence of the Settlement Market   Let us denote the competitive equilibrium contract without the settlement market by $\langle (Q_{1m}, F_{1m}), (Q_{2m}^H, F_{2m}^H), (Q_{2m}^L, F_{2m}^L) \rangle$, where we use subscript m to indicate "mortality risk." Consumers' overconfidence in their future mortality risk introduces an additional concern for the insurers when designing the equilibrium contracts relative to an environment where the consumer bias concerns the likelihood of persistence of bequest motives. Recall that from the objective function (3), when consumers underestimate the probability of losing their bequest motives, such bias only distorts the allocation of resources between period 1 and period 2. In contrast, overconfidence with regard to mortality risk additionally distorts the allocation of resources across states in the second period [see the objective function (A52) in the Appendix]. When policyholders believe that state-p_L is more likely to occur, the insurer has incentives to decrease the premium in state-p_L and increase the premium in state-p_H. The following lemma formalizes this intuition.

Lemma 9 *There exists a threshold $\bar{\Delta}_m < 1$ such that $Q_{2m}^H = Q_{2m}^{FI}(p_H)$ if $\Delta_m > \bar{\Delta}_m$. In addition, $Q_{2m}^L = 0$ if $\phi_L \in (0,1)$ is sufficiently small.*

From Lemma 9, consumers receive a contract with zero premium in state-p_L as they become sufficiently overconfident, whereas an actuarially fair full-event insurance is obtained in state-p_H. This contrasts to the result in Lemma 4, where all types receive zero premiums in the second period. The intuition is as follows. As we mentioned, when consumers are overconfident about their bequest motives, this behavioral bias only changes the dynamic trade-off between consumption in the first and second periods, and the allocation of consumption across different states in the second period remains optimal. Therefore, insurer has an incentive to make the period-2 contracts better *in all states* by decreasing the second-period premiums. As a result, as in the rational benchmark model of Hendel and Lizzeri (2003) and Fang and Kung (2020), consumers of a *low* risk type receive an *actuarially fair* contract and consumers of a *high* risk type receive an *actuarially favorable* contract. However, when consumers are overconfident with regard to their mortality risk, they put more weight on the low-risk state due to their bias. As a result, the insurer has incentives to exploit this bias by making the contract in state-p_L better in terms of a lower premium, and increasing premium in state-p_H. Lemma 9 shows that, in the extreme case, the state-p_L contract will have zero premium and positive actuarial value, while the state-p_H contract will coincide with the spot contract. Note that consumer bias concerning mortality risk reverses the predictions of the rational benchmark model of Hendel and Lizzeri (2003) and Fang and Kung (2020): consumers of a *high* risk type now receive an actuarially fair contract while consumers of a *low* risk type receive a contract with a positive actuarial value. As will be discussed in detail later, this new feature of the equilibrium contract limits the impact of the settlement market on the shape of the life insurance contract in equilibrium.

Equilibrium in the Presence of the Settlement Market   Denote the set of equilibrium contracts in the presence of the settlement market by $\langle (Q_{1ms}, F_{1ms}), (Q_{2ms}^H, F_{2ms}^H), (Q_{2ms}^L, F_{2ms}^L) \rangle$. The following result can be obtained.

Lemma 10 *In the presence of the settlement market,* $Q_{2ms}^H = Q_2^{FI}(p_H)$, $Q_{2ms}^L = 0$ *if* Δ_m *is sufficiently large and* $\phi_L \in (0,1)$ *is sufficiently small.*

Lemma 10 states that, for the case of overconfidence with respect to future mortality risk, the equilibrium pricing pattern for period-2 contracts in the presence of the settlement market is identical to that when there is no settlement market (as in Lemma 9). This is in stark contrast to Lemma 5, which predicts that, for the case of overconfidence with respect to bequest motives, zero-premium period-2 contracts do not emerge when there is a settlement market while they would in the absence of the settlement market. When consumers exhibit overconfidence in the persistence of their bequest motives, the settlement market protects them when they unexpectedly (due to overconfidence) lose their bequest motives *independent of* the second-period health state. The settlement market also makes it too costly for the insurer to offer a contract with zero premium for some states. This force does not apply as strongly when consumers' overconfidence concerns the distribution of period-2 mortality risk. In fact, the equilibrium contract will feature zero premium in state-p_L as predicted in Lemma 9; but the state-p_L is much less likely to actually occur than the consumer believes it is. Thus the actuarial value in the zero-premium contract for state-p_L in period 2 is somewhat immune from being exploited and threatened by the settlement market. More explicitly, consider the case where $\phi_L \in (0,1)$ is sufficiently small and $\Delta_m = 1$. In words, consumers subjectively care a lot more about the utility in state-p_L than they should in period 1, and in period 2 they almost always end up with state-p_H. In the absence of the life settlement market, the insurer commits to an actuarially fair contract in state-p_H and a contract with high actuarial value in state-p_L. Unlike the case where consumers are overconfident in the persistence of their bequest motives, the promised high actuarial value in state-p_L will only be cashed out by the settlement firm if the consumer actually ends up in state-p_L and loses his bequest motives, which occurs with probability $\phi_L q$, which is small. As a result, the positive effect of the settlement market whereby it allows biased consumers to correct their prior mistakes, is not as strong in the case of overconfidence with respect to future mortality risk. This in turn implies that life settlement is limited in its potential to unlock the actuarial value of a contract, and allows zero-premium contracts with large actuarial values in state-p_L to persist in equilibrium.

Welfare Comparison Lemmas 9 and 10 show that the role of the settlement market in changing the equilibrium contract and correcting the mistakes consumers made based on their incorrect beliefs is limited when consumers are overconfident about their future mortality risk. However, we will show that the life settlement market still constrains the life insurers' ability to exploit the consumers' biased beliefs in a different manner, and we provide an example in which the presence of the settlement market can again be welfare-improving.

We follow the discussions in Section 3 and assume that

$$u(c) = v(c) = \begin{cases} \frac{c^{1-\frac{1}{\rho}}-1}{1-\frac{1}{\rho}} & \text{if} \quad \rho > 0 \text{ and } \rho \neq 1, \\ \ln(c) & \text{if} \quad \rho = 1. \end{cases}$$

Thus, both utility functions exhibit constant IES of $\rho > 0$. To simplify the analysis, we shut down the channel of unlocking the actuarial value of a contract by assuming $\beta = 0$.[48]

Theorem 3 *(Welfare Comparison with Overconfidence Concerning Future Mortality* *Risk) Suppose that Assumption 1 is satisfied. The following statements hold:*

1. *Consumer welfare is weakly reduced by the presence of the life settlement market if Δ_m is* *sufficiently small.*

2. *Suppose that $u(\cdot) = v(\cdot)$, and both utility functions exhibit constant IES of $\rho > 0$. Moreover,* *$\beta = 0$. Then consumer welfare is higher in the presence of the settlement market than in its* *absence when $\Delta_m \in (0,1)$ is sufficiently large and $\phi_L \in (0,1)$ is sufficiently small.[49]*

The intuition for the first part of Theorem 3 resembles its counterpart in Theorem 1. Next, we provide the intuition for the second part. When $\Delta_m \in (0,1)$ is sufficiently large and $\phi_L \in (0,1)$ is sufficiently small, the consumers' welfare in the second period is mainly determined by the contract offered in state p_H (since ϕ_L is small); because Lemmas 9 and 10 show that the state-p_H contract terms are identical with or without the settlement market, the comparison of consumers' welfare mainly hinges on the utility they obtain from the first-period contract. Due to the threat from the settlement market on pricing, the primary insurance firms will offer a better (i.e., less front-loaded) first-period contract. Formally, we can show that $F_{1ms} > F_{1m}$ and $Q_{1ms} < Q_{1m}$. Such a difference in the equilibrium contracts protects consumers from being exploited due to their biased beliefs, and hence increases consumer welfare.

Interestingly, in contrast to Theorem 1, the limiting result of the welfare comparison established in Theorem 3 does not require the IES of the utility function be greater than one as in the baseline model in Section 2. To understand this result, let us delve into the equilibrium contracts in more details. Not surprisingly, in the absence of life settlement market, the insurer will cater to the consumer overconfidence by offering a favorable contract term in state-p_L in period 2 in the form of a higher face value F_{2m}^L. The promise of a high death benefit F_{2m}^L is not costly to the insurer because ϕ_L is low, but will be much valued by the consumer because Δ_m is high. Indeed, it can be shown that, in the absence of the life settlement market, the ratio between F_{2m}^L and F_{1m} is given by

$$\frac{F_{2m}^L}{F_{1m}} = \left(\frac{\tilde{\phi}_L}{\phi_L}\right)^\rho = \left(1 + \frac{1-\phi_L}{\phi_L}\Delta_m\right)^\rho. \tag{28}$$

The term $\left(\tilde{\phi}_L/\phi_L\right)^\rho > 1$ measures the distortion of the equilibrium contract that is due to consumer's biased belief. Intuitively, such distortion leads to greater welfare loss when the magnitude of overconfidence becomes large (i.e., large Δ_m and small ϕ_L), and when consumers have a weak propensity towards consumption-smoothing (i.e., $\rho > 1$). A similar argument applies, and the ratio

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[48]The result is robust when $\beta \in (0,1]$.

[49]Note that in Theorem 3, in order for the life settlement market to strictly affect the consumer welfare, ϕ_L must be positive and small. In the limit when $\Delta_m = 1$ and $\phi_L = 0$, it is clear that the contracts with and without the settlement market will exactly coincide. Thus, life settlement does not affect consumer welfare in the limit.

between the second-period and first-period face values depends on the IES parameter ρ when consumers sufficiently underestimate the probability of losing their bequest motives [see Equation (23)].

In the presence of the life settlement market, the ratio between F_{2ms}^L and F_{1ms} will now be more balanced because increasing the state-p_L face value in period 2 can actually be costly to the insurance company (when the consumer loses bequest motives in state-p_L); indeed, we can show that

$$\frac{F_{2ms}^L}{F_{1ms}} = \left[\frac{\tilde{\phi}_L}{\phi_L} \times (1 - q) \right]^\rho . \tag{29}$$

The additional term $(1 - q)$ captures the beneficial effect of the settlement market on disciplining the primary life insurance market, which is decreasing in q because q is the additional fraction of consumers to whom the life insurer needs to pay the face value due to the presence of the settlement market. Note that ρ enters the above expression, and the ratio F_{2ms}^L / F_{1ms} approaches infinity as $\Delta_m \nearrow 1$ and $\phi_L \searrow 0$. From Equations (28) and (29), we see that the IES parameter ρ influences the equilibrium pricing strategy of the primary insurer regardless of the presence of the settlement market. This implies that the welfare comparison does not hinge on the IES in the limit when policyholders are overconfident about their future mortality risk, as Theorem 3 predicts. In contrast, recall that when consumers exhibit overconfidence in the persistence of their bequest motives, an upper bound on the amount of front-loading exists when the settlement market is in place, indicating the existence of the lower bound of the first-period face value. Similarly, an upper bound on the second-period face values can be established.[50] Consequently, the ratio between the highest period-2 and period-1 face values cannot be arbitrarily large when consumers become sufficiently overconfident about their bequest motives, and is bounded from above by a threshold independent of ρ.

Role of IES of $u(\cdot)$ **and** $v(\cdot)$. Theorem 3 presents a limiting result of ϕ_L and Δ_m, assuming $u(\cdot) = v(\cdot)$. In this subsection we report numerical results to isolate the role of IES of $u(\cdot)$ and $v(\cdot)$ for intermediate values Δ_m. To proceed, we assume that $(y_1, y_2, \beta, q) = (1, 1, 0, 0.4)$, $(p_1, p_2^L, p_2^H) = (0.1, 0.2, 0.8)$, and $(\phi_L, \phi_H) = (0.5, 0.5)$. Consumers are assumed to exhibit constant IES of ρ_1 and ρ_2 for $u(\cdot)$ and $v(\cdot)$ respectively, as in (25).

Figure 5 graphically illustrates our numerical results. The dashed curve is the combination of (ρ_1, ρ_2) for which the consumer welfare in the presence of the settlement market is equal to that in its absence for $\Delta_m = 0.3$.[51] The region of (ρ_1, ρ_2) to the right (to the left, respectively) of the dashed curve depicts the combination of (ρ_1, ρ_2) for which introducing the life settlement is welfare-enhancing (welfare-reducing, respectively) under the equilibrium contract. Similarly, the solid curve represents the contour plot for $\Delta_m = 0.5$. Similar to the result when consumers are overconfident in the persistence of their bequest motives, the contour line is downward-sloping in

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[50]Formally, the first-order condition required by the second-period full-event insurance, i.e., $u'(y_2 - Q_{2s}(p_2)) = v'(F_{2s}(p_2))$, together with the fact that $Q_2(p_{2s}) \geq 0$, implies immediately that $F_{2s}(p_2) \leq v'^{-1}(u'(y_1))$ for all $p_2 \in (0, 1]$.

[51]Again, the contour plots are shown only for $(\rho_1, \rho_2) \in [0.1, 5.1] \times [0.1, 5.1]$.
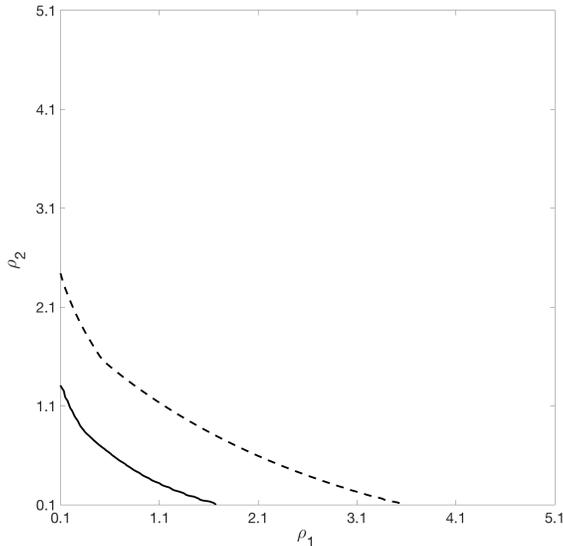
Figure 5: Welfare Comparison with Overconfidence Regarding Future Mortality Risk.
Each curve represents combinations of (ρ_1, ρ_2) for which the consumer welfare in the presence of the settlement market is equal to that in is absence, for different values of Δ_m. To the right (respectively, left) of the curve, consumer welfare is higher (respectively, lower) with the life settlement market than without. For the dashed curve, $\Delta_m = 0.3$; for the solid curve, $\Delta_m = 0.5$.

the (ρ_1, ρ_2) space, and shifts downwards as the consumer becomes more overconfident.

5 Discussions and Empirical Implications

In this section, we briefly discuss identification of the curvature of consumers' utility functions, as well as the empirical implications of our theoretical results.[52]

5.1 Identification of IES

Our welfare comparison hinges on the size of IES of the utility functions $u(\cdot)$ and $v(\cdot)$. Suppose that consumers exhibit constant IES of ρ_1 and ρ_2 for $u(\cdot)$ and $v(\cdot)$ respectively, as described in (25). Next, we briefly discuss how consumers' IES might be identified from the equilibrium contract without the settlement market. The analysis is similar for the case where the settlement market is present.

Fix an equilibrium long-term contract $\langle (Q_1, F_1), (Q_2(p_2), F_2(p_2)) : p_2 \in [0, 1] \rangle$ without the settlement market. First, consider the case where the second-period premiums are positive, i.e., $Q_2(p_2) > 0$ for all $p_2 \in (0, 1]$. The mortality risk threshold p_2^* can then be derived accordingly from the equilibrium contract. Moreover, it follows from Proposition 1 that policyholders obtain

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[52]We thank an associate editor for motivating the discussions in this section.

full-event insurance in both periods, i.e.,

$$u'(y_1 - Q_1) = v'(F_1),$$
$$u'\left(y_2 - Q_2(p_2^*)\right) = v'\left(F_2(p_2^*)\right).$$

Exploiting the constant IES functional form of $u(\cdot)$ and $v(\cdot)$, we know that ρ_1 solves

$$\log\left(1 - \frac{1}{\rho_1}\right) - \frac{1}{\rho_1}\log(y_1 - Q_1) - \log\left(1 - \frac{1}{\rho_1}\frac{\log\left(\frac{y_1 - Q_1}{y_2 - Q_2(p_2^*)}\right)}{\log\left(\frac{F_1}{F_2(p_2^*)}\right)}\right) + \frac{1}{\rho_1}\frac{\log\left(\frac{y_1 - Q_1}{y_2 - Q_2(p_2^*)}\right)}{\log\left(\frac{F_1}{F_2(p_2^*)}\right)}\log(F_1) = 0,$$

and ρ_2 can be derived as

$$\rho_2 = \rho_1 \log\left(\frac{F_1}{F_2(p_2^*)}\right) \Big/ \log\left(\frac{y_1 - Q_1}{y_2 - Q_2(p_2^*)}\right).$$

Next, let us consider the case of zero period-2 premiums, i.e., $Q_2(p_2) = 0$ for all $p_2 \in (0,1]$. By Proposition 1, policyholders lose full-event insurance in the second period in equilibrium and the period-2 face values are constant across all health states $p_2 \in (0,1]$. Note that the objective probability of a policyholder losing bequest motives q can be derived from the zero-profit condition (4). If, in addition, policyholders' subjective belief $\tilde{q}$ can be identified, then (ρ_1, ρ_2) can be derived from $u'(y_1 - Q_1) = v'(F_1)$ and $(1 - \tilde{q})u'(y_2) = (1 - q)u'(y_1 - Q_1)$.[53]

5.2 Overconfidence Regarding Bequest Motives vs. Overconfidence Regarding Mortality Risk

Thus far, we have investigated two types of consumer overconfidence. Section 2 addresses the case where policyholders are overconfident in the persistence of their bequest motives, while Section 4.3 addresses the case where consumers are overconfident in their mortality risk. It would be interesting to identify which model applies in practice through the different equilibrium insurance policies they predict.

To proceed, we assume that the second-period mortality risk p_2 can be either p_L or p_H, with $p_H > p_L > p_1$, as in Section 4.3. All results in Section 3 can be easily adapted to this model specification.

Policy Differences in the Absence of the Life Settlement Market Consider first the case without the settlement market. We follow the notation in Section 4.3 and denote the equilibrium contract by $\langle(Q_{1m}, F_{1m}), (Q_{2m}^H, F_{2m}^H), (Q_{2m}^L, F_{2m}^L)\rangle$ for the case where consumers are overconfident about their future mortality risk. Similarly, with slight abuse of notation, we denote the equilibrium contract by $\langle(Q_{1b}, F_{1b}), (Q_{2b}^H, F_{2b}^H), (Q_{2b}^L, F_{2b}^L)\rangle$ for the case where consumers exhibit overconfidence

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[53]See Equation (A10) in the Appendix.

in the persistence of their bequest motives, where we use the subscript b to indicate "bequest motives."

Proposition 1 demonstrates that only two possible patterns of second-period premiums can arise in equilibrium when consumers are overconfident about the persistence of their bequest motives: either $0 < Q_{2b}^L < Q_{2b}^H$ or $Q_{2b}^L = Q_{2b}^H$. Moreover, we must have $V_{2b}^L \equiv p_2^L F_{2b}^L - Q_{2b}^L < p_2^H F_{2b}^H - Q_{2b}^H \equiv V_{2b}^H$, i.e., the actuarial value of the second-period contract in the good health state is lower than that in the bad health state. In contrast, Lemma 9 demonstrates the possibility of $Q_{2m}^L = 0 < Q_{2m}^H$ and $V_{2m}^L \equiv p_2^L F_{2m}^L - Q_{2m}^L > p_2^H F_{2m}^H - Q_{2m}^H \equiv V_{2m}^H$.

Based on the above discussions, we can conclude that a model of overconfidence with respect to bequest motives is more likely if the second-period premiums remain constant for a set of health conditions. Further, a model of overconfidence regarding future mortality risk is more sensible if (i) policyholders pay zero premiums in a good health state, whereas they pay positive premiums in a bad health state; and/or (ii) the actuarial value of the second-period contracts *decreases* as policyholders' health condition deteriorates.

Policy Differences in the Presence of the Life Settlement Market   Next, let us turn to the situation without the settlement market. Proposition 4 predicts a similar pattern to that in Proposition 1: the actuarial value at state-p_H must be greater than that at state state-p_L. In addition, Lemma 5 demonstrates that zero premiums never emerge in equilibrium. In contrast, Lemma 10 asserts that the actuarial value of the second-period contract can decrease with policyholders' mortality risk and zero premiums can arise in equilibrium.

The above comparison indicates that a model of overconfidence regarding future mortality risk is more sensible in the presence of the settlement market if (i) the actuarial value of the second-period contracts *decreases* as policyholders' health condition deteriorates; and/or (ii) policyholders receive zero premiums in the second period.

5.3   Distinguishing the "Overconfident" Model from the "Rational" Model

Next, we elaborate on some features that can be used to distinguish an "overconfident" model from a "rational" one. A notable property of a model that includes consumer overconfidence is the possibility of zero premiums in the equilibrium contracts. To be more specific, consumers would obtain zero premiums in the second period, as Lemmas 3 and 9 predict, independent of the source of overconfidence. In contrast, such a feature never arises in a rational model. The following result can be obtained from Lemma A1 in the Appendix and Lemma 5:

Remark 1 *Suppose that policyholders are rational. Then the equilibrium second-period premiums always remain positive with or without the life settlement market.*

Therefore, zero premiums cannot be rationalized by a rational model, and are thus an indication of consumer overconfidence. Recently, life insurance firms (e.g., AIG, AAA, StateFarm, etc.) have begun to offer return of premium (ROP) life insurance, which is more expensive than the traditional term life insurance policy and shares some of the features predicted in Lemma 4. With an ROP

insurance policy, the insurers return all (or part) of the premiums into the plan if the policyholder outlives the term of the policy. This can be loosely interpreted as a zero or negative period-2 premium in our model. To see this, suppose that the insurer returns an amount $R > 0$ to a policyholder if he outlives the policy, in which case his period-2 consumption becomes $y_2 - Q_2(p_2) + R$. Clearly, the effective period-2 premium the policyholder pays is $Q_2(p_2) - R$, which could turn negative if R is sufficiently large.

In addition to insurance premiums, policyholders' propensity to sell their life insurance contracts on the secondary market may be used to identify which model applies. Recall that lapsation is motivated by bequest shocks in our baseline model. Suppose that policyholders are rational (i.e., $\tilde{q} = q$) and differ only in the probability of losing their bequest motives. Because q enters policyholders' expected utility but not firms' profit in the presence of the settlement market, pooling will not arise and different types would be offered different contracts in equilibrium.[54] In contrast, Proposition 7 shows that in a model with consumer overconfidence (i.e., $\tilde{q} < q$), policyholders of different objective probability q's—and thus different propensities for selling their policy in the second period—would opt for the same contract, given that they share the same belief $\tilde{q}$. Combining all the arguments, we suggest that a model that includes overconfidence is appropriate if the data shows that policyholders who purchase the same long-term contract initially exhibit sufficient heterogeneity in their propensity to sell their life insurance policies.[55]

5.4 Empirical Implications on Consumer Welfare

In this part, we discuss some implications on consumer welfare based on our theoretical results.[56] Recall that Propositions 2 and 5 both predict that, in equilibrium, there will be excessive front-loading for overconfident consumers relative to the rational benchmark, and the degree of front-loading increases as policyholders become more overconfident.[57] Further, Theorem 1 states that the presence of the settlement market is more likely to be welfare-enhancing when consumers are more overconfident and are more "vulnerable" in the sense that they are less risk-averse (or exhibit a higher IES). Combining these results, we can conclude the following: the more front-loaded the primary life insurance contract is, the more likely it is that the primary life insurers are exploiting overconfident consumers, which in turn implies that the introduction of the life settlement market

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[54]Equilibrium analysis in the absence of the settlement market is more complicated because q influences both consumers' expected utility and firms' profit. We conjecture that the competitive equilibrium (if it exists) will again feature separation through the standard argument in Rothschild and Stiglitz (1976).

[55]Empirically, count regression analysis can be used to predict the expected frequency of lapsation/surrender. See Denuit, Maréchal, Pitrebois, and Walhin (2007), Cameron and Trivedi (2013), and Knoller, Kraut, and Schoenmaekers (2016) for more details.

[56]Ideally, to quantify the welfare effects of the secondary markets, one needs to calibrate model parameters using data from the industry (see Chen, Esteban, and Shum (2013) for an example of such an exercise in the case of the secondary market for automobiles). This is beyond the scope of this paper and we leave the empirical exploration of the welfare consequences of the settlement market for future research.

[57]It should be noted that front-loading alone does not suffice to imply market inefficiency. In fact, it is the *excessive* front-loading caused by consumer overconfidence that leads to inefficiency. To see this, note that when policyholders are fully rational, front-loading arises in the equilibrium long-term contract and serves as a welfare-improving commitment device over the set of short-term contracts, which do not feature front-loading.

is more likely to improve consumer welfare.

At a more specific level, as we have mentioned in Section 5.3, many primary life insurance firms now offer "return of premium" (ROP) term life insurance, in which, if the policyholder survives the policy term, the insurer returns all or part of the premium that is paid. An ROP policy is significantly more expensive than the traditional term life insurance policy. It thus exhibits more front-loading than traditional life insurance policies, and is more likely to be selected by consumers with a higher degree of overconfidence, as Propositions 5 and 8 suggest. Our result thus indicates that consumers who purchase an ROP life insurance policy would potentially benefit more from the introduction of the life settlement market.

Also, we know that gender, occupation, and age are all important pricing factors to the primary life insurers. Also, previous studies find that these pricing factors are closely related to the consumers' biases and risk attitudes. Consider, for instance, gender. A plethora of empirical and experimental evidence suggests that women on average tend to exhibit higher risk aversion than men (e.g., Croson and Gneezy, 2009; Eckel and Grossman, 2008; Jianakoplos and Bernasek, 1998, among many others), and are less overconfident than men (e.g., Barber and Odean, 2001). Our theory then suggests that male policyholders will likely be offered insurance policies with a higher degree of front-loading in the absence of the settlement market, and as a result, introducing the settlement market is more likely to benefit male policyholders than female policyholders.

Age also plays an important role in explaining individual differences in risk attitudes (Dohmen, Falk, Huffman, Sunde, Schupp, and Wagner, 2011) and overconfidence (Sandroni and Squintani, 2004). According to Dohmen et al. (2011): "Willingness to take risks appears to decrease steadily with age for men, whereas for women willingness to take risks decreases more rapidly from the late teens to age 30, and then remains flat, until it begins to decrease again from the mid-50s onwards." Sandroni and Squintani (2004) documented that "overconfidence is particularly pervasive among young adults, but it does not vanish with learning and experience." Our theory predicts that, when all else is held constant, young (male) policyholders would be exploited by the insurance firms to a greater extent, and would thus benefit more from the introduction of the life settlement market.

6 Conclusion

In this paper, we investigate how the life settlement market—the secondary market for life insurance—may affect consumer welfare in a dynamic equilibrium model of life insurance with one-sided commitment and overconfident policyholders who may allow their policies to lapse when they lose their bequest motives. In the baseline model, policyholders may underestimate the probability of losing their bequest motives and the CSVs are restricted to zero. The actual and perceived probability of policyholders losing bequest motives are observed by firms. We show that, in the absence of the life settlement, overconfident consumers may buy "too much" reclassification risk insurance for later periods in the competitive equilibrium. The life settlement market can impose a limit on the extent to which overconfident consumers can be exploited by the primary insurers. In particular, we show that the life settlement market may increase the equilibrium welfare of over-

confident consumers when they are sufficiently vulnerable in the sense that they have a sufficiently large intertemporal elasticity of substitution of consumption.

In one extension, we alter the observability assumption of policyholders' actual or perceived probability of losing their bequest motives, and show that the equilibrium contracts feature the same property as in the baseline model. In another extension, we allow the primary insurers to include endogenous CSVs in the contract. We show that positive CSVs of the life insurance policies will not be utilized in equilibrium when the settlement market is absent. In contrast, when the settlement market is in place, CSVs are positive and equal the amount that can be obtained from the settlement market. We further generalize the model and consider another form of overconfidence: the policyholders may be overconfident about their future mortality risk. Unlike the case of overconfidence with respect to bequest motives, when consumers are overconfident about their future mortality risk in the sense that they put too high a subjective probability on the low-mortality state, the competitive equilibrium contract in the absence of life settlement exploits the consumer bias by offering them very high face values in the low-mortality state. In all three extensions, we show that our main result on the welfare comparison (i.e., Theorem 1) remains qualitatively unchanged.

There are several directions for future research. First, in this paper we study the role of consumer overconfidence in determining the shape of the equilibrium life insurance contract. It would be interesting to empirically test the existence of policyholders' overconfidence based on the predictions in this paper. Second, we follow Daily, Hendel, and Lizzeri (2008) and Fang and Kung (2020), and assume throughout the paper that policyholders may allow their insurance contracts to lapse when they lose their bequest motives. It is worthwhile to analyze the welfare implications of the settlement market in a unified framework where lapsation is driven by bequest motive shocks as well as by negative income shocks. Third, our model lasts for two periods as is commonly assumed in the literature for the sake of tractability, which in turn indicates that the loss of bequest motives can only occur at the beginning of the second period. Extending the model to multiple periods will enable us to investigate the impacts of the timing of loss of bequest motives (and thus the mistakes in the expectation of such shocks) on the shape of the equilibrium contracts. Fourth, as mentioned in Section 3.2, it would be important to inject consumer heterogeneity and private information into our model, and explore the welfare effects of naïveté-based discrimination in the spirit of Heidhues and Kőszegi (2017). Finally, in our paper we have identified the potential role of IES in the welfare analysis of the life insurance market when policyholders are not fully rational. Another intriguing research avenue would be to generalize the economic insights of IES and consumer vulnerability to other markets (e.g., the credit market and the labor market) and quasi-Bayesian models, and investigate the welfare impact and efficacy of different government policies under different market structures.

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Appendix: Proofs

Proof of Lemma 1

Proof. The first-order conditions for Problem (3) with respect to Q_1, F_1, $Q_2(p_2)$ $F_2(p_2)$ yield:

$$u'(y_1 - Q_1) = \mu, \tag{A1a}$$

$$v'(F_1) = \mu, \tag{A1b}$$

$$(1 - \tilde{q})u'\left(y_2 - Q_2(p_2)\right) = (1 - q)\mu + \frac{\lambda(p_2) + \gamma(p_2)}{(1 - p_1)\phi(p_2)}, \text{ for all } p_2 \in [0, 1], \tag{A1c}$$

$$(1 - \tilde{q})v'\left(F_2(p_2)\right) = (1 - q)\mu + \frac{\lambda(p_2)}{(1 - p_1)\phi(p_2)}, \text{ for all } p_2 \in [0, 1], \tag{A1d}$$

where μ, $\lambda(p_2)$ and $\gamma(p_2)$ are the Lagrange multipliers for constraints (4), (5), and (6); moreover, $\mu > 0$, $\lambda(p_2) \leq 0$ and $\gamma(p_2) \geq 0$ need to satisfy the complementary slackness conditions:

$$\lambda(p_2)\left[Q_2(p_2) - p_2 F_2(p_2)\right] = 0, \text{ for all } p_2 \in [0, 1], \tag{A2a}$$

$$\gamma(p_2)Q_2(p_2) = 0, \text{ for all } p_2 \in [0, 1]. \tag{A2b}$$

First, we show that $Q_2(p_2) \leq Q_2(p_2')$. The complementary slackness condition (A2a), together with the postulated $p_2' \in \mathcal{NB}$, implies that $\lambda(p_2') = 0$. Note that the Inada condition on $v(\cdot)$ implies that $F_2(p_2) > 0$; together with $p_2 \in \mathcal{B}$, we must have $Q_2(p_2) = p_2 F_2(p_2) > 0$, which in turn implies that $\gamma(p_2) = 0$ from (A2b). Therefore, we have that

$$\begin{aligned}
(1 - \tilde{q})u'\left(y_2 - Q_2(p_2)\right) &= (1 - q)\mu + \frac{\lambda(p_2) + \gamma(p_2)}{(1 - p_1)\phi(p_2)} \\
&\leq (1 - q)\mu + \frac{\lambda(p_2') + \gamma(p_2')}{(1 - p_1)\phi(p_2')} = (1 - \tilde{q})u'\left(y_2 - Q_2(p_2')\right),
\end{aligned}$$

where the two equalities follow from the first-order condition (A1c); and the inequality follows from $\lambda(p_2) + \gamma(p_2) \leq 0 \leq \lambda(p_2') + \gamma(p_2')$. From the strict concavity of $u(\cdot)$ and $\tilde{q} < 1$, we must have $Q_2(p_2) \leq Q_2(p_2')$. Similarly, it can be shown that $F_2(p_2) \geq F_2(p_2')$.

Next, we show that $p_2 < p_2'$. Suppose to the contrary that $p_2 \geq p_2'$. Then we have that

$$Q_2(p_2) \leq Q_2(p_2') < p_2' F_2(p_2') \leq p_2 F_2(p_2),$$

where the second inequality follows from $p_2' \in \mathcal{NB}$, and the last inequality follows from $F_2(p_2) \geq F_2(p_2')$ and the postulated $p_2 \geq p_2'$. Therefore, we have that $Q_2(p_2) < p_2 F_2(p_2)$ and thus $p_2 \in \mathcal{NB}$, which contradicts the postulated $p_2 \in \mathcal{B}$. This completes the proof. ∎

Proof of Lemma 2

Proof. See the proof of Proposition 1. ∎

Proof of Lemma 3

Proof. Suppose to the contrary that there exist two health states $\tilde{p}_2 \neq 0$ and $\tilde{p}'_2 \neq 0$ such that $Q_2(\tilde{p}'_2) > 0 = Q_2(\tilde{p}_2)$, then $\gamma(\tilde{p}_2) \geq 0 = \gamma(\tilde{p}'_2)$ from (A2b). Moreover, the Inada condition on $v(\cdot)$ implies that $F_2(\tilde{p}_2) > 0$. Therefore,

$$Q_2(\tilde{p}_2) - \tilde{p}_2 F_2(\tilde{p}_2) = 0 - \tilde{p}_2 F_2(\tilde{p}_2) < 0.$$

Together with (A2a), we must have $\lambda(\tilde{p}_2) = 0 \geq \lambda(\tilde{p}'_2)$, and thus

$$
\begin{aligned}
(1 - \tilde{q})u'\left(y_2 - Q_2(\tilde{p}'_2)\right) = (1 - q)\mu + & \frac{\lambda(\tilde{p}'_2) + \gamma(\tilde{p}'_2)}{(1 - p_1)\phi(\tilde{p}'_2)} \\
\leq (1 - q)\mu + & \frac{\lambda(\tilde{p}_2) + \gamma(\tilde{p}_2)}{(1 - p_1)\phi(\tilde{p}_2)} = (1 - \tilde{q})u'\left(y_2 - Q_2(\tilde{p}_2)\right),
\end{aligned}
$$

where the two equalities follow from (A1c); and the inequality follows from $\lambda(\tilde{p}'_2) + \gamma(\tilde{p}'_2) \leq 0 \leq \lambda(\tilde{p}_2) + \gamma(\tilde{p}_2)$. Thus, $Q_2(\tilde{p}'_2) \leq Q_2(\tilde{p}_2)$ from the strict concavity of $u(\cdot)$, which contradicts the postulated $Q_2(\tilde{p}'_2) > 0 = Q_2(\tilde{p}_2)$. This completes the proof. ∎

Proof of Lemma 4

Proof. The proof proceeds in the following four steps. First, we show that the second-period premiums for all non-zero health states are strictly positive when policyholders are rational (i.e., $\Delta = 0$) in Lemma A1. Second, we show in Lemma A2 that there exists a threshold overconfidence level above which (below which, respectively) $p_2^* = 0$ ($p_2^* > 0$, respectively). Third, we show in Lemma A3 that $p_2^* = 0$ occurs when q is sufficiently large, holding fixed the subjective belief $\tilde{q}$ about losing bequest motives. Last, fixing $\tilde{q}$, we prove the existence of an objective belief threshold (i.e., $q_0(\tilde{q})$) above which (below which, respectively) $p_2^* = 0$ ($p_2^* > 0$, respectively) in Lemma A4. Lemma 4 follows immediately from combining the aforementioned intermediary results.

Lemma A1 *If $\Delta = 0$, then $Q_2(p_2) > 0$ for all $p_2 \in (0, 1]$.*

Proof. Suppose to the contrary that there exists a health state $\hat{p}_2 \in (0, 1]$ such that $Q_2(\hat{p}_2) = 0$. Because $F_2(p_2) > 0$ for all $p_2 > 0$ from the Inada condition on $v(\cdot)$, we must have that $Q_2(\hat{p}_2) - \hat{p}_2 F_2(\hat{p}_2) < 0$, implying $\lambda(\hat{p}_2) = 0$. Combining (A1a) and (A1c) yields that

$$u'(y_2) = u'\left(y_2 - Q_2(\hat{p}_2)\right) = \frac{1 - q}{1 - \tilde{q}}\mu + \frac{1}{1 - \tilde{q}}\frac{\lambda(\hat{p}_2) + \gamma(\hat{p}_2)}{(1 - p_1)\phi(\hat{p}_2)} \geq \frac{1 - q}{1 - \tilde{q}}\mu = u'(y_1 - Q_1), \quad \text{(A3)}$$

where the inequality follows from $\lambda(\hat{p}_2) = 0$ and $\gamma(\hat{p}_2) \geq 0$, and the last equality follows from $\Delta = 0$. By Footnote 22, $Q_1 \geq p_1 F_1 > 0$; together with Assumption 1, we must have $Q_1 \geq \delta$ and thus

$$y_2 > y_1 - \delta \geq y_1 - Q_1, \quad \text{(A4)}$$

which in turn implies that $u'(y_2) < u'(y_1 - Q_1)$. This contradicts (A3) and completes the proof. ∎

Lemma A2 *Fix q and $p_2 \neq 0$. Denote the equilibrium period-2 premium in health state p_2 with respect to overconfidence Δ and Δ' by $Q_2(p_2)$ and $Q_2'(p_2)$ respectively. If $Q_2(p_2) > 0 = Q_2'(p_2)$, then $\Delta < \Delta'$.*

Proof. For notational convenience, we use the prime symbol to refer to the variables for which the degree of consumer overconfidence is Δ'. Suppose to the contrary that $\Delta \geq \Delta'$. It follows immediately that $\tilde{q} \equiv q(1 - \Delta) \leq q(1 - \Delta') \equiv \tilde{q}'$. From Lemma 3, $Q_2'(p_2) = 0$ for all $p_2 \in (0, 1]$, which implies that $\gamma'(p_2) \geq 0$ and $\lambda'(p_2) = 0$ for all $p_2 \in (0, 1]$. Similarly, $Q_2(p_2) > 0$ for all $p_2 \in (0, 1]$, implying $\gamma(p_2) = 0$, $\lambda(p_2) \leq 0$, and thus $u'(y_2 - Q_2(p_2)) = v'(F_2(p_2))$ for all $p_2 \in (0, 1]$.

We first show that the period-2 face value under Δ' is strictly greater than that under Δ for all $p_2 \in (0, 1]$. Combining the first-order conditions (A1c) and (A1d), we have that

$$
\begin{aligned}
v'\left(F_2'(p_2)\right) &= u'\left(y_2 - Q_2'(p_2)\right) - \frac{1}{(1 - \tilde{q}')} \times \frac{\gamma'(p_2)}{(1 - p_1)\phi(p_2)} \\
&< u'\left(y_2 - Q_2(p_2)\right) = v'\left(F_2(p_2)\right), \quad \text{for all } p_2 \in (0, 1],
\end{aligned}
$$

where the strict inequality follows from $\gamma'(p_2) \geq 0$ and $Q_2(p_2) > 0 = Q_2'(p_2)$. Thus, $F_2'(p_2) > F_2(p_2)$ for all $p_2 \in (0, 1]$ from the strict concavity of $v(\cdot)$.

Next, we show that the period-1 face value under Δ' is strictly greater than that under Δ. Fixing $p_2 \neq 0$, combining conditions (A1b) and (A1c) yields

$$
\begin{aligned}
(1 - q)v'(F_1) &= (1 - \tilde{q})u'\left(y_2 - Q_2(p_2)\right) - \frac{\lambda(p_2) + \gamma(p_2)}{(1 - p_1)\phi(p_2)} \\
&\geq (1 - \tilde{q})u'\left(y_2 - Q_2(p_2)\right) \\
&> (1 - \tilde{q}')u'\left(y_2 - Q_2'(p_2)\right) \\
&\geq (1 - \tilde{q}')u'\left(y_2 - Q_2'(p_2)\right) - \frac{\lambda'(p_2) + \gamma'(p_2)}{(1 - p_1)\phi(p_2)} = (1 - q)v'(F_1').
\end{aligned}
$$

The first inequality follows from $\gamma(p_2) = 0$ and $\lambda(p_2) \leq 0$; the second inequality follows from $Q_2(p_2) > 0 = Q_2'(p_2)$; and the third inequality follows from $\gamma'(p_2) \geq 0$ and $\lambda'(p_2) = 0$. The above inequality clearly implies that $F_1' > F_1$ and $Q_1' < Q_1$.

To complete the proof, notice that the expected profit under Δ' can be bounded from above by

$$
\begin{aligned}
&(Q_1' - p_1 F_1') + (1 - p_1)(1 - q) \int_0^1 \left[Q_2'(p_2) - p_2 F_2'(p_2)\right] d\Phi(p_2) \\
&< (Q_1 - p_1 F_1) + (1 - p_1)(1 - q) \int_0^1 \left[Q_2(p_2) - p_2 F_2(p_2)\right] d\Phi(p_2) = 0,
\end{aligned}
$$

which is a contradiction to the zero-profit condition (4) when the degree of consumer overconfidence is Δ'. This completes the proof. ∎

Lemma A3 *Fixing $\tilde{q} \in [0, 1)$, there exists $q \in (\tilde{q}, 1)$ such that $Q_2(p_2) = 0$ for some $p_2 \neq 0$.*

Proof. Suppose to the contrary that there exists $\tilde{q}$ such that $Q_2(p_2) > 0$ for all $q \in (\tilde{q}, 1)$. This implies that $\lambda(p_2) \leq 0$ and $\gamma(p_2) = 0$ for all $p_2 \in (0, 1]$. Then we must have

$$v'\left(F_2(p_2)\right) = u'\left(y_2 - Q_2(p_2)\right) > u'(y_2), \text{ for all } p_2 \in (0, 1]. \tag{A5}$$

Therefore, $F_2(p_2)$ is bounded from above by $v'^{-1}\left(u'(y_2)\right)$; and the period-1 expected profit is bounded from above by

$$Q_1 - p_1 F_1 = -(1 - p_1)(1 - q) \int_0^1 \left[Q_2(p_2) - p_2 F_2(p_2)\right] d\Phi(p_2)$$
$$< (1 - p_1)(1 - q)\bar{p}_2 v'^{-1}\left(u'(y_2)\right),$$

where

$$\bar{p}_2 := \int_0^1 p_2 d\Phi(p_2) \tag{A6}$$

is the expected period-2 mortality risk. Moreover, it follows from Footnote 22 that $Q_1 - p_1 F_1 \geq 0$. Therefore, we have that

$$0 \leq Q_1 - p_1 F_1 < (1 - p_1)(1 - q)\bar{p}_2 v'^{-1}\left(u'(y_2)\right). \tag{A7}$$

Taking limits on all sides of (A7) as $q \nearrow 1$ yields

$$0 \leq \lim_{q \nearrow 1}(Q_1 - p_1 F_1) \leq \lim_{q \nearrow 1}(1 - p)(1 - q)\bar{p}_2 v'^{-1}\left(u'(y_2)\right) = 0.$$

This clearly implies that $\lim_{q \nearrow 1} F_1 = F_1^{FI}$ and $\lim_{q \nearrow 1} Q_1 = Q_1^{FI}$, where (Q_1^{FI}, F_1^{FI}) is the unique solution to the following pair of equations:

$$u'\left(y_1 - Q_1^{FI}\right) = v'\left(F_1^{FI}\right),$$
$$p_1 F_1^{FI} - Q_1^{FI} = 0.$$

It is straightforward to verify that $Q_1^{FI} = \delta$ and $F_1^{FI} = \delta/p_1$, where δ is defined in Assumption 1. To complete the proof, notice that

$$(1 - \tilde{q})u'(y_2) < (1 - \tilde{q})v'(F_2\left(p_2\right)) = (1 - q)v'(F_1) + \frac{\lambda(p_2)}{(1 - p_1)\phi(p_2)} \leq (1 - q)v'(F_1),$$

where the first inequality follows from (A5); the equality follows from (A1b) and (A1d); and the last inequality follows from $\lambda(p_2) \leq 0$. Therefore,

$$(1 - \tilde{q})u'(y_2) = \lim_{q \nearrow 1}(1 - \tilde{q})u'(y_2) \leq \lim_{q \nearrow 1}(1 - q)v'(F_1) = 0 \times v'(F_1^{FI}) = 0,$$

a contradiction. This completes the proof. ∎

Lemma A4 *Fixing $\tilde{q}$, there exists a threshold $q_0(\tilde{q}) \in (\tilde{q}, 1)$ such that $Q_2(p_2) = 0$ for all $p_2 \in (0, 1]$ if $q > q_0(\tilde{q})$, and $Q_2(p_2) > 0$ for all $p_2 \in (0, 1]$ if $q < q_0(\tilde{q})$. Moreover, $q_0(\tilde{q})$ is weakly increasing in $\tilde{q}$.*

Proof. From Lemma A1, $Q_2(p_2) > 0$ for all $p_2 \in (0, 1]$ if $\Delta = 0$, or equivalently, if $q = \tilde{q}$. Moreover, Lemma A3, together with Lemma 3, indicates that there exists $q \in (\tilde{q}, 1)$ such that $Q_2(p_2) = 0$ for all $p_2 \in (0, 1]$.

We first prove the existence of the threshold $q_0(\tilde{q})$. Fixing $\tilde{q}$, suppose to the contrary that there exist q'' and q with $q'' > q$ such that $Q_2(p_2) = 0 < Q_2''(p_2)$ for all $p_2 \in (0, 1]$. With slight abuse of notation, we use the double prime symbol to refer to the variables when the consumer's objective probability of losing bequest motives is q''. It follows immediately that $\lambda(p_2) = 0$, $\lambda''(p_2) \leq 0$, $\gamma(p_2) \geq 0$, and $\gamma''(p_2) = 0$ for all $p_2 \in (0, 1]$. From the first-order conditions (A1c) and (A1d), we have that

$$v'\left(F_2(p_2)\right) = u'\left(y_2 - Q_2(p_2)\right) - \frac{1}{1-\tilde{q}} \times \frac{\gamma(p_2)}{(1-p_1)\phi(p_2)}$$
$$< u'\left(y_2 - Q_2''(p_2)\right)$$
$$= v'\left(F_2''(p_2)\right), \text{ for all } p_2 \in (0, 1].$$

The strict inequality follows from the fact that $\gamma(p_2) \geq 0$ and $Q_2''(p_2) > 0 = Q_2(p_2)$; and the last equality follows from $\gamma''(p_2) = 0$. Therefore, $F_2(p_2) > F_2''(p_2)$ for all $p_2 \in (0, 1]$ from the strict concavity of $v(\cdot)$. Combining the first-order conditions (A1b) and (A1c) yields that

$$v'(F_1) = \frac{1-\tilde{q}}{1-q} u'\left(y_2 - Q_2(p_2)\right) - \frac{\lambda(p_2) + \gamma(p_2)}{(1-p_1)(1-q)\phi(p_2)}$$
$$< \frac{1-\tilde{q}}{1-q''} u'\left(y_2 - Q_2''(p_2)\right) - \frac{\lambda''(p_2) + \gamma''(p_2)}{(1-p_1)(1-q'')\phi(p_2)} = v'(F_1''),$$

where the strict inequality follows from $q < q''$, $Q_2(p_2) = 0 < Q_2''(p_2)$, and $\lambda(p_2) + \gamma(p_2) \geq 0 \geq \lambda''(p_2) + \gamma''(p_2)$. Therefore, we have that $F_1 > F_1''$, $Q_1 < Q_1''$, and

$$0 = (Q_1'' - p_1 F_1'') + (1-p_1)(1-q'') \int_0^1 \left[Q_2''(p_2) - p_2 F_2''(p_2)\right] d\Phi(p_2)$$
$$> (Q_1 - p_1 F_1) + (1-p_1)(1-q) \int_0^1 \left[Q_2(p_2) - p_2 F_2(p_2)\right] d\Phi(p_2) = 0,$$

which is a contradiction.

Next, we show that $q_0(\tilde{q})$ is weakly increasing in $\tilde{q}$. Suppose to the contrary that there exist $\tilde{q}_1$ and $\tilde{q}_2$ such that $\tilde{q}_1 > \tilde{q}_2$ and $q_0(\tilde{q}_1) < q_0(\tilde{q}_2)$. Note that $\tilde{q}_1 < q_0(\tilde{q}_1)$ from the above argument. Therefore, we have that

$$\tilde{q}_2 < \tilde{q}_1 < q_0(\tilde{q}_1) < q_0(\tilde{q}_2).$$

Let

$$\acute{q} := \frac{q_0(\tilde{q}_1) + q_0(\tilde{q}_2)}{2} \in \left(q_0(\tilde{q}_1), q_0(\tilde{q}_2)\right).$$

Fix $q = \acute{q}$. Because $\acute{q} < q_0(\tilde{q}_2)$, the period-2 premiums in all non-zero health states are positive for $(q, \tilde{q}) = (\acute{q}, \tilde{q}_2)$. By the same argument, the period-2 premiums in all health states are zero for $(q, \tilde{q}) = (\acute{q}, \tilde{q}_1)$ because $\acute{q} > q_0(\tilde{q}_1)$. Therefore, it follows instantly that $\frac{\acute{q} - \tilde{q}_1}{\acute{q}} > \frac{\acute{q} - \tilde{q}_2}{\acute{q}}$ from Lemma A2, or equivalently, $\tilde{q}_1 < \tilde{q}_2$, which contradicts the postulated $\tilde{q}_2 < \tilde{q}_1$. This completes the proof. ∎

Now we can prove Lemma 4. Define $\bar{q}$ as $\bar{q} := q_0(0)$. We consider two cases that depend on the relationship between q and $\bar{q}$.

Case I: $q < \bar{q} \equiv q_0(0)$. It suffices to show that $Q_2(p_2) > 0$ for all $p_2 \in (0, 1]$ and $\Delta \in [0, 1]$. Suppose to the contrary that there exists $\tilde{q} \in [0, 1]$ such that $Q_2(p_2) = 0$ for some $p_2 \in (0, 1]$. Then $Q_2(p_2) = 0$ for all $p_2 \in (0, 1]$ from Lemma 3. It follows immediately that $q \geq q_0(\tilde{q}) \geq q_0(0) \equiv \bar{q}$ from Lemma A4, a contradiction.

Case II: $q > \bar{q} \equiv q_0(0)$. For $\tilde{q} = 0$ (i.e., $\Delta = 1$), it follows instantly that $Q_2(p_2) = 0$ for all $p_2 \in (0, 1]$ from Lemma A4. Similarly, Lemma A1 implies that $Q_2(p_2) > 0$ for all $p_2 \in (0, 1]$ if $\tilde{q} = q$ (i.e., $\Delta = 0$). Therefore, it follows from Lemma A2 that there exists a threshold $\bar{\Delta}(q) \in (0, 1)$ such that $Q_2(p_2) = 0$ for all $p_2 \in (0, 1]$ if $\Delta > \bar{\Delta}(q)$, and $Q_2(p_2) > 0$ for all $p_2 \in (0, 1]$ if $\Delta < \bar{\Delta}(q)$. This completes the proof. ∎

Proof of Proposition 1

Proof. Lemma 3 narrows down the set of period-2 equilibrium premiums to one of two possibilities: (i) $Q_2(p_2) > 0$ for all $p_2 \in (0, 1]$; or (ii) $Q_2(p_2) = 0$ for all $p_2 \in (0, 1]$.

Under case (i), constraint (6) is not binding, thus $\gamma(p_2) = 0$ for all $p_2 \in (0, 1]$. Hence, the first-order conditions (A1c) and (A1d) imply that Equation (9) holds for all $p_2 \in (0, 1]$.

Under case (ii), $Q_2(p_2) = 0$ implies that constraint (5) is not binding, and thus $\lambda(p_2) = 0$ for all $p_2 \in (0, 1]$. Combining the first-order conditions (A1b) and (A1d), we must have that

$$(1 - \tilde{q})v'\left(F_2(p_2)\right) = (1 - q)v'(F_1) \text{ for all } p_2 \in (0, 1], \tag{A8}$$

which in turn implies that $F_2(p_2)$ must be constant for all $p_2 \in (0, 1]$.

Now we consider the implications of Lemmas 1 and 3. As we mentioned in the main text, Lemma 1 implies that there exists a threshold death probability p_2^* in period 2 that divides the set $\mathcal{B}$ from $\mathcal{NB}$. There are three possibilities: (a) $p_2^* = 0$; (b) $p_2^* = 1$; and (c) $p_2^* \in (0, 1)$.

First, consider the case where $p_2^* = 1$. This implies that the no-lapsation condition (5) binds for all period-2 health states, i.e., $Q_2(p_2) - p_2 F_2(p_2) = 0$ for all $p_2 \in (0, 1]$. Because $F_2(p_2) > 0$ for all $p_2 \in (0, 1]$ by the Inada condition on $v(\cdot)$, $Q_2(p_2) = p_2 F_2(p_2) > 0$ for all $p_2 \in (0, 1]$. This in turn implies that condition (9) also holds for all $p_2 \in (0, 1]$. Thus, in this case, the set of equilibrium period-2 contracts corresponds to the fair premium and face value full-event spot insurance contracts defined by (8a)-(8b).

Second, consider the case where $p_2^* = 0$. We first argue that $p_2^* = 0$ implies that:

$$Q_2(p_2) = 0 \text{ for all } p_2 \in (0, 1]. \tag{A9}$$

A6

To see this, suppose to the contrary that $Q_2(p_2) > 0$ for some $p_2 \in (0, 1]$; then by Lemma 3, it must be that $Q_2(p_2) > 0$ for all $p_2 \in (0, 1]$. Hence (9) holds. Moreover, by Lemma 1, $p_2^* = 0$ implies that $p_2 \in \mathcal{NB}$ for all $p_2 \in (0, 1]$, hence $\lambda(p_2) = 0$ for all $p_2 \in (0, 1]$. Thus, the first-order conditions (A1) imply that

$$u'\left(y_2 - Q_2(p_2)\right) = v'\left(F_2(p_2)\right) = \frac{1-q}{1-\tilde{q}} u'\left(y_1 - Q_1\right) \text{ for all } p_2 \in (0, 1]. \tag{A10}$$

Thus, $F_2(p_2)$ and $Q_2(p_2)$ must be constant for all $p_2 \in (0, 1]$. However, when p_2 is sufficiently small, $Q_2(p_2) - p_2 F_2(p_2)$ must be strictly positive, contradicting (5). Therefore, if $p_2^* = 0$, then the set of equilibrium contracts is fully characterized by (4), (7), (A8) and (A9).

Third, consider the case where $p_2^* \in (0, 1)$. If $p_2 < p_2^*$, then the no-lapsation constraint (5) binds. Because $F_2(p_2) > 0$ by the Inada condition on $v(\cdot)$, it follows immediately that $Q_2(p_2) = p_2 F_2(p_2) > 0$ for $p_2 \in (0, p_2^*)$ and thus constraint (6) is not binding, implying $\gamma(p_2) = 0$. Therefore, (A1c) and (A1d) imply that $u'\left(y_2 - Q_2(p_2)\right) = v'(F_2(p_2))$ for all $p_2 < p_2^*$. This, together with the binding constraint (5), implies that $\langle Q_2(p_2), F_2(p_2)\rangle = \langle Q_2^{FI}(p_2), F_2^{FI}(p_2)\rangle$ for all $p_2 < p_2^*$, where $\langle Q_2^{FI}(p_2), F_2^{FI}(p_2)\rangle$ is characterized by (8). If $p_2 > p_2^*$, then $p_2 \in \mathcal{NB}$, hence $\lambda(p_2) = 0$. Thus, the first-order condition (A1d) implies that $F_2(p_2)$ must be constant in p_2. Moreover, from the discussion above for the case where $p_2 < p_2^*$, we know that $Q_2(p_2) > 0$ if $p_2 < p_2^*$. Therefore, $\gamma(p_2) = 0$ for all $p_2 \in (0, 1]$ from Lemma 3, indicating that $u'\left(y_2 - Q_2(p_2)\right) = v'\left(F_2(p_2)\right)$ for all $p_2 \in (0, 1]$. Because $F_2(p_2)$ is a constant for $p_2 > p_2^*$, it must be that $Q_2(p_2)$ is a constant for $p_2 > p_2^*$ as well and moreover, the premiums are front-loaded in the sense that $Q_2(p_2) < Q_2^{FI}(p_2)$ for $p_2 > p_2^*$. That is, the insurance firms charge the policyholders a level period-2 premium for health states $p_2 > p_2^*$ below the corresponding fair premium, so as to insure the policyholders against reclassification risk. In addition, we must have that $\langle Q_2(p_2^*), F_2(p_2^*)\rangle = \langle Q_2^{FI}(p_2^*), F_2^{FI}(p_2^*)\rangle$ at p_2^* by continuity, and

$$(1 - \tilde{q})u'\left(y_2 - Q_2^{FI}(p_2^*)\right) = (1 - q)u'(y_1 - Q_1). \tag{A11}$$

The equilibrium long-term contract in this case is fully characterized by (4), (7), (9) and (10). Equation (A11) also provides an explicit unique characterization for p_2^* provided that p_2^* lies strictly in $(0, 1)$ because $Q_2^{FI}(\cdot)$ as defined by (8) is monotonically increasing. From Equation (A11), it is clear that when q is sufficiently close to 1 and $\tilde{q}$ is sufficiently close to 0, the left-hand side of Equation (A11) will be higher than the right-hand side even if $Q_2^{FI}(p_2^*) = 0$. When this occurs, p_2^* will be 0. This completes the proof. ∎

Proof of Proposition 2

Proof. For notational convenience, we use the hat symbol to refer to the variables for which the degree of consumer overconfidence is $\hat{\Delta}$. We first show that it must be the case that $\hat{Q}_1 > Q_1$. Suppose to the contrary that $\hat{\Delta} > \Delta$ (i.e., $\hat{\tilde{q}} < \tilde{q}$) and $\hat{Q}_1 \leq Q_1$. Equation (7) clearly implies that $\hat{F}_1 \geq F_1$.

We first show that $\hat{Q}_2(p_2) - p_2 \hat{F}_2(p_2) \leq Q_2(p_2) - p_2 F_2(p_2)$ for all $p_2 \in (0, 1]$. It is clear that if health state $p_2 \in \mathcal{B}$ under Δ, then $Q_2(p_2) - p_2 F_2(p_2) = 0 \geq \hat{Q}_2(p_2) - p_2 \hat{F}_2(p_2)$; and it remains

to consider the case where $p_2 \in \mathcal{NB}$ under Δ. By definition, $\lambda(p_2) = 0$. In addition, from the first-order conditions (A1b) and (A1d), we have that

$$
\begin{aligned}
(1 - \tilde{q})v'\left(F_2(p_2)\right) &= (1 - q)v'(F_1) + \frac{\lambda(p_2)}{(1 - p_1)\phi(p_2)} \\
&\geq (1 - q)v'(\hat{F}_1) + \frac{\hat{\lambda}(p_2)}{(1 - p_1)\phi(p_2)} \\
&= (1 - \hat{\tilde{q}})v'\left(\hat{F}_2(p_2)\right) > (1 - \tilde{q})v'\left(\hat{F}_2(p_2)\right),
\end{aligned}
$$

where the first inequality follows from $\hat{F}_1 \geq F_1$ and $\lambda(p_2) = 0 \geq \hat{\lambda}(p_2)$; and the second inequality follows from the postulated $\hat{\tilde{q}} < \tilde{q}$. Therefore, $\hat{F}_2(p_2) > F_2(p_2)$ if $p_2 \in \mathcal{NB}$ under Δ.

Similarly, we can show that $\hat{Q}_2(p_2) \leq Q_2(p_2)$. To see this, notice that $\hat{Q}_2(p_2) = 0 \leq Q_2(p_2)$ if $\hat{\gamma}(p_2) > 0$. If $\hat{\gamma}(p_2) = 0$, from the first-order conditions (A1b) and (A1c), we can obtain that

$$
\begin{aligned}
(1 - \tilde{q})u'\left(y_2 - Q_2(p_2)\right) &= (1 - q)v'(F_1) + \frac{\lambda(p_2) + \gamma(p_2)}{(1 - p_1)\phi(p_2)} \\
&\geq (1 - q)v'(\hat{F}_1) + \frac{\hat{\lambda}(p_2) + \hat{\gamma}(p_2)}{(1 - p_1)\phi(p_2)} \\
&= (1 - \hat{\tilde{q}})u'\left(y_2 - \hat{Q}_2(p_2)\right) > (1 - \tilde{q})u'\left(y_2 - \hat{Q}_2(p_2)\right),
\end{aligned}
$$

where the first inequality follows from $\hat{F}_1 \geq F_1$, the postulated $\hat{\lambda}(p_2) \leq 0 = \lambda(p_2)$, and $\hat{\gamma}(p_2) = 0 \leq \gamma(p_2)$; and the second inequality follows directly from $\hat{\tilde{q}} < \tilde{q}$. Therefore, $\hat{Q}_2(p_2) < Q_2(p_2)$ if $p_2 \in \mathcal{NB}$ under Δ; together with $\hat{F}_2(p_2) > F_2(p_2)$, we have that $\hat{Q}_2(p_2) - p_2\hat{F}_2(p_2) < Q_2(p_2) - p_2 F_2(p_2)$ for all $p_2 \in \mathcal{NB}$ under Δ.

An insurance firm's expected profit under Δ is

$$
\begin{aligned}
&(Q_1 - p_1 F_1) + (1 - p_1)(1 - q)\int_0^1 \left[Q_2(p_2) - p_2 F_2(p_2)\right] d\Phi(p_2) \\
&> (\hat{Q}_1 - p_1\hat{F}_1) + (1 - p_1)(1 - q)\int_0^1 \left[\hat{Q}_2(p_2) - p_2\hat{F}_2(p_2)\right] d\Phi(p_2) = 0,
\end{aligned}
$$

where the strict inequality follows from the postulated $p_2^* < 1$, a contradiction to the zero-profit condition (4). Therefore, we must have $\hat{Q}_1 > Q_1$ and thus $\hat{F}_1 < F_1$.

Next, we prove that $\hat{p}_2^* < p_2^*$. Suppose to the contrary that $\hat{p}_2^* \geq p_2^*$. It follows immediately that $\hat{Q}_2(\hat{p}_2^*) \geq Q_2(p_2^*)$ and $\hat{F}_2(\hat{p}_2^*) \leq F_2(p_2^*)$ from Equation (10). Moreover, we have shown that $\hat{Q}_1 > Q_1$ and $\hat{F}_1 < F_1$. Therefore, an insurance firm's expected profit in equilibrium under Δ can

be bounded from above by

$$(Q_1 - p_1 F_1) + (1 - p_1)(1 - q) \left\{ \int_0^{p_2^*} \left[Q_2(p_2) - p_2 F_2(p_2) \right] d\Phi(p_2) + \int_{p_2^*}^1 \left[Q_2(p_2) - p_2 F_2(p_2) \right] d\Phi(p_2) \right\}$$

$$= (Q_1 - p_1 F_1) + (1 - p_1)(1 - q) \int_{p_2^*}^1 \left[Q_2(p_2^*) - p_2 F_2(p_2^*) \right] d\Phi(p_2)$$

$$= (Q_1 - p_1 F_1) + (1 - p_1)(1 - q) \left\{ \int_{p_2^*}^{\hat{p}_2^*} \left[Q_2(p_2^*) - p_2 F_2(p_2^*) \right] d\Phi(p_2) + \int_{\hat{p}_2^*}^1 \left[Q_2(p_2^*) - p_2 F_2(p_2^*) \right] d\Phi(p_2) \right\}$$

$$\leq (Q_1 - p_1 F_1) + (1 - p_1)(1 - q) \left\{ 0 + \int_{\hat{p}_2^*}^1 \left[\hat{Q}_2(p_2^*) - p_2 \hat{F}_2(p_2^*) \right] d\Phi(p_2) \right\}$$

$$< (\hat{Q}_1 - p_1 \hat{F}_1) + (1 - p_1)(1 - q) \left\{ \int_0^{\hat{p}_2^*} \left[\hat{Q}_2(p_2) - p_2 \hat{F}_2(p_2) \right] d\Phi(p_2) + \int_{\hat{p}_2^*}^1 \left[\hat{Q}_2(p_2) - p_2 \hat{F}_2(p_2) \right] d\Phi(p_2) \right\}$$

$$= 0,$$

where the first inequality follows from $Q_2(p_2^*) - p_2 F_2(p_2^*) \leq Q_2(p_2^*) - p_2^* F_2(\hat{p}_2^*) = 0$ for $p_2 \geq p_2^*$, and $Q_2(p_2^*) - p_2 F_2(p_2^*) \leq \hat{Q}_2(p_2^*) - p_2 \hat{F}_2(p_2^*)$; and the second inequality follows from $\hat{Q}_1 > Q_1$ and $\hat{F}_1 < F_1$. This contradicts to the zero-profit condition (4) under Δ, and completes the proof. ∎

Proof of Proposition 3

Proof. Fixing $q \in (0, 1)$ and $\Delta \in (0, 1)$, we consider the following three cases:

Case I: $p_2^* = 1$. From Proposition 1, the equilibrium contract coincides with the spot contracts. Similar to the proof in Proposition 2, we can show that decreasing Δ does not change the shape of the equilibrium contracts, and hence consumer welfare remains unchanged.

Case II: $p_2^* = 0$. In this case, $Q_2(p_2; q, \Delta) = 0$ for all $p_2 \in (0, 1]$, and $F_2(p_2; q, \Delta)$ is constant over p_2. Define $F_2(q, \Delta)$ as $F_2(q, \Delta) := F_2(p_2; q, \Delta)$. Then $\langle Q_1(q, \Delta), F_1(q, \Delta), F_2(q, \Delta) \rangle$ is the solution to the following system of equations:

$$(1 - \tilde{q}) v' \left(F_2(q, \Delta) \right) = (1 - q) v' \left(F_1(q, \Delta) \right), \tag{A12}$$

$$v' \left(F_1(q, \Delta) \right) = u' \left(y_1 - Q_1(q, \Delta) \right), \tag{A13}$$

$$Q_1(q, \Delta) - p_1 F_1(q, \Delta) = (1 - p_1)(1 - q) \bar{p}_2 F_2(q, \Delta), \tag{A14}$$

where $\bar{p}_2$ is the average period-2 mortality risk as defined in (A6). Taking the partial derivative of (A14) with respect to Δ yields

$$\frac{\partial Q_1(q, \Delta)}{\partial \Delta} - p_1 \frac{\partial F_1(q, \Delta)}{\partial \Delta} = (1 - p_1)(1 - q) \bar{p}_2 \frac{\partial F_2(q, \Delta)}{\partial \Delta}. \tag{A15}$$

A9

Therefore, the partial derivative of $W(q, \Delta)$ with respect to Δ can be simplified as

$$
\begin{aligned}
\frac{\partial W(q, \Delta)}{\partial \Delta} &= -u'\left(y_1 - Q_1(q, \Delta)\right) \frac{\partial Q_1(q, \Delta)}{\partial \Delta} + p_1 v'\left(F_1(q, \Delta)\right) \frac{\partial F_1(q, \Delta)}{\partial \Delta} \\
&\quad + (1 - p_1)(1 - q)\bar{p}_2 v'\left(F_2(q, \Delta)\right) \frac{\partial F_2(q, \Delta)}{\partial \Delta} \\
&= v'\left(F_1(q, \Delta)\right) \left(-\frac{\partial Q_1(q, \Delta)}{\partial \Delta} + p_1 \frac{\partial F_1(q, \Delta)}{\partial \Delta}\right) \\
&\quad + (1 - p_1)(1 - q)\bar{p}_2 v'\left(F_2(q, \Delta)\right) \frac{\partial F_2(q, \Delta)}{\partial \Delta} \\
&= -\left[v'\left(F_1(q, \Delta)\right) - v'\left(F_2(q, \Delta)\right)\right] \times \left(\frac{\partial Q_1(q, \Delta)}{\partial \Delta} - p_1 \frac{\partial F_1(q, \Delta)}{\partial \Delta}\right),
\end{aligned}
$$

where the second equality follows from (A13); and the third equality follows from (A15). It follows from equation (A12) that

$$
v'\left(F_1(q, \Delta)\right) - v'\left(F_2(q, \Delta)\right) = \frac{q - \tilde{q}}{1 - q} v'\left(F_2(q, \Delta)\right) \geq 0.
$$

Moreover, Proposition 2 implies that

$$
\frac{\partial F_1(q, \Delta)}{\partial \Delta} < 0, \text{ and } \frac{\partial Q_1(q, \Delta)}{\partial \Delta} > 0.
$$

Therefore, we must have that $\frac{\partial W(q, \Delta)}{\partial \Delta} \leq 0$.

Case III: $0 < p_2^* < 1$. It follows from Proposition 2 that p_2^* is strictly decreasing in Δ. Thus, there exists a one-to-one mapping between Δ and p_2^*. In addition, the set of equilibrium contracts is pinned down once p_2^* is determined. Therefore, to show that $W(q, \Delta)$ is decreasing in Δ is equivalent to showing that $W^p(p_2^*)$ is increasing in p_2^*, where $W^p(p_2^*)$ is defined as

$$
\begin{aligned}
W^p(p_2^*) := &\left[u\left(y_1 - Q_1(p_2^*)\right) + p_1 v\left(F_1(p_2^*)\right)\right] \\
&+ (1 - p_1)(1 - q) \int_0^{p_2^*} \left[u\left(y_2 - Q_2^{FI}(p_2)\right) + p_2 v\left(F_2^{FI}(p_2)\right)\right] d\Phi(p_2) \\
&+ (1 - p_1)(1 - q) \int_{p_2^*}^1 \left[u\left(y_2 - Q_2^{FI}(p_2^*)\right) + p_2 v\left(F_2^{FI}(p_2^*)\right)\right] d\Phi(p_2).
\end{aligned}
$$

In the above expression, $\langle Q_1(p_2^*), F_1(p_2^*)\rangle$ is the solution to the following pair of equations:

$$
\begin{aligned}
u'\left(y_1 - Q_1(p_2^*)\right) &= v'\left(F_1(p_2^*)\right), &\text{(A16)} \\
Q_1(p_2^*) - p_1 F_1(p_2^*) &= (1 - p_1)(1 - q) \int_{p_2^*}^1 \left[p_2 F_2^{FI}(p_2^*) - Q_2^{FI}(p_2^*)\right] d\Phi(p_2). &\text{(A17)}
\end{aligned}
$$

Taking the derivative of (A17) with respect to p_2^* yields

$$(1-p_1)(1-q)\int_{p_2^*}^{1}\left(p_2\frac{dF_2^{FI}(p_2^*)}{dp_2^*}-\frac{dQ_2^{FI}(p_2^*)}{dp_2^*}\right)d\Phi(p_2)=-\left(p_1\frac{dF_1(p_2^*)}{dp_2^*}-\frac{dQ_1(p_2^*)}{dp_2^*}\right). \quad\text{(A18)}$$

With slight abuse of notation, we drop p_2^* in $Q_1(\cdot)$, $F_1(\cdot)$, $Q_2^{FI}(\cdot)$, and $F_2^{FI}(\cdot)$ in what follows. Taking the derivative of $W^p(p_2^*)$ with respect to p_2^* yields

$$\begin{aligned}\frac{dW^p(p_2^*)}{dp_2^*}=&v'(F_1)\left(p_1\frac{dF_1}{dp_2^*}-\frac{dQ_1}{dp_2^*}\right)+(1-p_1)(1-q)\left[u\left(y_2-Q_2^{FI}\right)+p_2^*v\left(F_2^{FI}\right)\right]\phi(p_2)\\ &+(1-p_1)(1-q)\int_{p_2^*}^{1}v'(F_2)\left(p_2\frac{dF_2^{FI}}{dp_2^*}-\frac{dQ_2^{FI}}{dp_2^*}\right)d\Phi(p_2)\\ &-(1-p_1)(1-q)\left[u\left(y_2-Q_2^{FI}\right)+p_2^*v\left(F_2^{FI}\right)\right]\phi(p_2)\\ =&v'(F_1)\left(p_1\frac{dF_1}{dp_2^*}-\frac{dQ_1}{dp_2^*}\right)+(1-p_1)(1-q)\int_{p_2^*}^{1}v'(F_2)\left(p_2\frac{dF_2^{FI}}{dp_2^*}-\frac{dQ_2^{FI}}{dp_2^*}\right)d\Phi(p_2)\\ =&\left[v'(F_1)-v'(F_2)\right]\times\left(p_1\frac{dF_1}{dp_2^*}-\frac{dQ_1}{dp_2^*}\right),\end{aligned}$$

where the third equality follows from (A18). By the same argument as in Case II, it can be verified that $v'(F_1)-v'(F_2)>0$. Moreover, it follows from Proposition 2 that both p_2^* and F_1 are strictly decreasing in Δ, and Q_1 is strictly increasing in Δ. Therefore, $\frac{dF_1}{dp_2^*}>0$ and $\frac{dQ_1}{dp_2^*}<0$, implying that $\frac{dW^p(p_2^*)}{dp_2^*}>0$. This completes the proof. ∎

Proof of Lemma 5

Proof. Similar to the case where there is no life settlement market, the Kuhn-Tucker conditions are necessary and sufficient for the global maximum due to the fact that the objective function (13) is concave and the constraints (14), (15), and (16) are all linear. Let $\mu_s>0$, $\lambda_s(p_2)\le 0$, and $\gamma_s(p_2)\ge 0$ denote the Lagrange multipliers for constraints (14), (15), and (16) respectively, the first-order conditions for Problem (13) with respect to Q_{1s}, F_{1s}, $Q_{2s}(p_2)$, and $F_{2s}(p_2)$ are:

$$u'(y_1-Q_{1s})=\mu_s, \quad\text{(A19a)}$$

$$v'(F_{1s})=\mu_s, \quad\text{(A19b)}$$

$$(1-\tilde{q})u'\left(y_2-Q_{2s}(p_2)\right)+\beta\tilde{q}u'\left(y_2+\beta V_{2s}(p_2)\right)=\mu_s+\frac{\lambda_s(p_2)+\gamma_s(p_2)}{(1-p_1)\phi(p_2)}, \quad\text{(A19c)}$$

$$(1-\tilde{q})v'\left(F_{2s}(p_2)\right)+\beta\tilde{q}u'\left(y_2+\beta V_{2s}(p_2)\right)=\mu_s+\frac{\lambda_s(p_2)}{(1-p_1)\phi(p_2)}. \quad\text{(A19d)}$$

Note that the second term $\beta\tilde{q}u'\left(y_2+\beta V_{2s}(p_2)\right)$ on the left-hand side of (A19c) and (A19d) results from the cash payment that policyholders receive from the settlement firm.

A11

Suppose to the contrary that there exists a tuple (q, Δ) such that $Q_{2s}(p_2) = 0$ for some $p_2 \in (0, 1]$. This implies that $Q_{2s}(p_2) - p_2 F_{2s}(p_2) < 0$ and thus $\lambda_s(p_2) = 0$. From the first-order conditions (A19a) and (A19c), we have that

$$
\begin{aligned}
(1 - \tilde{q}) u'(y_2) + \beta \tilde{q} u' \left(y_2 + \beta V_{2s}(p_2)\right) &= (1 - \tilde{q}) u' \left(y_2 - Q_{2s}(p_2)\right) + \beta \tilde{q} u' \left(y_2 + \beta V_{2s}(p_2)\right) \\
&= u' \left(y_1 - Q_{1s}\right) + \frac{\lambda_s(p_2) + \gamma_s(p_2)}{(1 - p_1)\phi(p_2)} \\
&\geq u' \left(y_1 - Q_{1s}\right),
\end{aligned}
$$

where the inequality follows from $\lambda_s(p_2) = 0$ and $\gamma_s(p_2) \geq 0$. Next, we show that the above inequality cannot hold in equilibrium. By the same argument used in (A4), we can show that $y_2 > y_1 - Q_{1s}$. This in turn implies that

$$
(1 - \tilde{q}) u'(y_2) + \beta \tilde{q} u' \left(y_2 + \beta V_{2s}(p_2)\right) \leq [1 - (1 - \beta)\tilde{q}] u'(y_2) < u'(y_1 - Q_{1s}).
$$

This completes the proof. ∎

Proof of Proposition 4

Proof. Lemma 5 implies that $\gamma_s(p_2) = 0$ for all $p_2 \in (0, 1]$. Thus the first-order conditions (A19) imply that in equilibrium $\langle (Q_{1s}, F_{1s}), (Q_{2s}(p_2), F_{2s}(p_2)) : p_2 \in [0, 1] \rangle$ must satisfy the following full-event insurance conditions:

$$
u'(y_1 - Q_{1s}) = v'(F_{1s}), \tag{A20}
$$
$$
u' \left(y_2 - Q_{2s}(p_2)\right) = v' \left(F_{2s}(p_2)\right) \text{ for all } p_2 \in (0, 1]; \tag{A21}
$$

that is, policyholders obtain full-event insurance in both period 1 and all health states in period 2 in the presence of the life settlement market.

As we did in the analysis used in the absence of the settlement market, we can again partition the period-2 health states into two subsets $\mathcal{B}_s$ and $\mathcal{NB}_s$, depending on whether the no-lapsation constraint (15) binds. The following result, which is similar to Lemma 1, can then be obtained:

Lemma A5 *If $p_2 \in \mathcal{B}_s$ and $p_2' \in \mathcal{NB}_s$, then $p_2 < p_2'$ and $Q_{2s}(p_2) < Q_{2s}(p_2')$.*

Lemma A5 implies that there is a threshold p_{2s}^* such that $p_2 \in \mathcal{B}_s$ if $p_2 < p_{2s}^*$ and $p_2 \in \mathcal{NB}_s$ if $p_2 > p_{2s}^*$. If $p_{2s}^* = 1$, then it is obvious that the equilibrium period-2 contracts degenerate to the set of spot contracts. The following lemma characterizes the set of period-2 premiums $Q_{2s}(p_2)$ provided that $p_{2s}^* \in (0, 1)$.

Lemma A6 *If $p_{2s}^* \in (0, 1)$, then the equilibrium period-2 premiums $Q_{2s}(p_2)$ satisfy:*

   1. for $p_2 \leq p_{2s}^*$, $Q_{2s}(p_2) = Q_2^{FI}(p_2)$;

2. *for $p_2 > p_{2s}^*$, $Q_{2s}(p_2)$ solves:*

$$(1-\tilde{q})u'\left(y_2 - Q_{2s}(p_2)\right) + \beta\tilde{q}u'\left(y_2 + \beta V_{2s}(p_2)\right) = (1-\tilde{q})u'\left(y_2 - Q_2^{FI}(p_{2s}^*)\right) + \beta\tilde{q}u'(y_2). \quad \text{(A22)}$$

The proof is trivial and is omitted for brevity. Note that $V_{2s}(p_{2s}^*) \equiv p_{2s}^* F_{2s}(p_{2s}^*) - Q_{2s}(p_{2s}^*) = p_{2s}^* F_2^{FI}(p_{2s}^*) - Q_2^{FI}(p_{2s}^*) = 0$ from part 1 of Lemma A6. Equation (A22) states that in a competitive equilibrium, premium and face value are chosen to equalize the marginal utility of consumption across all period-2 health states above p_{2s}^*. By Lemma A6, the set of period-2 contracts is fully characterized by p_{2s}^* alone. Moreover, it can be shown from (A22) that both $Q_{2s}(p_2)$ and $V_{2s}(p_2)$ are strictly increasing in p_2 if $\beta > 0$. From the first-order conditions (A19a), (A19c), and Lemma A6, the period-1 premium Q_{1s} is the solution to:

$$u'(y_1 - Q_{1s}) = (1 - \tilde{q})u'\left(y_2 - Q_2^{FI}(p_{2s}^*)\right) + \beta\tilde{q}u'(y_2). \quad \text{(A23)}$$

To characterize the set of equilibrium insurance contracts, it remains to pin down p_{2s}^*, which is determined by the zero-profit condition (14). This completes the proof. ∎

Proof of Proposition 5

Proof. We use the hat symbol to refer to the variables for which the degree of consumer overconfidence is $\hat{\Delta}$. Suppose to the contrary that $\hat{\Delta} > \Delta$ (i.e., $\hat{\tilde{q}} < \tilde{q}$) and $\hat{F}_{1s} \geq F_{1s}$. It follows directly that $\hat{Q}_{1s} \leq Q_{1s}$ from Equation (A20). Fixing a health state $p_2 \in (0, 1]$, we first compare firm's expected profits under Δ and $\hat{\Delta}$ depending on whether $p_2 \in \mathcal{B}_s$ under $\hat{\Delta}$.

Case I: $p_2 \in \mathcal{B}_s$ under Δ. It is clear that $Q_{2s}(p_2) - p_2 F_{2s}(p_2) = 0 \geq \hat{Q}_{2s}(p_2) - p_2\hat{F}_{2s}(p_2)$.

Case II: $p_2 \in \mathcal{NB}_s$ under Δ. It follows directly that $\lambda_s(p_2) = 0$. From (A19b) and (A19d), we have that

$$(1 - \tilde{q})v'\left(F_{2s}(p_2)\right) + \beta\tilde{q}u'\left(y_2 + \beta V_{2s}(p_2)\right) = v'(F_{1s}) + \frac{\lambda_s(p_2)}{(1 - p_1)\phi(p_2)}$$

$$\geq v'(\hat{F}_{1s}) + \frac{\hat{\lambda}_s(p_2)}{(1 - p_1)\phi(p_2)}$$

$$= (1 - \hat{\tilde{q}})v'\left(\hat{F}_{2s}(p_2)\right) + \beta\hat{\tilde{q}}u'\left(y_2 + \beta\hat{V}_{2s}(p_2)\right),$$

$$\text{(A24)}$$

where the inequality follows from the postulated $\hat{F}_{1s} \geq F_{1s}$ and $\hat{\lambda}_s(p_2) \leq 0 = \lambda_s(p_2)$. Note that (A24) implies that $\hat{F}_{2s}(p_2) > F_{2s}(p_2)$. To see this, suppose to the contrary that $\hat{F}_{2s}(p_2) \leq F_{2s}(p_2)$. Then it follows that $\hat{Q}_{2s}(p_2) \geq Q_{2s}(p_2)$ from (A21), and hence $\hat{V}_{2s}(p_2) \leq V_{2s}(p_2)$. Therefore, we have that

$$(1 - \tilde{q})v'\left(F_{2s}(p_2)\right) + \beta\tilde{q}u'\left(y_2 + \beta V_{2s}(p_2)\right) < (1 - \hat{\tilde{q}})v'\left(F_{2s}(p_2)\right) + \beta\hat{\tilde{q}}u'\left(y_2 + \beta V_{2s}(p_2)\right)$$

$$\leq (1 - \hat{\tilde{q}})v'\left(\hat{F}_{2s}(p_2)\right) + \beta\hat{\tilde{q}}u'\left(y_2 + \beta\hat{V}_{2s}(p_2)\right),$$

A13

where the first inequality follows from $\beta \leq 1$, the postulated $\hat{\tilde{q}} < \tilde{q}$, and $u'(y_2 + \beta V_{2s}(p_2)) < u'(y_2 - Q_{2s}(p_2)) = v'(F_{2s}(p_2))$; and the second inequality follows from the postulated $\hat{F}_{2s}(p_2) \leq F_{2s}(p_2)$ and $\hat{V}_{2s}(p_2) \leq V_{2s}(p_2)$, which contradicts (A24). Therefore, when $p_2 \in \mathcal{NB}_s$ under Δ, it must be the case that $\hat{F}_{2s}(p_2) > F_{2s}(p_2)$ and $\hat{Q}_{2s}(p_2) < Q_{2s}(p_2)$, which in turn implies that $Q_{2s}(p_2) - p_2 F_{2s}(p_2) > \hat{Q}_{2s}(p_2) - p_2 \hat{F}_{2s}(p_2)$.

Next we consider an insurance firm's expected profits under Δ, which can be bounded from above by

$$
(Q_{1s} - p_1 F_{1s}) + (1 - p_1) \int_0^1 \left[Q_{2s}(p_2) - p_2 F_{2s}(p_2) \right] d\Phi(p_2)
$$
$$
> \left(\hat{Q}_{1s} - p_1 \hat{F}_{1s} \right) + (1 - p_1) \int_0^1 \left[\hat{Q}_{2s}(p_2) - p_2 \hat{F}_{2s}(p_2) \right] d\Phi(p_2) = 0,
$$

where the strict inequality follows from the postulated $\hat{Q}_{1s} \leq Q_{1s}$ and the observation that the set $\mathcal{NB}_s$ under Δ is non-empty ($p_{2s}^* < 1$), a contradiction to the zero-profit condition (14) under $\hat{\Delta}$. This completes the proof. ■

Proof of Proposition 6

Proof. Define
$$
\mathcal{H}(p_2) := Q_2^{FI}(p_2) - p_1 F_2^{FI}(p_2) + \delta.
$$

It is evident that $\mathcal{H}(p_2)$ is strictly increasing in p_2. Moreover, we have that

$$
\lim_{p_2 \searrow 0} \mathcal{H}(p_2) = 0 - p_1 v'^{-1} \left(u'(y_2) \right) + \delta = p_1 \left[-v'^{-1} \left(u'(y_2) \right) + \frac{\delta}{p_1} \right] < p_1 \left[-v'^{-1} \left(u'(y_1 - \delta) \right) + \frac{\delta}{p_1} \right] = 0,
$$

where the strict inequality follows from the monotonicity of $u(\cdot)$ and $v'(\cdot)$, and the last inequality follows from Assumption 1. Next, note that

$$
\mathcal{H}(p_1) = Q_2^{FI}(p_1) - p_1 F_2^{FI}(p_1) + \delta = \delta > 0.
$$

Therefore, there exists a unique solution to $\mathcal{H}(p_2) = 0$ for $p_2 \in (0, p_1)$, which we denote by $\underline{p}_{2s}^{\star}$.

Before proving the proposition, it is useful to state two intermediate results.

Lemma A7 *For all $(q, \Delta) \in (0, 1) \times [0, 1]$, $p_{2s}^* \geq \underline{p}_{2s}^{\star}$.*

Proof. Suppose to the contrary that there exists a tuple (q, Δ) such that $p_{2s}^* < \underline{p}_{2s}^{\star}$, then it follows from Lemma A5 that the no-lapsation condition (15) at $p_2 = \underline{p}_{2s}^{\star}$ does not bind, i.e., $Q_{2s}(\underline{p}_{2s}^{\star}) - \underline{p}_{2s}^{\star} F_{2s}(\underline{p}_{2s}^{\star}) < 0$ and $\lambda_s(\underline{p}_{2s}^{\star}) = 0$. Moreover, we have that

$$
v' \left(F_{2s}(\underline{p}_{2s}^{\star}) \right) = u' \left(y_2 - Q_{2s}(\underline{p}_{2s}^{\star}) \right) > (1 - \tilde{q}) u' \left(y_2 - Q_{2s}(\underline{p}_{2s}^{\star}) \right) + \beta \tilde{q} u' \left(y_2 + \beta V_{2s}(\underline{p}_{2s}^{\star}) \right)
$$
$$
\geq u' \left(y_1 - Q_{1s} \right) = v' \left(F_{1s} \right),
$$

A14

where the first and second equality follow from (A21) and (A20) respectively; the second inequality follows from (A19a), (A19c), $\lambda_s(\underline{p}_{2s}^\star) = 0$, and $\gamma_s(\underline{p}_{2s}^\star) \geq 0$; and the first inequality follows from $y_2 - Q_{2s}(\underline{p}_{2s}^\star) < y_2 + \beta V_{2s}(\underline{p}_{2s}^\star)$ and $\beta \in [0,1]$. Therefore, we must have $y_2 - Q_{2s}(\underline{p}_{2s}^\star) < y_1 - Q_{1s}$ from the strict concavity of $u(\cdot)$, or equivalently, $Q_{1s} < Q_{2s}(\underline{p}_{2s}^\star) + y_1 - y_2$; and $F_{1s} > F_{2s}(\underline{p}_{2s}^\star)$ from the strict concavity of $v(\cdot)$.

An insurance firm's period-1 expected profits can then be bounded from above by

$$Q_{1s} - p_1 F_{1s} < \left(Q_{2s}(\underline{p}_{2s}^\star) + y_1 - y_2 \right) - p_1 F_{2s}(\underline{p}_{2s}^\star) < Q_{2s}(\underline{p}_{2s}^\star) - p_1 F_{2s}(\underline{p}_{2s}^\star) + \delta = 0,$$

where the second inequality follows from Assumption 1, and the equality follows from the definition of $\underline{p}_{2s}^\star$; a contradiction to (14). This completes the proof. ∎

Lemma A8 *Let p_{2s}^* and $\breve{p}_{2s}^*$ be the equilibrium threshold above which the second-period premiums are actuarially favorable under Δ and $\breve{\Delta}$, respectively. Fixing $q \in (0,1)$, if $p_{2s}^* < 1$ and $\breve{p}_{2s}^* = 1$, then $\Delta > \breve{\Delta}$.*

Proof. For notational convenience, we use the hat symbol to refer to the variables for which the degree of consumer overconfidence is $\breve{\Delta}$. Suppose to the contrary that $\Delta \leq \breve{\Delta}$, or equivalently $\tilde{q} \geq \breve{\tilde{q}}$. First, note that $\breve{p}_{2s}^* = 1$ implies that the period-2 contracts under $\breve{\Delta}$ are spot contracts for all $p_2 \in (0,1]$, and hence $\breve{Q}_{2s}(p_2) = Q_2^{FI}(p_2)$ and $\breve{Q}_{1s} = Q_1^{FI}$, where Q_1^{FI} is defined in the proof of Lemma A3. Next, consider the period-2 health state $p_2 = p_{2s}^*$. From the first-order conditions (A19a) and (A19c), we have that

$$(1 - \breve{\tilde{q}})u'\left(y_2 - Q_2^{FI}(p_{2s}^*) \right) + \beta \breve{\tilde{q}} u'(y_2) \leq u'\left(y_1 - \breve{Q}_{1s} \right). \tag{A25}$$

Lemma A6, together with (A19a) and (A19c), implies that

$$(1 - \tilde{q})u'\left(y_2 - Q_{2s}(p_{2s}^*) \right) + \beta \tilde{q} u'\left(y_2 + \beta V_{2s}(p_{2s}^*) \right) = u'(y_1 - Q_{1s}). \tag{A26}$$

Because $p_{2s}^* < 1$, we must have $Q_{1s} > Q_1^{FI} = \breve{Q}_{1s}$ from (14), implying that

$$u'(y_1 - Q_{1s}) > u'(y_1 - \breve{Q}_{1s}).$$

The above inequality, together with (A25) and (A26), implies that

$$(1 - \breve{\tilde{q}})u'\left(y_2 - Q_2^{FI}(p_{2s}^*) \right) + \beta \breve{\tilde{q}} u'(y_2) < (1 - \tilde{q})u'\left(y_2 - Q_{2s}(p_{2s}^*) \right) + \beta \tilde{q} u'\left(y_2 + \beta V_{2s}(p_{2s}^*) \right),$$

which is a contradiction because the left-hand side of the above inequality must be no less than the right-hand side. To see this, note that

$$(1 - \breve{\tilde{q}})u'\left(y_2 - Q_2^{FI}(p_{2s}^*) \right) + \beta \breve{\tilde{q}} u'(y_2) \geq (1 - \tilde{q})u'\left(y_2 - Q_{2s}^{FI}(p_{2s}^*) \right) + \beta \tilde{q} u'(y_2)$$

$$= (1 - \tilde{q})u'\left(y_2 - Q_{2s}(p_{2s}^*) \right) + \beta \tilde{q} u'\left(y_2 + \beta V_{2s}(p_{2s}^*) \right),$$

A15

where the inequality follows from $u'\left(y_2 - Q_{2s}^{FI}(p_{2s}^*)\right) \geq u'(y_2)$, $\beta \in [0,1]$, and the postulated $\tilde{q} \geq \check{q}$; and the equality follows from Lemma A6. This completes the proof. ∎

Now we can prove Proposition 6. Lemma A7 implies that $p_{2s}^*(q, \Delta) \geq \underline{p}_{2s}^\star > 0$, and hence rules out the possibility that $p_{2s}^*(q, \Delta) = 0$. Lemma A8 indicates the existence of a threshold overconfidence level $\bar{\Delta} \in [0,1]$ such that $p_{2s}^*(q, \Delta) < 1$ for $\Delta > \bar{\Delta}$ and $p_{2s}^*(q, \Delta) = 1$ for $\Delta < \bar{\Delta}$. Therefore, it suffices to consider the following two cases.

Case I: $\bar{\Delta} = 1$. That is, $p_{2s}^* = 1$ for all $\Delta \in [0,1]$, implying that the period-2 equilibrium contracts are spot contracts for all $p_2 \in (0,1]$. As a result, $W_s(q, \Delta)$ is constant over Δ.

Case II: $\bar{\Delta} < 1$. If $\Delta < \bar{\Delta}$, then the argument in Case I applies. If $\Delta > \bar{\Delta}$, by the implicit function theorem, $p_{2s}^*(q, \Delta)$ is continuous and differentiable in both arguments. Because $\langle Q_{2s}(p_2; q, \Delta), F_{2s}(p_2; q, \Delta) \rangle = \langle Q_2^{FI}(p_2), F_2^{FI}(p_2) \rangle$ for $p_2 < p_{2s}^*(q, \Delta)$ from Proposition 4, the zero-profit condition (14) can be rewritten as

$$\left[Q_{1s}(q, \Delta) - p_1 F_{1s}(q, \Delta)\right] + (1 - p_1) \int_{p_{2s}^*(q,\Delta)}^1 [Q_{2s}(p_2; q, \Delta) - p_2 F_{2s}(p_2; q, \Delta)] d\Phi(p_2) = 0.$$

With slight abuse of notation, we drop q and Δ in $Q_{1s}(\cdot)$, $F_{1s}(\cdot)$, $Q_{2s}(\cdot)$, $F_{2s}(\cdot)$, $p_{2s}^*(\cdot)$, and $V_{2s}(\cdot)$ in what follows. Taking the partial derivative of the above equality with respect to Δ yields that

$$\left(\frac{\partial Q_{1s}}{\partial \Delta} - p_1 \frac{\partial F_{1s}}{\partial \Delta}\right) + (1 - p_1) \int_{p_{2s}^*}^1 \left(\frac{\partial Q_{2s}(p_2)}{\partial \Delta} - p_2 \frac{\partial F_{2s}(p_2)}{\partial \Delta}\right) d\Phi(p_2) = 0. \qquad \text{(A27)}$$

Similarly, taking the partial derivative of $W_s(q, \Delta)$ with respect to Δ yields that

$$\frac{\partial W_s(q, \Delta)}{\partial \Delta} = v'(F_{1s}) \left(p_1 \frac{\partial F_{1s}}{\partial \Delta} - \frac{\partial Q_{1s}}{\partial \Delta}\right)$$

$$+ (1 - p_1) \int_{p_{2s}^*}^1 \left[\begin{array}{c}(1-q)v'\left(F_{2s}(p_2)\right) \\ +\beta q u'\left(y_2 + \beta V_{2s}(p_2)\right)\end{array}\right] \left(p_2 \frac{\partial F_{2s}(p_2)}{\partial \Delta} - \frac{\partial Q_{2s}(p_2)}{\partial \Delta}\right) d\Phi(p_2)$$

$$= (1 - p_1) \int_{p_{2s}^*}^1 \left(\frac{\partial Q_{2s}(p_2)}{\partial \Delta} - p_2 \frac{\partial F_{2s}(p_2)}{\partial \Delta}\right) \left[\begin{array}{c}v'(F_{1s}) - (1-q)v'\left(F_{2s}(p_2)\right) \\ -\beta q u'\left(y_2 + \beta V_{2s}(p_2)\right)\end{array}\right] d\Phi(p_2)$$

$$= (1 - p_1)(q - \tilde{q}) \int_{p_{2s}^*}^1 \left(\frac{\partial Q_{2s}(p_2)}{\partial \Delta} - p_2 \frac{\partial F_{2s}(p_2)}{\partial \Delta}\right) \left[\begin{array}{c}u'\left(y_2 - Q_{2s}(p_2)\right) \\ -\beta u'\left(y_2 + \beta V_{2s}(p_2)\right)\end{array}\right] d\Phi(p_2),$$

where the second equality follows from (A27), and the third equality follows from the fact that $(1 - \tilde{q})v'\left(F_{2s}(p_2)\right) + \beta \tilde{q} u'\left(y_2 + \beta V_{2s}(p_2)\right) = \mu_s = v'(F_{1s})$ for $p_2 \geq p_{2s}^*$. To proceed, let us define $x(p_2)$ and $z(p_2)$ as follows:

$$x(p_2) := \frac{\partial Q_{2s}(p_2)}{\partial \Delta} - p_2 \frac{\partial F_{2s}(p_2)}{\partial \Delta},$$

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and
$$z(p_2) := u'\left(y_2 - Q_{2s}(p_2)\right) - \beta u'\left(y_2 + \beta V_{2s}(p_2)\right).$$

Because $Q_{2s}(p_2) > 0$ and $V_{2s}(p_2) \geq 0$, we must have $z(p_2) > (1-\beta)u'(y_2) \geq 0$. Next, we divide the set $\mathcal{NB}_s$ into two subsets $\mathcal{NB}_s^+$ and $\mathcal{NB}_s^-$ depending on the sign of $x(p_2)$. Specifically, let

$$\mathcal{NB}_s^+ := \left\{ p_2 \,\middle|\, p_2 \in \mathcal{NB}_s, x(p_2) \geq 0 \right\},$$

and

$$\mathcal{NB}_s^- := \left\{ p_2 \,\middle|\, p_2 \in \mathcal{NB}_s, x(p_2) < 0 \right\}.$$

Note that $\lambda_s(p_2) = \gamma_s(p_2) = 0$ for $p_2 \in \mathcal{NB}_s$. Combining (A19b) and (A19c) yields that

$$(1 - \tilde{q})u'\left(y_2 - Q_{2s}(p_2)\right) + \beta \tilde{q} u'\left(y_2 + \beta V_{2s}(p_2)\right) = v'(F_{1s}). \tag{A28}$$

Recall that $\tilde{q} = q(1 - \Delta)$. Taking the partial derivative of (A28) with respect to Δ and rearranging yields that

$$q\left[u'\left(y_2 - Q_{2s}(p_2)\right) - \beta u'\left(y_2 + \beta V_{2s}(p_2)\right) \right]$$
$$= v''(F_{1s})\frac{\partial F_{1s}}{\partial \Delta} + \beta^2 \tilde{q} u''\left(y_2 + \beta V_{2s}(p_2)\right)\left(\frac{\partial Q_{2s}(p_2)}{\partial \Delta} - p_2 \frac{\partial F_{2s}(p_2)}{\partial \Delta} \right)$$
$$+ (1 - \tilde{q})u''\left(y_2 - Q_{2s}(p_2)\right)\frac{\partial Q_{2s}(p_2)}{\partial \Delta}. \tag{A29}$$

Suppose that there exist two health states, denoted by p_i and p_j, such that $p_i \in \mathcal{NB}_s^+$ and $p_j \in \mathcal{NB}_s^-$, then we must have $x(p_i) \geq 0 > x(p_j)$ by definition. Next, we show that $z(p_i) < z(p_j)$. To see this, first note that (A21) implies that the partial derivative of $Q_{2s}(p_2)$ and $F_{2s}(p_2)$ with respect to Δ must be of different signs. Therefore, it follows immediately from the postulated $x(p_i) \geq 0 > x(p_j)$ that

$$\frac{\partial Q_{2s}(p_i)}{\partial \Delta} \geq 0 \geq \frac{\partial F_{2s}(p_i)}{\partial \Delta},$$

and

$$\frac{\partial Q_{2s}(p_j)}{\partial \Delta} < 0 < \frac{\partial F_{2s}(p_j)}{\partial \Delta}.$$

The above inequalities, together with (A29) and the fact that $u''(\cdot) < 0$, imply that

$$
\begin{aligned}
qz(p_i) &\equiv q\left[u'\left(y_2 - Q_{2s}(p_i)\right) - \beta u'\left(y_2 + \beta V_{2s}(p_i)\right)\right] \\
&= v''(F_{1s})\frac{\partial F_{1s}}{\partial \Delta} + \beta^2 \tilde{q}\underbrace{u''\left(y_2 + \beta V_{2s}(p_i)\right)x(p_i)}_{\leq 0} + (1-\tilde{q})\underbrace{u''\left(y_2 - Q_{2s}(p_i)\right)\frac{\partial Q_{2s}(p_i)}{\partial \Delta}}_{\leq 0} \\
&< v''(F_{1s})\frac{\partial F_{1s}}{\partial \Delta} + \beta^2 \tilde{q}\underbrace{u''\left(y_2 + \beta V_{2s}(p_j)\right)x(p_j)}_{>0} + (1-\tilde{q})\underbrace{u''\left(y_2 - Q_{2s}(p_j)\right)\frac{\partial Q_{2s}(p_j)}{\partial \Delta}}_{>0} \\
&\equiv qz(p_j),
\end{aligned}
$$

and thus $z(p_i) < z(p_j)$. Recall that $z(p_2) > 0$. Define $\overline{z}$ and $\underline{z}$ as the following:

$$
\overline{z} := \sup_{p_2 \in \mathcal{NB}_s^+} z(p_2), \text{ and } \underline{z} := \inf_{p_2 \in \mathcal{NB}_s^-} z(p_2).
$$

It follows immediately that $\underline{z} \geq \overline{z} \geq 0$. Therefore, the partial derivative of $W_s(q, \Delta)$ with respect to Δ can be further simplified as

$$
\begin{aligned}
\frac{\partial W_s(q, \Delta)}{\partial \Delta} &= (1 - p_1)(q - \tilde{q})\int_{p_{2s}^*}^1 x(p_2)z(p_2)d\Phi(p_2) \\
&= (1 - p_1)(q - \tilde{q})\left(\int_{p_2 \in \mathcal{NB}_s^+} x(p_2)z(p_2)d\Phi(p_2) + \int_{p_2 \in \mathcal{NB}_s^-} x(p_2)z(p_2)d\Phi(p_2)\right) \\
&\leq (1 - p_1)(q - \tilde{q})\left(\int_{p_2 \in \mathcal{NB}_s^+} x(p_2)\overline{z}d\Phi(p_2) + \int_{p_2 \in \mathcal{NB}_s^-} x(p_2)\underline{z}d\Phi(p_2)\right) \\
&\leq (1 - p_1)(q - \tilde{q})\overline{z}\int_{p_{2s}^*}^1 x(p_2)d\Phi(p_2) \\
&= (q - \tilde{q})\overline{z}\left(p_1\frac{\partial F_{1s}}{\partial \Delta} - \frac{\partial Q_{1s}}{\partial \Delta}\right),
\end{aligned}
$$

where the last equality follows from (A27). From Proposition 5, we have that $p_1\frac{\partial F_{1s}}{\partial \Delta} - \frac{\partial Q_{1s}}{\partial \Delta} \leq 0$; together with the fact that $q \geq \tilde{q}$ and $\overline{z} \geq 0$, we must have that $\frac{\partial W_s(q, \Delta)}{\partial \Delta} \leq 0$. This completes the proof. ∎

Proof of Theorem 1

Proof. It can be shown that Proposition 4 in Fang and Kung (2020) is robust to the more general income dynamics that satisfy Assumption 1 when policyholders are rational, i.e.,

$$
W(q, 0) \geq W_s(q, 0), \tag{A30}
$$

using the notation for equilibrium welfare $W(\cdot)$ and $W_s(\cdot)$ in Definitions 1 and 2. The first part of Theorem 1 follows directly from the continuity of $W(\cdot, \cdot)$ and $W_s(\cdot, \cdot)$ and Equation (A30), and

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it remains to prove the second part.

To proceed, we state several useful intermediary results. With slight abuse of notation, we add q into $Q_1(\cdot)$, $F_1(\cdot)$, $Q_2(\cdot)$, and $F_2(\cdot)$ in the proof of Lemmas A9 to A11 below, to emphasize the fact that the set of equilibrium contracts depends on $(q, \tilde{q})$, or equivalently, (q, Δ).

Lemma A9 *Fixing* $\tilde{q} < 1$, $\lim_{q \nearrow 1} (1 - p_1)(1 - q) \int_0^1 \left[u \left(y_2 - Q_2(p_2) \right) + p_2 v \left(F_2(p_2) \right) \right] d\Phi(p_2) = 0$.

Proof. It is clear that the result holds if $\lim_{c \nearrow \infty} v(\cdot) < \infty$. Thus, it suffices to consider the case where $\lim_{c \nearrow \infty} v(c) = \infty$. First, it follows from Lemma A4 that $Q_2(p_2; q) = 0$ for all $p_2 \in [0, 1]$ if $q > q_0(\tilde{q})$, where the threshold $q_0(\tilde{q})$ is defined in the proof of Lemma A4. Therefore, we have that

$$\lim_{q \nearrow 1} Q_2(p_2; q) = 0, \text{ and thus } \lim_{q \nearrow 1} u \left(y_2 - Q_2(p_2; q) \right) = u(y_2).$$

Second, there is a strictly positive lower bound of $F_2(p_2; q)$, denoted by κ. To see this, note that the first-order conditions (A1c) and (A1d) imply that

$$v' \left(F_2(p_2; q) \right) \le u' \left(y_1 - Q_2(p_2; q) \right) \le u' \left(y_1 - Q_2^{FI}(1) \right),$$

where the last inequality follows from the fact that $Q_2(p_2; q) \le Q_2^{FI}(p_2) \le Q_2^{FI}(1)$. Therefore, we have that

$$F_2(p_2; q) \ge v'^{-1} \left(u' \left(y_1 - Q_2^{FI}(1) \right) \right) =: \kappa.$$

The zero-profit condition (4), together with the fact that $0 \le Q_1(q) - p_1 F_1(q) \le y_1$, implies that

$$\int_0^1 p_2 F_2(p_2; q) d\Phi(p_2) = \frac{Q_1(q) - p_1 F_1(q)}{(1 - p_1)(1 - q)} \le \frac{y_1}{(1 - p_1)(1 - q)}. \tag{A31}$$

Recall that $\bar{p}_2 \equiv \int_0^1 p_2 d\Phi(p_2)$ from (A6). Therefore, we have that

$$
\begin{aligned}
\bar{p}_2 v(\kappa) \equiv \int_0^1 p_2 v(\kappa) \Phi(p_2) &\le \int_0^1 p_2 v \left(F_2(p_2; q) \right) d\Phi(p_2) \\
&\le \int_0^1 \left[v \left(p_2 F_2(p_2; q) + (1 - p_2)\kappa \right) - (1 - p_2)v(\kappa) \right] d\Phi(p_2) \\
&\le v \left(\int_0^1 p_2 F_2(p_2; q) d\Phi(p_2) + \int_0^1 (1 - p_2)\kappa d\Phi(p_2) \right) - (1 - \bar{p}_2) v(\kappa) \\
&\le v \left(\frac{y_1}{(1 - p_1)(1 - q)} + (1 - \bar{p}_2)\kappa \right) - (1 - \bar{p}_2) v(\kappa),
\end{aligned}
$$

where the second and the third inequality follow from the concavity of $v(\cdot)$; and the last inequality follows from (A31). Multiplying the above inequality by $(1 - p_1)(1 - q)$ and taking limits as $q \nearrow 1$

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on all sides yields that

$$0 = \lim_{q \nearrow 1}(1-p_1)(1-q)\bar{p}_2 v(\kappa)$$

$$\leq \lim_{q \nearrow 1}(1-p_1)(1-q)\int_0^1 p_2 v\left(F_2(p_2)\right) d\Phi(p_2)$$

$$\leq \lim_{q \nearrow 1}(1-p_1)(1-q)v\left(\frac{y_1}{(1-p_1)(1-q)} + (1-\bar{p}_2)\kappa\right) - \lim_{q \nearrow 1}(1-p_1)(1-q)(1-\bar{p}_2)v(\kappa) = 0.$$

The last equality holds due to the Inada condition $\lim_{c \nearrow \infty} v'(c) = 0$. Specifically,

$$\lim_{q \nearrow 1}(1-p_1)(1-q)v\left(\frac{y_1}{(1-p_1)(1-q)} + (1-\bar{p}_2)\kappa\right) = \lim_{t \searrow 0}\frac{v\left(\frac{y_1}{t} + (1-\bar{p}_2)\kappa\right)}{\frac{1}{t}}$$

$$= y_1 \cdot \lim_{t \searrow 0} v'\left(\frac{y_1}{t} + (1-\bar{p}_2)\kappa\right) = 0,$$

where the second equality follows from L'Hospital's rule. Therefore, we have that

$$\lim_{q \nearrow 1}(1-p_1)(1-q)\int_0^1 p_2 v\left(F_2(p_2)\right) d\Phi(p_2) = 0,$$

which in turn implies that,

$$\lim_{q \nearrow 1}(1-p_1)(1-q)\int_0^1 \left[u\left(y_2 - Q_2(p_2)\right) + p_2 v\left(F_2(p_2)\right)\right] d\Phi(p_2) = 0.$$

This completes the proof. ∎

Lemma A10 *Denote the consumer equilibrium welfare in the absence of the life settlement market for the environment $(q, \tilde{q})$ by $W^\dagger(q, \tilde{q})$. Suppose that $\eta(\cdot)$ is positively bounded away from one, i.e., there exists $\alpha > 1$ such that $\eta(c) \geq \alpha$ for all $c > 0$. Then $\lim_{q \nearrow 1} W^\dagger(q, \tilde{q}) = \left[u(0) + p_1 v(0)\right] + (1 - p_1)u(y_2)$ for all $\tilde{q} \in [0, 1)$.*

Proof. It is clear that $W^\dagger(q, \tilde{q}) \equiv W(q, \frac{q-\tilde{q}}{q})$, where $W(\cdot, \cdot)$ is defined in (12). Fix $\tilde{q}$. It follows from Lemma A4 that $Q_2(p_2; q) = 0$ and $\lambda(p_2) = 0$ for all $p_2 \in (0, 1]$ when $q > q_0(\tilde{q})$. Combining (A1b) and (A1d) yields that

$$(1-\tilde{q})v'\left(F_2(p_2; q)\right) = (1-q)v'\left(F_1(q)\right); \tag{A32}$$

together with $\tilde{q} \leq q$, we have that $F_2(p_2; q) \geq F_1(q)$ for all $p_2 \in (0, 1]$ when $q > q_0(\tilde{q})$. From the postulated $\eta(c) \equiv -\frac{v'(c)}{cv''(c)} \geq \alpha$, we have that

$$\frac{dc^{\frac{1}{\alpha}}v'(c)}{dc} = c^{\frac{1}{\alpha}-1}\left[\frac{1}{\alpha}v'(c) + cv''(c)\right] \geq 0.$$

Therefore, the function $c^{\frac{1}{\alpha}}v'(c)$ is weakly increasing in c; together with $F_2(p_2; q) \geq F_1(q)$, we can

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obtain

$$\left[F_2(p_2;q)\right]^{\frac{1}{\alpha}} \times v'\left(F_2(p_2;q)\right) \geq \left[F_1(q)\right]^{\frac{1}{\alpha}} \times v'\left(F_1(q)\right). \tag{A33}$$

Equation (A32), together with (A33), implies that

$$\frac{1-q}{1-\tilde{q}} = \frac{v'\left(F_2(p_2;q)\right)}{v'\left(F_1(q)\right)} \geq \left(\frac{F_1(q)}{F_2(p_2;q)}\right)^{\frac{1}{\alpha}}.$$

Rearranging the above inequality yields that

$$F_2(p_2;q) \geq F_1(q)\left(\frac{1-\tilde{q}}{1-q}\right)^{\alpha}.$$

The above inequality, together with the zero-profit condition (4), implies that

$$p_1 F_1(q) + (1-p_1)(1-q)\bar{p}_2 F_1(q)\left(\frac{1-\tilde{q}}{1-q}\right)^{\alpha} \leq p_1 F_1(q) + (1-p_1)(1-q)\int_0^1 p_2 F_2(p_2;q)d\Phi(p_2)$$

$$= Q_1(q) \leq y_1, \text{ for } q > q_0(\tilde{q}).$$

From the Inada condition on $v(\cdot)$, we have $F_1(q) > 0$; together with the above inequality, we can obtain that

$$0 < F_1(q) \leq \frac{y_1}{p_1 + (1-p_1)(1-q)\bar{p}_2\left(\frac{1-\tilde{q}}{1-q}\right)^{\alpha}}, \text{ for } q > q_0(\tilde{q}).$$

Taking limits as $q \nearrow 1$ on all sides of the above inequality yields that

$$0 \leq \lim_{q \nearrow 1} F_1(q) \leq \lim_{q \nearrow 1} \frac{y_1}{p_1 + (1-p_1)(1-q)\bar{p}_2\left(\frac{1-\tilde{q}}{1-q}\right)^{\alpha}} = 0.$$

This indicates that $\lim_{q \nearrow 1} F_1(q) = 0$ and thus $\lim_{q \nearrow 1} Q_1(q) = y_1$ from (7). Therefore, the consumer equilibrium welfare in the limit as $q \nearrow 1$ can be derived as

$$\lim_{q \nearrow 1} W^{\dagger}(q,\tilde{q}) := \lim_{q \nearrow 1}\left[u\left(y_1 - Q_1(q)\right) + p_1 v\left(F_1(q)\right)\right]$$

$$+ (1-p_1)\lim_{q \nearrow 1}\int_0^1\left\{(1-q)\left[u\left(y_2 - Q_2(p_2;q)\right) + p_2 v\left(F_2(p_2;q)\right)\right] + qu(y_2)\right\}d\Phi(p_2)$$

$$= \left[u(0) + p_1 v(0)\right] + (1-p_1)u(y_2),$$

where the second equality follows from Lemma A9. This completes the proof. ∎

Lemma A11 *Denote the consumer equilibrium welfare in the presence of the life settlement market for the environment $(q,\tilde{q})$ by $W_s^{\dagger}(q,\tilde{q})$. Fix $\tilde{q} \in [0,1)$. If $\eta(c) \geq \alpha > 1$ for all $c > 0$, then there exists a threshold $\underline{q}(\tilde{q})$ such that $W_s^{\dagger}(q,\tilde{q}) > W^{\dagger}(q,\tilde{q})$ for $q \geq \underline{q}(\tilde{q})$.*

Proof. It is evident that $W_s^{\dagger}(q,\tilde{q}) \equiv W_s(q, \frac{q-\tilde{q}}{q})$, where $W_s(\cdot,\cdot)$ is defined in (19). Fixing $\tilde{q}$, note that the equilibrium contract in the presence of the life settlement market does not depend on q;

and we drop q in $F_{1s}(\cdot)$, $Q_{2s}(\cdot)$, and $V_{2s}(\cdot)$ in what follows. It follows directly from (19) that

$$\lim_{q \nearrow 1} W_s^\dagger(q, \tilde{q}) = \left[u(y_1 - Q_{1s}) + p_1 v(F_{1s})\right] + (1 - p_1) \int_0^1 u\left(y_2 + \beta V_{2s}(p_2)\right) d\Phi(p_2).$$

Because $F_{1s} > 0$ and $Q_{1s} < y_1$, we must have that

$$u(y_1 - Q_{1s}) + p_1 v(F_{1s}) > u(0) + p_1 v(0).$$

The above inequality, together with the fact that $u(y_2) \leq u\left(y_2 + \beta V_{2s}(p_2)\right)$, implies that

$$\lim_{q \nearrow 1} W^\dagger(q, \tilde{q}) = \left[u(0) + p_1 v(0)\right] + (1 - p_1) u(y_2)$$

$$< \left[u(y_1 - Q_{1s}) + p_1 v(F_{1s})\right] + (1 - p_1) \int_0^1 u\left(y_2 + \beta V_{2s}(p_2)\right) d\Phi(p_2) = \lim_{q \nearrow 1} W_s^\dagger(q, \tilde{q}),$$

where the first equality follows from Lemma A10.

Fixing $\tilde{q} \in [0, 1)$, we have that $W^\dagger(\tilde{q}, \tilde{q}) \geq W_s^\dagger(\tilde{q}, \tilde{q})$ from (A30). Moreover, $\lim_{q \nearrow 1} W^\dagger(q, \tilde{q}) < \lim_{q \nearrow 1} W_s^\dagger(q, \tilde{q})$. By the continuity of $W^\dagger(\cdot, \cdot)$ and $W_s^\dagger(\cdot, \cdot)$, there must exist a threshold $\underline{q}(\tilde{q}) \in (\tilde{q}, 1)$ such that $W_s^\dagger(q, \tilde{q}) > W^\dagger(q, \tilde{q})$ for $q \geq \underline{q}(\tilde{q})$. This completes the proof. ∎

Now we can prove the second part of Theorem 1. Let $\check{q} := \underline{q}(0)$. It follows immediately from Lemma A11 that

$$W_s(q, 1) \equiv W_s^\dagger(q, 0) > W^\dagger(q, 0) \equiv W(q, 1), \text{ for } q \geq \underline{q}(0) \equiv \check{q}.$$

Moreover, we have that $W_s(q, 0) \leq W(q, 0)$ from (A30). Fix $q \geq \check{q}$. Because $W(q, \Delta)$ and $W_s(q, \Delta)$ are both continuous in Δ, we must have that $W_s(q, \Delta) > W(q, \Delta)$ if Δ is sufficiently close to one. This completes the proof. ∎

Proof of Proposition 7

Proof. See main text. ∎

Proof of Proposition 8

Proof. See main text. ∎

Proof of Lemma 6

Proof. The long-term equilibrium contract $\langle (Q_1^s, F_1^s), (Q_2^s(p_2), F_2^s(p_2), S_2^s(p_2)) : p_2 \in [0, 1] \rangle$ solves

the following maximization problem:

$$\max \left[u\left(y_1 - Q_1^s\right) + p_1 v\left(F_1^s\right) \right] \tag{A34}$$

$$+ (1 - p_1) \int_0^1 \left\{ (1 - \tilde{q}) \left[u\left(y_2 - Q_2^s(p_2)\right) + p_2 v\left(F_2^s(p_2)\right) \right] + \tilde{q} u\left(y_2 + S_2^s(p_2)\right) \right\} d\Phi(p_2)$$

$$\text{s.t.} \quad \left(Q_1^s - p_1 F_1^s\right) + (1 - p_1) \int_0^1 \left\{ (1 - q) \left[Q_2^s(p_2) - p_2 F_2^s(p_2) \right] - q S_2^s(p_2) \right\} d\Phi(p_2) = 0, \tag{A35}$$

$$Q_2^s(p_2) - p_2 F_2^s(p_2) + S_2^s(p_2) \leq 0 \quad \text{for all } p_2 \in [0, 1], \tag{A36}$$

$$S_2^s(p_2) \geq 0 \quad \text{for all } p_2 \in [0, 1], \tag{A37}$$

$$Q_2^s(p_2) \geq 0 \quad \text{for all } p_2 \in [0, 1]. \tag{A38}$$

Condition (A36) guarantees that there will be no lapsation among policyholders whose bequest motives persist in the second period. To see this, suppose that (A36) is violated; then we have that $p_2 F_2^s(p_2) - Q_2^s(p_2) < S_2^s(p_2)$. This implies that the actuarial value of the contract at health state p_2 is less than the CSV. Under such a scenario, the competing insurance firms can cast a spot contract to attempt to convince the policyholders whose bequest motives remain in place to surrender their contracts and purchase a spot contract.

The first-order conditions for Problem (A34) with respect to Q_1^s, F_1^s, $Q_2^s(p_2)$, $F_2^s(p_2)$, and $S_2^s(p_2)$ yield that

$$u'(y_1 - Q_1^s) = \mu^s, \tag{A39a}$$

$$v'(F_1^s) = \mu^s, \tag{A39b}$$

$$(1 - \tilde{q})u'\left(y_2 - Q_2^s(p_2)\right) = (1 - q)\mu^s + \frac{\lambda^s(p_2) + \gamma^s(p_2)}{(1 - p_1)\phi(p_2)}, \tag{A39c}$$

$$(1 - \tilde{q})v'\left(F_2^s(p_2)\right) = (1 - q)\mu^s + \frac{\lambda^s(p_2)}{(1 - p_1)\phi(p_2)}, \tag{A39d}$$

$$\tilde{q}u'\left(y_2 + S_2^s(p_2)\right) = q\mu^s - \frac{\lambda^s(p_2) + \eta^s(p_2)}{(1 - p_1)\phi(p_2)}, \tag{A39e}$$

where μ^s, $\lambda^s(p_2)$, $\eta^s(p_2)$, and $\gamma^s(p_2)$ are the Lagrange multipliers for constraints (A35), (A36), (A37), and (A38), with $\mu^s > 0$, $\lambda^s(p_2) \leq 0$, $\eta^s(p_2) \geq 0$, and $\gamma^s(p_2) \geq 0$ satisfying complementary slackness conditions:

$$\lambda^s(p_2) \left[Q_2^s(p_2) - p_2 F_2^s(p_2) - S_2^s(p_2) \right] = 0, \tag{A40a}$$

$$\eta^s(p_2) S_2^s(p_2) = 0, \tag{A40b}$$

$$\gamma^s(p_2) Q_2^s(p_2) = 0. \tag{A40c}$$

Suppose to the contrary that $S_2^s(p_2) > 0$ for some $p_2 \in [0, 1]$. Then $\eta^s(p_2) = 0$. Combining (A39a) and (A39e) yields that

$$u'\left(y_2 + S_2^s(p_2)\right) = \frac{q}{\tilde{q}} \times \mu^s - \frac{\lambda^s(p_2) + \eta^s(p_2)}{(1 - p_1)\tilde{q}\phi(p_2)} = \frac{q}{\tilde{q}} \times \mu^s - \frac{\lambda^s(p_2)}{(1 - p_1)\tilde{q}\phi(p_2)} \geq \mu^s = u'(y_1 - Q_1^s). \tag{A41}$$

Note that $Q_1^s - p_1 F_1^s \geq 0$ by (A35) and (A36); together with (A39a) and (A39b), we must have that $Q_1^s \geq Q_1^{FI} \equiv \delta$. This indicates that

$$y_2 + S_2^s(p_2) > y_2 > y_1 - \delta \geq y_1 - Q_1^s,$$

where the second inequality follows from Assumption 1. Therefore, we must have that $u'(y_2 + S_2^s(p_2)) < u'(y_1 - Q_1^s)$ from the strict concavity of $u(\cdot)$, which contradicts (A41). This completes the proof. ∎

Proof of Lemma 7

Proof. The long-term equilibrium contract $\langle (Q_{1s}^s, F_{1s}^s), (Q_{2s}^s(p_2), F_{2s}^s(p_2), S_{2s}^s(p_2)) : p_2 \in [0,1] \rangle$ solves the following maximization problem:

$$\max \left[u\left(y_1 - Q_{1s}^s\right) + p_1 v\left(F_{1s}^s\right) \right] \tag{A42}$$

$$+ (1 - p_1) \int_0^1 \left\{ (1 - \tilde{q}) \left[\begin{array}{c} u\left(y_2 - Q_{2s}^s(p_2)\right) \\ + p_2 v\left(F_{2s}^s(p_2)\right) \end{array} \right] + \tilde{q} u\left(y_2 + S_{2s}^s(p_2)\right) \right\} d\Phi(p_2)$$

$$\text{s.t. } (Q_{1s}^s - p_1 F_{1s}^s) + (1 - p_1) \int_0^1 \left\{ (1 - q) \left[Q_{2s}^s(p_2) - p_2 F_{2s}^s(p_2) \right] - q S_{2s}^s(p_2) \right\} d\Phi(p_2) = 0, \tag{A43}$$

$$Q_{2s}^s(p_2) - p_2 F_{2s}^s(p_2) \leq 0 \text{ for all } p_2 \in [0,1], \tag{A44}$$

$$S_{2s}^s(p_2) - \beta V_{2s}^s(p_2) \geq 0 \text{ for all } p_2 \in [0,1], \tag{A45}$$

$$Q_{2s}^s(p_2) \geq 0 \text{ for all } p_2 \in [0,1], \tag{A46}$$

where

$$V_{2s}^s(p_2) \equiv p_2 F_{2s}^s(p_2) - Q_{2s}^s(p_2) \tag{A47}$$

is the actuarial value of the period-2 contract at health state p_2. By the no-lapsation condition (A44), $V_{2s}^s(p_2)$ is always non-negative.

In fact, the complete set of constraints should include (A43), (A45), (A46), $V_2^s(p_2) \geq S_2^s(p_2)$, and $S_2^s(p_2) \geq 0$. Constraint $V_2^s(p_2) \geq S_2^s(p_2)$ guarantees that policyholders with bequest motives will not surrender the contract and buy a spot contract; and constraint $S_2^s(p_2) \geq 0$ requires the CSV to be non-negative. It is evident that these two constraints automatically imply (A44). Therefore, the above maximization problem is indeed a relaxed problem. It can be shown that the solution to the relaxed maximization problem also satisfies $V_2^s(p_2) \geq S_2^s(p_2)$ and $S_2^s(p_2) \geq 0$, implying that it is without loss of generality to focus on the above relaxed problem.

Let $\mu_s^s > 0$, $\lambda_s^s(p_2) \leq 0$, $\eta_s^s(p_2) \geq 0$, and $\gamma_s^s(p_2) \geq 0$ denote the Lagrange multipliers for constraints (A43), (A44), (A45), and (A46) respectively. The first-order conditions for Problem (A42) with respect to Q_{1s}^s, F_{1s}^s, $Q_{2s}^s(p_2)$, $F_{2s}^s(p_2)$, and $S_{2s}^s(p_2)$ yield:

$$u'(y_1 - Q_{1s}^s) = \mu_s^s, \tag{A48a}$$

$$v'(F_{1s}^s) = \mu_s^s, \tag{A48b}$$

$$(1 - \tilde{q})u'\left(y_2 - Q_{2s}^s(p_2)\right) = (1 - q)\mu_s^s + \frac{\lambda_s^s(p_2) + \beta\eta_s^s(p_2) + \gamma_s^s(p_2)}{(1 - p_1)\phi(p_2)}, \tag{A48c}$$

$$(1 - \tilde{q})v'\left(F_{2s}^s(p_2)\right) = (1 - q)\mu_s^s + \frac{\lambda_s^s(p_2) + \beta\eta_s^s(p_2)}{(1 - p_1)\phi(p_2)}, \tag{A48d}$$

$$\tilde{q}u'\left(y_2 + S_{2s}^s(p_2)\right) = q\mu_s^s - \frac{\eta_s^s(p_2)}{(1 - p_1)\phi(p_2)}. \tag{A48e}$$

Suppose to the contrary that $S_{2s}^s(p_2) > \beta V_{2s}^s(p_2)$ for some health state p_2. Then $\eta_s^s(p_2) = 0$; together with (A48a) and (A48e), we have that

$$u'\left(y_2 + S_{2s}^s(p_2)\right) = \frac{q}{\tilde{q}} \times \mu_s^s - \frac{\eta_s^s(p_2)}{(1 - p_1)\tilde{q}\phi_s(p_2)} = \frac{q}{\tilde{q}} \times \mu_s^s \geq \mu_s^s = u'\left(y_1 - Q_{1s}^s\right). \tag{A49}$$

Note that $Q_{1s}^s - p_1 F_{1s}^s \geq 0$ by (A43), (A44), and (A45); together with (A48a) and (A48b), we must have that $Q_{1s}^s \geq Q_1^{FI} \equiv \delta$, and thus $y_2 + S_{2s}^s(p_2) > y_2 > y_1 - \delta \geq y_1 - Q_{1s}^s$, where the second inequality follows from Assumption 1. Therefore, $u'(y_2 + S_{2s}^s(p_2)) < u'(y_1 - Q_{1s}^s)$ from the strict concavity of $u(\cdot)$, which contradicts (A49). This completes the proof. $\blacksquare$

Proof of Lemma 8

Proof. Substituting (A48e) and $S_{2s}^s(p_2) = \beta V_{2s}^s(p_2)$ into (A48c) and (A48d) yields the following:

$$(1 - \tilde{q})u'\left(y_2 - Q_{2s}^s(p_2)\right) + \beta\tilde{q}u'\left(y_2 + \beta V_{2s}^s(p_2)\right) = (1 - q + \beta q)\mu_s^s + \frac{\lambda_s^s(p_2) + \gamma_s^s(p_2)}{(1 - p_1)\phi(p_2)}, \tag{A50a}$$

$$(1 - \tilde{q})v'\left(F_{2s}^s(p_2)\right) + \beta\tilde{q}u'\left(y_2 + \beta V_{2s}^s(p_2)\right) = (1 - q + \beta q)\mu_s^s + \frac{\lambda_s^s(p_2)}{(1 - p_1)\phi(p_2)}. \tag{A50b}$$

Unlike the case in which the life settlement market exists and CSVs are not restricted to zero, both q and $\tilde{q}$ enter the optimization problem and therefore also enter the above conditions.

Define $\overline{\beta}$ as

$$\overline{\beta} := \max\left\{1 - \frac{u'(y_1 - Q_1^{FI}) - u'(y_2)}{qu'(y_1 - Q_1^{FI}) - \tilde{q}u'(y_2)}, 0\right\}.$$

It is straightforward to verify that $\overline{\beta} \in [0, 1)$. Fix $\beta > \overline{\beta}$. Suppose to the contrary that there exists a tuple (q, Δ) such that $Q_{2s}^s(p_2) = 0$ for some $p_2 \in (0, 1]$. This implies that $\gamma_s^s(p_2) \geq 0$ and $\lambda_s^s(p_2) = 0$. From the first-order conditions (A48a) and (A50a), we have that

$$(1 - \tilde{q})u'(y_2) + \beta\tilde{q}u'\left(y_2 + \beta V_{2s}^s(p_2)\right) = (1 - \tilde{q})u'\left(y_2 - Q_{2s}^s(p_2)\right) + \beta\tilde{q}u'\left(y_2 + \beta V_{2s}^s(p_2)\right)$$

$$= \left[1 - (1 - \beta)q\right] \times u'\left(y_1 - Q_{1s}^s\right) + \frac{\lambda_s^s(p_2) + \gamma_s^s(p_2)}{(1 - p_1)\phi(p_2)}$$

$$\geq \left[1 - (1 - \beta)q\right] \times u'\left(y_1 - Q_{1s}^s\right),$$

which cannot hold if $\beta > \bar{\beta}$. To see this, note that $\beta > \bar{\beta}$ and Assumption 1 imply that

$$\left[1 - (1 - \beta)\tilde{q}\right] \times u'(y_2) < \left[1 - (1 - \beta)q\right] \times u'\left(y_1 - Q_1^{FI}\right). \tag{A51}$$

Therefore, we have that

$$
\begin{aligned}
(1 - \tilde{q})u'(y_2) + \beta\tilde{q}u'\left(y_2 + \beta V_{2s}^s(p_2)\right) &\leq \left[1 - (1 - \beta)\tilde{q}\right] \times u'(y_2) \\
&< \left[1 - (1 - \beta)q\right] \times u'\left(y_1 - Q_1^{FI}\right) \\
&\leq \left[1 - (1 - \beta)q\right] \times u'\left(y_1 - Q_{1s}^s\right),
\end{aligned}
$$

where the first and third inequality follow from $V_{2s}^s(p_2) \geq 0$, $Q_{1s}^s \geq Q_1^{FI}$, and $u''(\cdot) < 0$; and the second inequality follows directly from (A51). This completes the proof. ■

Proof of Theorem 2

Proof. It can be verified that Proposition 7 in Fang and Kung (2020) holds under Assumption 1. The first part of Theorem 2 follows directly from the aforementioned result, and the fact that consumer welfare in the absence/presence of the settlement market is continuous in Δ. The proof of the second part closely follows that of Theorem 1, and is omitted for brevity. ■

Proof of Lemma 9

Proof. The equilibrium contract $\langle(Q_{1m}, F_{1m}), (Q_{2m}^H, F_{2m}^H), (Q_{2m}^L, F_{2m}^L)\rangle$ solves the following maximization problem:

$$\max \left[u(y_1 - Q_{1m}) + p_1 v(F_{1m})\right] \tag{A52}$$

$$+ (1 - p_1) \sum_{i=H,L} \tilde{\phi}_i \left\{(1 - q)\left[u\left(y_2 - Q_{2m}^i\right) + p_i v\left(F_{2m}^i\right)\right] + qu(y_2)\right\}$$

$$\text{s.t. } (Q_{1m} - p_1 F_{1m}) + (1 - p_1)(1 - q) \sum_{i=H,L} \phi_i \left(Q_{2m}^i - p_i F_{2m}^i\right) = 0, \tag{A53}$$

$$Q_{2m}^i - p_i F_{2m}^i \leq 0, \text{ for } i \in \{H, L\}, \tag{A54}$$

$$Q_{2m}^i \geq 0, \text{ for } i \in \{H, L\}. \tag{A55}$$

The first-order conditions for Problem (A52) with respect to Q_{1m}, F_{1m}, Q_{2m}^i, and F_{2m}^i yield:

$$u'(y_1 - Q_{1m}) = \mu_m, \tag{A56a}$$

$$v'(F_{1m}) = \mu_m, \tag{A56b}$$

$$u'\left(y_2 - Q_{2m}^i\right) = \frac{\phi_i}{\tilde{\phi}_i}\mu_m + \frac{\lambda_m^i + \gamma_m^i}{(1 - p_1)(1 - q)\tilde{\phi}_i}, \text{ for } i \in \{H, L\}, \tag{A56c}$$

$$v'\left(F_{2m}^i\right) = \frac{\phi_i}{\tilde{\phi}_i}\mu_m + \frac{\lambda_m^i}{(1-p_1)(1-q)\tilde{\phi}_i}, \text{ for } i \in \{H, L\}, \tag{A56d}$$

where μ_m, λ_m^i, and γ_m^i are the Lagrange multipliers for constraints (A53), (A54), and (A55), with $\mu_m > 0$, $\lambda_m^i \leq 0$ and $\gamma_m^i \geq 0$ satisfying the following complementary slackness conditions:

$$\begin{aligned}
\lambda_m^i \left[Q_{2m}^i - p_i F_{2m}^i\right] &= 0, \text{ for } i \in \{H, L\}, \\
\gamma_m^i Q_{2m}^i &= 0, \text{ for } i \in \{H, L\}.
\end{aligned}$$

We first show that $Q_{2m}^H = Q_2^{FI}(p_H)$ if Δ_m is sufficiently large. Note that $u'\left(y_2 - Q_{2m}^H\right) \geq v'\left(F_{2m}^H\right)$ from (A56c) and (A56d); together with $Q_{2m}^H - p_H F_{2m}^H \leq 0$ from (A54), we have that $F_{2m}^H \geq F_2^{FI}(p_H)$. Similarly, we have that $F_{1m} \leq F_1^{FI}$, where F_1^{FI} is defined in the proof of Lemma A3. Next, define $\bar{\Delta}_m$ as follows:

$$\bar{\Delta}_m := \max\left\{1 - v'\left(F_1^{FI}\right) \Big/ v'\left(F_2^{FI}(p_H)\right), 0\right\}.$$

For $\Delta_m > \bar{\Delta}_m$, we have that

$$\begin{aligned}
\frac{\lambda_m^H}{(1-p_1)(1-q)\tilde{\phi}_H} &= v'\left(F_{2m}^H\right) - \frac{\phi_H}{\tilde{\phi}_H}\mu_m = v'\left(F_{2m}^H\right) - \frac{1}{1-\Delta_m}v'(F_{1m}) \\
&\leq v'\left(F_2^{FI}(p_H)\right) - \frac{1}{1-\Delta_m}v'\left(F_1^{FI}\right) < 0,
\end{aligned}$$

where the first equality follows from (A56d); the second equality follows from (A56b) and the definition of ϕ_H and $\tilde{\phi}_H$; the first inequality follows from $F_{2m}^H \geq F_2^{FI}(p_H)$ and $v'(F_{1m}) \geq v'\left(F_1^{FI}\right)$; and the last inequality follows directly from the definition of $\bar{\Delta}_m$. Therefore, we must have that $\lambda_m^H < 0$ when $\Delta_m > \bar{\Delta}_m$, implying $Q_{2m}^H = p_H F_{2m}^H > 0$ and hence $Q_{2m}^H = Q_2^{FI}(p_H)$.

Next, we show that $Q_2^L = 0$ if $\Delta_m > \bar{\Delta}_m$ and ϕ_L is sufficiently small. Denote the unique solution to $u'(y_1 - Q_1) = v'(F_1)$ and $Q_1 - p_1 F_1 = \frac{y_1}{2}$ by $(\check{Q}_1, \check{F}_1)$, and let

$$\bar{\phi}_L := \min\left\{1, \frac{y_1}{2p_L(1-p_1)(1-q)v'^{-1}\left(u'(y_2)\right)}, \frac{\bar{\Delta}_m u'(y_2)}{u'(y_1 - \check{Q}_1) - (1-\bar{\Delta}_m)u'(y_2)}\right\}.$$

Fix $\Delta_m > \bar{\Delta}_m$ and $\phi_L < \bar{\phi}_L$. Suppose to the contrary that $Q_{2m}^L > 0$, then $\gamma_m^L = 0$ and full-event insurance is obtained for state-p_L, which in turn implies that

$$F_{2m}^L = v'^{-1}\left(u'\left(y_2 - Q_{2m}^L\right)\right) < v'^{-1}\left(u'(y_2)\right).$$

From the zero-profit condition (A53), we have that

$$Q_{1m} - p_1 F_{1m} = -(1-p_1)(1-q) \sum_{i=H,L} \phi_i \left[Q_{2m}^i - p_i F_{2m}^i \right]$$

$$= -(1-p_1)(1-q)\phi_L \left[Q_{2m}^L - p_L F_{2m}^L \right]$$

$$< -(1-p_1)(1-q)\phi_L \left[0 - p_L v'^{-1} \left(u'(y_2) \right) \right] < \frac{y_1}{2},$$

where the second equality follows from the fact that $Q_{2m}^H = Q_2^{FI}(p_H)$ and $F_{2m}^H = F_2^{FI}(p_H)$ for $\Delta_m > \bar{\Delta}_m$; the first inequality follows from $Q_{2m}^L > 0$ and $F_{2m}^L < v'^{-1}(u'(y_2))$; and the last inequality follows from $\phi_L < \bar{\phi}_L$. This implies that $Q_{1m} < \check{Q}_1$; together with the first-order conditions (A56a) and (A56c), we have that

$$\frac{\lambda_m^L + \gamma_m^L}{(1-p_1)(1-q)\tilde{\phi}_L} = u'\left(y_2 - Q_{2m}^L \right) - \frac{\phi_L}{\tilde{\phi}_L}\mu_m$$

$$= u'\left(y_2 - Q_{2m}^L \right) - \frac{\phi_L}{\phi_L + \Delta_m(1-\phi_L)}u'(y_1 - Q_{1m})$$

$$\geq u'(y_2) - \frac{\phi_L}{\phi_L + \bar{\Delta}_m(1-\phi_L)}u'(y_1 - \check{Q}_1) > 0,$$

where the first inequality follows from $\Delta_m > \bar{\Delta}_m$, $Q_{2m}^L > 0$ and $Q_{1m} < \check{Q}_1$; and the second inequality follows from $\phi_L < \bar{\phi}_L$. Because $\lambda_m^L \leq 0$, we must have $\gamma_m^L > 0$. This implies that $Q_{2m}^L = 0$, which contradicts the postulated $Q_{2m}^L > 0$. This completes the proof. ∎

Proof of Lemma 10

Proof. The equilibrium competitive contract $\langle (Q_{1ms}, F_{1ms}), (Q_{2ms}^H, F_{2ms}^H), (Q_{2ms}^L, F_{2ms}^L) \rangle$ solves the following maximization problem:

$$\max \left[u(y_1 - Q_{1ms}) + p_1 v(F_{1ms}) \right] \tag{A58}$$

$$+ (1-p_1) \sum_{i=H,L} \tilde{\phi}_i \left\{ (1-q) \left[u\left(y_2 - Q_{2ms}^i \right) + p_i v\left(F_{2ms}^i \right) \right] + q u\left(y_2 + \beta V_{2ms}^i \right) \right\}$$

$$\text{s.t. } (Q_{1ms} - p_1 F_{1ms}) + (1-p_1) \sum_{i=H,L} \phi_i \left(Q_{2ms}^i - p_i F_{2ms}^i \right) = 0, \tag{A59}$$

$$Q_{2ms}^i - p_i F_{2ms}^i \leq 0, \text{ for } i \in \{H, L\}, \tag{A60}$$

$$Q_{2ms}^i \geq 0, \text{ for } i \in \{H, L\}. \tag{A61}$$

Again, $V_{2ms}^i \equiv p_i F_{2ms}^i - Q_{2ms}^i$ is the actuarial value of the period-2 contract at health state p_i, with $i \in \{H, L\}$.

The first-order conditions with respect to Q_{1ms}, F_{1ms}, Q_{2ms}^i, and F_{2ms}^i yield:

$$u'(y_1 - Q_{1ms}) = \mu_{ms}, \tag{A62a}$$

$$v'(F_{1ms}) = \mu_{ms}, \tag{A62b}$$

$$(1-q)u'\left(y_2 - Q^i_{2ms}\right) + \beta q u'\left(y_2 + \beta V^i_{2ms}\right) = \frac{\phi_i}{\tilde{\phi}_i}\mu_{ms} + \frac{\lambda^i_{ms} + \gamma^i_{ms}}{(1-p_1)\tilde{\phi}_i}, \text{ for } i \in \{H, L\}, \tag{A62c}$$

$$(1-q)v'\left(F^i_{2ms}\right) + \beta q u'\left(y_2 + \beta V^i_{2ms}\right) = \frac{\phi_i}{\tilde{\phi}_i}\mu_{ms} + \frac{\lambda^i_{ms}}{(1-p_1)\tilde{\phi}_i}, \text{ for } i \in \{H, L\}, \tag{A62d}$$

where μ_{ms}, λ^i_{ms} and γ^i_{ms} are the Lagrange multipliers for constraints (A59), (A60), and (A61), with $\mu_{ms} > 0$, $\lambda^i_{ms} \leq 0$, and $\gamma^i_{ms} \geq 0$ satisfying the following complementary slackness conditions:

$$\lambda^i_{ms}\left[Q^i_{2ms} - p_i F^i_{2ms}\right] = 0, \text{ for } i \in \{H, L\},$$
$$\gamma^i_{ms}Q^i_{2ms} = 0, \text{ for } i \in \{H, L\}.$$

By the same argument as in the proof of Lemma 9, we have that $F^H_{2ms} \geq F^{FI}_2(p_H)$ and $F_{1ms} \leq F^{FI}_1$, which in turn implies that

$$v'(F^H_{2ms}) \leq v'\left(F^{FI}_2(p_H)\right), \text{ and } v'(F_{1ms}) \geq v'(F^{FI}_1). \tag{A63}$$

Next, define $\bar{\bar{\Delta}}_m$ as

$$\bar{\bar{\Delta}}_m := \max\left\{1 - \frac{v'(F^{FI}_1)}{(1-q)v'\left(F^{FI}_2(p_H)\right) + \beta q u'(y_2)}, 0\right\}.$$

For $\Delta_m > \bar{\bar{\Delta}}_m$, we have that

$$\frac{\lambda^H_{ms}}{(1-p_1)\tilde{\phi}_H} = (1-q)v'\left(F^H_{2ms}\right) + \beta q u'\left(y_2 + \beta V^H_{2ms}\right) - \frac{\phi_H}{\tilde{\phi}_H}v'(F_{1ms})$$
$$\leq (1-q)v'\left(F^{FI}_2(p_H)\right) + \beta q u'(y_2) - \frac{1}{1-\Delta_m}v'\left(F^{FI}_1\right) < 0,$$

where the equality follows from (A62b) and (A62d); the first inequality follows from $V^H_{2ms} \geq 0$ and (A63); and the second inequality follows directly from the definition of $\bar{\bar{\Delta}}_m$. Therefore, $\lambda^H_{ms} < 0$, implying that $Q^H_{2ms} = p_H F^H_{2ms} > 0$ and hence $Q^H_{2ms} = Q^{FI}_2(p_H)$. The proof for $Q^L_{2ms} = 0$ is similar to that of the counterpart in Lemma 9, and is omitted for brevity. This completes the proof. ∎

Proof of Theorem 3

Proof. Fixing (ϕ_L, Δ_m), we first define the consumer equilibrium welfare in the absence of the settlement market and in its presence, which we denote by $W_m(\phi_L, \Delta_m)$ and $W_{ms}(\phi_L, \Delta_m)$ respec-

tively, using the objective distribution of period-2 mortality risk:

$$
W_m(\phi_L, \Delta_m) := \left[u\left(y_1 - Q_{1m}\left(\phi_L, \Delta_m\right)\right) + p_1 v\left(F_{1m}\left(\phi_L, \Delta_m\right)\right) \right]
$$

$$
+ (1 - p_1) \sum_{i=H,L} \phi_i \left\{ (1 - q) \left[u\left(y_2 - Q_{2m}^i\left(\phi_L, \Delta_m\right)\right) + p_i v\left(F_{2m}^i\left(\phi_L, \Delta_m\right)\right) \right] + qu(y_2) \right\},
$$

$$(A64)$$

and

$$
W_{ms}(\phi_L, \Delta_m) := \left[u\left(y_1 - Q_{1ms}\left(\phi_L, \Delta_m\right)\right) + p_1 v\left(F_{1ms}\left(\phi_L, \Delta_m\right)\right) \right]
$$

$$
+ (1 - p_1) \sum_{i=H,L} \phi_i \left\{ \begin{array}{c} (1 - q)\left[u\left(y_2 - Q_{2ms}^i\left(\phi_L, \Delta_m\right)\right) + p_i v\left(F_{2ms}^i\left(\phi_L, \Delta_m\right)\right) \right] \\ + qu\left(y_2 + \beta V_{2ms}^i\left(\phi_L, \Delta_m\right)\right) \end{array} \right\}.
$$

$$(A65)$$

We add ϕ_L and Δ_m into the equilibrium contracts in the above definitions to emphasize that the equilibrium contracts depend on (ϕ_L, Δ_m). The first part of Theorem 3 follows directly from Equation (A30) and the fact that $W_m(\cdot, \cdot)$ and $W_{ms}(\cdot, \cdot)$ are both continuous in Δ_m; and it remains to prove the second part. Lemmas 9 and 10 state that $Q_{2m}^L(\phi_L, \Delta_m) = Q_{2ms}^L(\phi_L, \Delta_m) = 0$, $Q_{2m}^H(\phi_L, \Delta_m) = Q_{2ms}^H(\phi_L, \Delta_m) = Q_2^{FI}(p_H)$, and $F_{2m}^H(\phi_L, \Delta_m) = F_{2ms}^H(\phi_L, \Delta_m) = F_2^{FI}(p_H)$ if Δ_m is sufficiently large and ϕ_L is sufficiently small. Therefore, in order to calculate consumer equilibrium welfare, it remains to pin down the first-period contract and the second-period face value in state-p_L.

In the absence of the life settlement market, the first-period contract and the second-period face value in state-p_L, which we denote by $\langle Q_{1m}, F_{1m}, F_{2m}^L \rangle$, are fully characterized by the following system of equations:

$$
\begin{aligned}
u'(y_1 - Q_{1m}) &= v'(F_{1m}), \\
Q_{1m} - p_1 F_{1m} &= (1 - p_1)(1 - q)\phi_L p_L F_{2m}^L, \\
v'\left(F_{2m}^L\right) &= \frac{\phi_L}{\tilde{\phi}_L} v'(F_{1m}).
\end{aligned}
$$

It follows immediately from the postulated $u(\cdot) = v(\cdot)$ that $Q_{1m} = y_1 - F_{1m}$. Moreover, exploiting the constant IES functional form of $u(\cdot)$ and $v(\cdot)$, $\langle F_{1m}, F_{2m}^L \rangle$ can be solved as follows:

$$
F_{2m}^L = \frac{y_1}{(1 + p_1)\left(\frac{\phi_L}{\tilde{\phi}_L}\right)^\rho + (1 - p_1)(1 - q)\phi_L p_L}, \quad \text{and } F_{1m} = \left(\frac{\phi_L}{\tilde{\phi}_L}\right)^\rho F_{2m}^L.
$$

Therefore, consumer equilibrium welfare in the absence of the life settlement market can be derived

as

$$W_m(\phi_L, \Delta_m) = \left[u(y_1 - Q_{1m}) + p_1 v(F_{1m})\right]$$

$$+ (1 - p_1) \sum_{i=H,L} \phi_i \left\{ (1 - q)\left[u\left(y_2 - Q_{2m}^i\right) + p_i v\left(F_{2m}^i\right)\right] + qu(y_2) \right\}$$

$$= (1 + p_1)v\left(F_{1m}\right) + (1 - p_1)\phi_L(1 - q)p_L v\left(F_{2m}^L\right) + \mathcal{M},$$

where

$$\mathcal{M} := (1 - p_1)\phi_L u(y_2) + (1 - p_1)\phi_H \left\{ (1 - q)\left[u\left(y_2 - Q_2^{FI}(p_H)\right) + p_H v\left(F_2^{FI}(p_H)\right)\right] + qu(y_2) \right\}.$$

Similarly, the first-period contract and the second-period face value in state-p_L in the presence of the life settlement market, which we denote by $\langle Q_{1ms}, F_{1ms}, F_{2ms}^L \rangle$, are fully characterized by the following system of equations:

$$
\begin{aligned}
u'(y_1 - Q_{1ms}) &= v'(F_{1ms}), \\
Q_{1ms} - p_1 F_{1ms} &= (1 - p_1)\phi_L p_L F_{2ms}^L, \\
(1 - q)v'\left(F_{2ms}^L\right) + \beta q u'\left(y_2 + \beta V_{2ms}^L\right) &= \frac{\phi_L}{\tilde{\phi}_L}v'(F_{1ms}).
\end{aligned}
$$

Again, we have $Q_{1ms} = y_1 - F_{1ms}$ from the postulated $u(\cdot) = v(\cdot)$; exploiting the postulated $\beta = 0$ and the constant IES functional form of $u(\cdot)$ and $v(\cdot)$, $\langle F_{1ms}, F_{2ms}^L \rangle$ can be solved as follows:

$$F_{2ms}^L = \frac{y_1}{(1 + p_1)\left(\frac{\phi_L}{\tilde{\phi}_L} \times \frac{1}{1-q}\right)^\rho + (1 - p_1)(1 - q)\phi_L p_L}, \quad \text{and} \quad F_{1ms} = \left(\frac{\phi_L}{\tilde{\phi}_L} \times \frac{1}{1-q}\right)^\rho F_{2ms}^L.$$

Therefore, consumer equilibrium welfare in the presence of the life settlement market is given by

$$W_{ms}(\phi_L, \Delta_m) = \left[u(y_1 - Q_{1ms}) + p_1 v(F_{1ms})\right]$$

$$+ (1 - p_1) \sum_{i=H,L} \phi_i \left\{ (1 - q)\left[u\left(y_2 - Q_{2ms}^i\right) + p_i v\left(F_{2ms}^i\right)\right] + qu\left(y_2 + \beta V_{2ms}^i\right) \right\}$$

$$= (1 + p_1)v(F_{1ms}) + (1 - p_1)\phi_L(1 - q)p_L v\left(F_{2ms}^L\right) + \mathcal{M}.$$

Carrying out the algebra, we see that

$$W_{ms}(\phi_L, \Delta_m) - W_m(\phi_L, \Delta_m)$$

$$= \left[(1+p_1)v\left(F_{1ms}\right) + (1-p_1)\phi_L(1-q)p_L v\left(F_{2ms}^L\right)\right] - \left[(1+p_1)v(F_{1m}) + (1-p_1)\phi_L(1-q)p_L v\left(F_{2m}^L\right)\right]$$

$$= \frac{\rho}{\rho-1} \times \left(F_{2ms}^L\right)^{\frac{\rho-1}{\rho}} \times \left[(1+p_1)\left(\frac{\phi_L}{\tilde{\phi}_L} \times \frac{1}{1-q}\right)^{\rho-1} + (1-p_1)\phi_L(1-q)p_L\right]$$

$$- \frac{\rho}{\rho-1} \times \left(F_{2m}^L\right)^{\frac{\rho-1}{\rho}} \times \left[(1+p_1)\left(\frac{\phi_L}{\tilde{\phi}_L}\right)^{\rho-1} + (1-p_1)\phi_L(1-q)p_L\right].$$

Next, we prove the theorem for the case $\rho > 1$; the analysis for $\rho \leq 1$ is similar. Carrying out the algebra, it is equivalent to show that the following inequality holds when ϕ_L is sufficiently small and Δ_m is sufficiently large:

$$\left[\frac{(1+p_1)\left(\frac{\phi_L}{\tilde{\phi}_L} \times \frac{1}{1-q}\right)^{\rho} + (1-p_1)(1-q)\phi_L p_L}{(1+p_1)\left(\frac{\phi_L}{\tilde{\phi}_L}\right)^{\rho} + (1-p_1)(1-q)\phi_L p_L}\right]^{1-\frac{1}{\rho}} < \frac{(1+p_1)\left(\frac{\phi_L}{\tilde{\phi}_L} \times \frac{1}{1-q}\right)^{\rho-1} + (1-p_1)\phi_L(1-q)p_L}{(1+p_1)\left(\frac{\phi_L}{\tilde{\phi}_L}\right)^{\rho-1} + (1-p_1)\phi_L(1-q)p_L}.$$

By continuity, it suffices to show that the above inequality holds when ϕ_L is sufficiently small and $\Delta_m = 1$ (i.e., $\tilde{\phi}_L = 1$), or equivalently,

$$\left[\frac{(1+p_1)\left(\phi_L \times \frac{1}{1-q}\right)^{\rho} + (1-p_1)(1-q)\phi_L p_L}{(1+p_1)(\phi_L)^{\rho} + (1-p_1)(1-q)\phi_L p_L}\right]^{1-\frac{1}{\rho}} < \frac{(1+p_1)\left(\phi_L \times \frac{1}{1-q}\right)^{\rho-1} + (1-p_1)\phi_L(1-q)p_L}{(1+p_1)(\phi_L)^{\rho-1} + (1-p_1)\phi_L(1-q)p_L}.$$

$$\text{(A66)}$$

Case I: $1 < \rho \leq 2$. From L'Hospital's rule, the left-hand side of (A66) approaches one as $\phi_L \searrow 0$. Similarly, as $\phi_L \searrow 0$, the right-hand side of (A66) approaches

$$\left(\frac{1}{1-q}\right)^{\rho-1} > 1, \text{ if } 1 < \rho < 2;$$

and is equal to

$$\frac{\frac{1+p_1}{1-q} + (1-p_1)(1-q)p_L}{(1+p_1) + (1-p_1)(1-q)p_L} > 1, \text{ if } \rho = 2.$$

Therefore, (A66) holds for sufficiently small ϕ_L.

Case II: $\rho > 2$. Denote $\frac{1-p_1}{1+p_1}(1-q)p_L$ by τ. Then (A66) is equivalent to

$$g(\phi_L) := \log\left[\frac{(\phi_L)^{\rho-2}}{(1-q)^{\rho-1}} + \tau\right] - \log\left[(\phi_L)^{\rho-2} + \tau\right]$$

$$- \frac{\rho-1}{\rho}\left\{\log\left[\frac{(\phi_L)^{\rho-1}}{(1-q)^{\rho}} + \tau\right] - \log\left[(\phi_L)^{\rho-1} + \tau\right]\right\} > 0.$$

Carrying out the algebra, $g'(\phi_L) > 0$ is equivalent to

$$\phi_L < \frac{p(p-2)}{(p-1)^2}\left[\frac{\frac{1}{(1-q)^{\rho-1}}}{\frac{\phi_L^{\rho-2}}{(1-q)^{\rho-1}} + \tau} - \frac{1}{\phi_L^{\rho-2} + \tau}\right] \Bigg/ \left[\frac{\frac{1}{(1-q)^{\rho}}}{\frac{\phi_L^{\rho-1}}{(1-q)^{\rho}} + \tau} - \frac{1}{\phi_L^{\rho-1} + \tau}\right]. \qquad (A67)$$

Note that as $\phi_L \searrow 0$, the right-hand side of the above inequality approaches

$$\frac{p(p-2)}{(p-1)^2}\left[\frac{1}{(1-q)^{\rho-1}} - 1\right] \Bigg/ \left[\frac{1}{(1-q)^{\rho}} - 1\right] > 0,$$

whereas the left-hand side approaches zero. Therefore, (A67) holds, or equivalently, $g(\phi_L)$ is strictly increasing in ϕ_L if ϕ_L is sufficiently small; together with the fact that $g(0) = 0$, there exists a threshold of ϕ_L below which $g(\phi_L) > 0$. This completes the proof. ∎