

# Interbank Networks in the Shadows of the Federal Reserve Act\*

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## Abstract

Public liquidity provision is segmented. Some financial intermediaries have direct access through a central bank, while the rest have indirect access through interbank connections. We construct a historical database of individual balance sheet, payments and funding relationships of Virginia banks. We show that the creation of the Federal Reserve changed the nature of interbank networks, which mutated from channeling deposits to channeling short-term funding among banks that hold less liquidity. It also changed its anatomy, with a network more geographically decentralized. We develop a model that captures this transformation and shows that public liquidity enhances stability but accumulates systemic risk.

Keywords: Dual Banking System, Federal Reserve Act, Shadow Banking, Interbank Networks, Systemic Risk

JEL Classification: G20, E50, N22

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# 1 Introduction

The large infusions of public liquidity by the Federal Reserve System (and several other central banks around the world) during the financial crisis of 2007-2009 and the 2020 COVID-19 pandemic revitalized the discussion about regulating more stringently financial intermediaries that get access to public funds. Yet, the provision of public liquidity not only affects the behavior of those intermediaries, but also the way they connect and interact with intermediaries that do not get direct access to public liquidity. How does public liquidity affects the operations and connections of financial intermediaries, both those under the umbrella of central banks and not? How those effects translate into financial fragility?

Answering these questions is challenging. One obstacle is the ubiquitous provision of public liquidity in the world, which prevents a comparison of interbank networks with and without central banks. Another obstacle is the complexity of modern financial markets, which obstruct a clean characterization of changes in the interbank network as a response of changes in central bank policies. A third obstacle is identifying changes in expectation about the extent and breath of public liquidity provision under different circumstances and shocks. In this paper, we tackle these challenges in two fronts. First, on the empirical front, we build a unique dataset that allows us to compare banks' portfolios and connections before and after the creation of the Federal Reserve System in 1913. This was perhaps the only event in the U.S. that captures a clean and dramatic change in the provision of public liquidity in the presence of a banking network simple enough to identify payment and funding services. Second, on a theoretical front, we construct an endogenous network model that provides us with a framework to interpret this data and draw implications for financial stability.

The Federal Reserve Act, which created the Federal Reserve System, was passed to offer liquidity to member banks through a discount window, with the precondition that members would accept stricter regulations. The Act made membership compulsory for national banks but voluntary for state banks, under the presumption that most state banks would choose to join. Most of them did not. While nonmembers operated under less stringent state regulations (relative to the ones imposed on member banks), yet had indirect access to public liquidity through correspondent member banks (CQ Researcher (1923)).<sup>1</sup> This is, the Federal Reserve Act may have created what we now call a *shadow banking system*: a system of banks, commercial or other, without direct access to liquidity facilities or bailout promises, and with more relaxed oversight and regulation.

We first build a model to obtain insights about how public liquidity provision could affect the

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<sup>1</sup>Banks placing deposits in other banks were called respondents and banks receiving deposits were called correspondents. Correspondent banks were generally located in financial centers.

structure of the interbank network and the behavior of its participants. Armed with these insights the model provides i) testable implications and ii) insights on financial stability and the buildup of systemic vulnerabilities. In our model, we have reserve city banks and country banks. Banks in a reserve city (as was the case of New York) have investment projects, collect deposits from country banks, and pay interest in return. Country banks also have investment projects, and receive deposits from local households and firms. This captures the *in-core* nature of the *national New York City-based core-periphery* structure of the pre-Federal Reserve monetary system, which serve as a hub to insure across regions. If country banks face the risk of withdrawal shocks, they have incentives to diversify following a pecking order: lending first, then depositing in core banks and finally holding cash.

With the access to public liquidity, member banks (as we will show usually those in the core) naturally react by reducing their liquidity holding. Interestingly, however, the indirect access of nonmembers to public liquidity through borrowing from members reduces the needs for diversification, inducing less holdings of cash and less deposits in members banks. In this way, the endogenous reaction of nonmember banks is to accumulate more risks in their own portfolios. We show that the creation of the Fed may have introduced a new source of fragility in the banking system by combining two factors. The first factor is an increase on interbank short-term borrowing activity between member and nonmember banks, which in turn increased the complexity of the network, the exposure to short-term funding runs and the extent of contagion. The second factor is a decline in the needs to hold private liquidity (cash and deposits in other banks). In few words, the *funding network* displaced a *payments network*, transforming the nature of interbank networks, from providing a *tool for diversification* to providing a *pipeline to public funds*.

Beyond the fragility implied by the accumulation of risk in nonmember banks, the increase in interbank short-term borrowing between member and nonmember banks makes the overall network more stable under normal circumstances. The system's vulnerability to shocks, however, increases: without public liquidity ex-post, there would be more liquidations of productive assets. Naturally, if public liquidity is not free or guaranteed, more stability comes at the cost of more vulnerability and, ultimately, more intensive use of costly public liquidity financed by public funds, or large distress in the absence of such liquidity.

The model also shows that public liquidity provision may have changed the structure of the interbank system by decentralizing it geographically (*crowd-out of private inter-regional insurance*). While deemed as a source for financial instability, the concentration of reserves in New York City also allowed banks to smooth local liquidity shocks. As New York City banks pooled reserves of a large number of banks across different regions, the interbank network

was able to diversify regional shocks that were not correlated (Gilbert (1983)). The Fed’s introduction of public liquidity induced banks to rely more on their local correspondents at lower costs, because of shorter distances, better information, stronger relations, etc. However, the emergence of decentralized interbank relationships made the banking system more vulnerable to regional liquidity shocks. In short, financial center banks transformed from being *a provider of private liquidity insurance* to *a conduit for public liquidity insurance*.

To test these model implications, we had to overcome a lack of detailed information about individual balance sheets and interbank relations around the passage of the Federal Reserve Act. Bank balance sheet information for this period from existing studies only report the total amounts of interbank items, not disaggregated by individual debtor or creditor correspondent bank, nor the extent of those relations.<sup>2</sup> Commercial bank directories such as *Rand McNally* and *Polk*, for instance, provide information on self-reported correspondent linkages (and sometimes the names of counterparties), but not the types of interbank transactions nor the amounts associated with these transactions.

To overcome these limitations, we construct a dataset of state banks in Virginia that were listed in the examination reports for the years 1911 and 1922 (that is, before and after passage of the Federal Reserve Act). We focus on banks who choose not to become members, which were the large majority of state banks. Examination reports provide their assets and liabilities as well as detailed information on their correspondents. On the asset side, we observe cash holdings, interbank deposits on each correspondent bank, bonds and loans. On the liability side, we observe equity, deposits (from households and firms and from other banks) and short-term borrowing from each correspondent bank. Our dataset is unique in that it captures both banks’ *funding and payment networks*, this is the correspondents from which they borrow and to which they deposit, respectively.<sup>3</sup>

Consistent with the model, we find that the introduction of the Federal Reserve System altered the role of the interbank system, reduced the liquidity within the banking system, and changed the structure of interbank networks. This is because nonmember banks could borrow indirectly from the Federal Reserve through member banks. First, the funding role of interbank relations superseded their payment role, with banks relying more on short-term borrowing and less on interbank deposits to manage liquidity. Second, the banking system became less liquid, with banks holding less cash and deposits in other banks. Third, there

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<sup>2</sup>“Notes and bills rediscounted,” “bills payable,” “due from other banks,” and “due to other banks” are usually indicators these studies use to infer the existence of interbank relationships. This does not provide much quantitative information about the nature of such relationships though.

<sup>3</sup>While several studies have examined how interbank deposits contributed to financial contagion, only recently attention has been paid to the role of interbank borrowing, such as Lockhart (1921a), Lockhart (1921b), Calomiris and Carlson (2014), and Redenius and Weiman (2020).

was less diversification on the payment network, with banks increasing their exposures by depositing more on a single counterparty. Lastly, we find that the interbank network became less concentrated on a core, with banks reducing their connectivity to financial center banks (this is also documented by Jaremski and Wheelock (2020)). This last finding, however, is driven by banks that enter after the Fed, as existing banks did not sever their relationships with old correspondents in financial centers.

While previous studies have recognized the danger of having a large number of banks operating outside the realm of the Federal Reserve System, they highlight the constraints on the ability to implement monetary policy or to steer the system during crises. Instead we highlight the endogenous reaction of nonmembers in terms of their portfolio choices and their connectivity to other banks, which puts the whole system in a more vulnerable position.<sup>4</sup> Our study has then important implications for policy today. Policies implemented by the central bank will have implications for all financial institutions. Hence, restricting access to public liquidity without controlling the nature of interbank links may backfire by accumulating systemic risk that may ultimately force the hand of the central bank for larger than optimal interventions.

**Related Literature:** Our paper contributes to a rich literature that studies the consequences of the Federal Reserve Act. Previous work has found that the creation of the Federal Reserve reduced financial volatility by smoothing seasonal liquidity pressures on the banking system (see Miron (1986), Mankiw et al. (1987), Bernstein et al. (2010), Carlson and Wheelock (2018b)). We show that, even though the creation of the Federal Reserve may have stabilized the functioning of the system in “normal times,” in the background it was building a shadow system that relied too much on public funds and guarantees, held too little liquidity, and connected too much on fragile borrowing. The creation of the Federal Reserve may have endogenously increased systemic risk, planting the seed for larger collapses.

Recent empirical studies, such as Das et al. (2018), Mitchener and Richardson (2019), Carlson and Wheelock (2018a), and Calomiris et al. (2019) have documented the importance accumulated systemic risk in accounting for the Great Depression. In particular, Carlson and Wheelock (2018a) analyze interbank deposits *of member banks* and show that the system became less resilient to liquidity shocks after the Fed’s founding. As member banks became less liquid and increased dependence on non-member bank deposits, they became more vulnerable to withdrawals by non-member banks. Mitchener and Richardson (2019) provide

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<sup>4</sup>Some studies, for instance, have shown that the inability of nonmember banks to access central bank liquidity magnified the severity of the Great Depression, leading to the creation of new and more extensive lending facilities, such as the Reconstruction Finance Corporation (Wicker (2000), Anbil and Vossmeier (2017)).

empirical evidence for this dynamic during the Depression period. While these studies focus on contagion risk posed by non-member banks to member banks by withdrawing deposits during distress, here we show that this is a more comprehensive reaction, not only during distress, setting the scenario for a tail event.

Much can be learned from the initial years of the Federal Reserve System, but few studies have investigated the effects of introducing Fed’s liquidity provision. Some studies examine the effect of the Federal Reserve liquidity provision on seasonal liquidity pressures.<sup>5</sup> Others document the changes in the structure of *interbank deposit networks* after the introduction of the Fed and study how the structure contributed to the severity of banking crises during the Great Depression.<sup>6</sup> This research, however, has not combined these results on payment networks with funding networks. Recently, Jaremski and Wheelock (2020) use correspondent linkage information in *Rand McNally Directory* and document changes in the structure of the US interbank network after the Fed Act. However, they focus on changes at the extensive margin without incorporating balance sheet information and changes at the intensive margin.

Our paper also contributes to the recent literature on the rise of shadow banking- both by regulatory arbitrage (Ordonez (2018)) and by restricted public liquidity provision (Bengui et al. (2019)). In this paper, we also argue these two factors have been critical to the structure and growth of perhaps the first shadow banking system in the U.S. We show this by using the fact that some banks chose to operate outside the Federal Reserve System (be a nonmember bank) and exploited indirect access to public liquidity through interbank connections. The modern application of our insights contribute to the recent literature on the transmission of monetary policy on the shadow banking sector (Adrian and Shin (2009), Freixas et al. (2011), Chen et al. (2018) and Bianchi and Bigio (2020)). According to our results, the monetary policy that introduces liquidity in the system has nontrivial effects on the relationship between traditional banks and shadow banks, the composition of their portfolios, and the overall stability of the financial system.<sup>7</sup>

On the theoretical front, we apply a network structure to understand how interlinkages (both intensive, focusing on the degree of payment and funding, and extensive, focusing on the existence and anatomy of links) react to government interventions. There are recent studies that endogenize the effects of public interventions to the functioning of banking networks. Erol and Ordoñez (2017), for example, study the reaction of an interbank network to banking regulations. They show that liquidity and capital requirements that are intended to provide

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<sup>5</sup>See Miron (1986), Mankiw et al. (1987), Bernstein et al. (2010), and Carlson and Wheelock (2018b)).

<sup>6</sup>See Mitchener and Richardson (2019) and Carlson and Wheelock (2018b).

<sup>7</sup>Gorton and Ordonez (2020) studies the role of monetary policy on systemic risk through the impact on collateral information in the system, not through the impact on interbank relations.

stability may also make the system more prone to collapse by discouraging the functioning of a network structure that insures against financial shocks. In this paper we study how facilities that lend to certain banks may also unintentionally harm both network functionality and total stability, and we provide evidence of those forces.

In terms of financial network theory, the closest work to ours is Erol (2018), who argues that uncapped ex-post liquidity provision induces a more centralized network by mitigating the insolvency contagion through core banks. Instead we show that liquidity provision induces a less centralized network by reducing the value of liquidity coinsurance by core banks. In this sense, we also contribute to the literature of interbank networks and their effects on systemic risk, as in Allen et al. (2012), Acemoglu et al. (2015) and Chang and Zhang (2019). Empirically, Anderson et al. (2019) show how the concentration of interbank deposits affected systemic risk during the National Banking Era. This paper bridges also these theoretical insights with empirical evidence.

The remainder of the paper is organized as follows: Section 2 provides historical background of the interbank system functioning before and after passage of the Federal Reserve Act. Section 3 presents and studies a benchmark model of a correspondence relationship between two banks, and then extends this benchmark gradually with a central bank and with a richer endogenous network structure. All proofs for this section are contained in Appendix D. Section 4 discusses our novel historical dataset and presents empirical evidence of (1) an increase in banks' reliance of short-term borrowing, (2) a reduction in the liquidity of nonmember banks, and (3) changes in the geographical properties of the core-periphery network. We conclude with some final remarks.

## 2 Historical Background

During the National Banking Era (1864-1912), the U.S. banking system exhibited seasonal spikes in loan interest rates and frequent episodes of banking panics. Short-term interest rates displayed strong seasonal fluctuations due to large increases in the supply of deposits during agricultural harvest seasons and the demand for credit during agricultural planting seasons. As a result, banks faced liquidity pressures in spring and fall, and panics occurred at times of the year in which these pressures peaked.

The interbank system of the period, through the network of correspondent deposits and short-term funding, played an important role in relaxing those liquidity pressures. The reserve structure during the National Banking Era involved national and state banks and was described as an inverted pyramid: rural banks (country banks in agricultural regions)

held their reserves in the form of correspondent balances (mostly, but not exclusively) in banks in central reserve cities, especially New York City.<sup>8</sup> The concentration of interbank deposits in New York City banks effectively transformed them into core banks to reallocate liquidity across regions. When rural banks faced seasonal demands, they withdrew their interbank deposits from financial centers, with those funds coming from other banks in areas where seasonal demands were less pressing. The geographical regional differences in demand produced somewhat offsetting flows of interbank deposits in New York City banks, which effectively provided private insurance across regions (Kemmerer (1910) and Carlson and Wheelock (2018a)). The interbank system helped banks meet seasonal liquidity pressures not only by allowing banks to cross-share deposits but also by allowing them to borrow short-term funds from correspondents. Country banks borrowed the most, reserve-city banks borrowed rarely, and central reserve-city banks borrowed hardly at all.

Although the interbank system helped soften the seasonal demands on banks in both its payment and funding facets, it did not create additional liquidity. As a result, the cash demands of country banks drained cash balances from New York City banks and led to seasonal spikes in interest rates. Contemporaries thought these seasonal swings contributed to bank panics and instability, and this belief prompted calls for reform to create an *elastic currency* that would make the reallocation of funds across regions less dependent on interbank relationships (Sprague (1910)).

In response to this financial landscape, the Federal Reserve System was created in 1913 (under the Federal Reserve Act) with three primary objectives: to eliminate the concentration of bank reserves in New York City banks by establishing 12 regional reserve banks; to create an elastic currency and thereby reduce seasonal volatility; and to prevent panics (Calomiris (1994)). To achieve these goals, the Federal Reserve offered member banks access to public funds through discount windows in 12 regional Federal Reserve Banks, but required those members to meet reserve requirements only by holding vault cash or placing deposits in those Federal Reserve Banks instead of reserve-city and central reserve-city banks.<sup>9</sup> To reduce the

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<sup>8</sup>The interbank system developed to overcome branching restrictions and facilitate interregional payments of goods and services. The National Banking Act institutionalized the interbank system by classifying banks in state banks and national banks and setting up a location-based three-tier system of national banks: central reserve-city banks (those located in New York City, Chicago, or St. Louis), reserve-city banks (banks in selected other large cities), and country banks (banks in all other locations). Central reserve-city banks were required to hold cash reserves equal to 25% of their deposits. Reserve-city banks were also required to hold reserves equal to 25% of their deposits, of which one-half could be deposits with a correspondent bank in a central reserve city. Country banks were required to hold reserves equal to 15% of their deposits, but they could keep three-fifths of the 15% as deposits with a correspondent bank in reserve and/or central reserve cities. State bank regulators subsequently passed similar laws.

<sup>9</sup>Even though only member banks were given access to Federal Reserve services, including the discount window, the Act made it possible to extend the discount window to nonmember banks in special circumstances



concentration of bank deposits in New York City, interbank deposits were no longer counted toward reserve requirements, reducing the incentives to hold those deposits.<sup>10</sup>

Although the creators of the Federal Reserve System hoped to bring state banks under a more unified system of regulation and supervision, they failed to do so because most state banks chose not to join the system. The Federal Reserve Act made it compulsory for national banks to join, whereas it made it voluntary for state banks. By June 1915, only 17 state banks had chosen to join. In 1917, the Federal Reserve Act was amended to encourage state banks to participate. After the amendment, membership grew slowly, eventually reaching a peak of 1,648 state member banks in 1922 (compared with 19,141 state banks who remained nonmembers, according to Committee Branch Group (1935)).<sup>11</sup> Even by 1929, only 5% of state banks in all U.S. had chosen to become members, with more than 60% of all banks remaining outside the realm of the Federal Reserve System.<sup>12</sup>

Two reasons for this lack of participation are usually highlighted. First, the Act mandated members to hold reserves in cash or in reserves within the Federal Reserve System, which did not pay interest. In contrast, state regulators allowed nonmembers to hold reserves in interbank deposits, which earned 2% interest (CQ Researcher (1923)). Second, member banks were subject to more stringent supervision and regulation than most nonmember banks.

The benefits of having direct access to the Federal Reserve's discount window were not large enough, however, to outweigh the costs of joining the system, as nonmember banks were able to gain access *indirectly* to the discount window through their member correspondent banks. Before 1923, the Federal Reserve System was subordinated to the Treasury's goal of supporting World War I by issuing Liberty Bonds, so it allowed member banks to act as *agents* and rediscount for nonmember banks if government bonds were used as collateral (Federal Reserve Board (1917)).<sup>13</sup> In 1923, the privilege given to member banks to act as agents of nonmember banks was revoked in order to encourage state banks to join.<sup>14</sup>

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with the approval of the Federal Reserve Board of Governors (see Carlson and Wheelock (2015)).

<sup>10</sup>The Federal Reserve Act retained for member banks the three-tier classification of central reserve city banks, reserve city banks, and country banks, but changed their reserve requirements. Member banks were required to hold 13%, 10% and 7%, respectively, of *demand deposits* and 3% of *time deposits* within the Federal Reserve Banks. These reserve requirements were first introduced in 1913, took effect in 1914 and were amended in 1917.

<sup>11</sup>In terms of relative size, member banks tended to be larger than nonmembers but nonmembers still held a sizable fraction of total deposits. In 1923, for instance, nonmember banks held more than a third of total U.S. commercial bank deposits (\$10.6 billion of a total of \$37.7 in the whole system).

<sup>12</sup>While there was some heterogeneity across states in terms of membership, in most states less than 4% of state banks joined the Federal Reserve System. The exceptions were northeastern states where membership among state banks topped 30%.

<sup>13</sup>Between June 15th and July 15th of 1917, nonmember banks could access the discount window directly, just with the endorsement of a member bank if they used government bonds as collateral.

<sup>14</sup>After 1923, member banks were allowed to rediscount paper of nonmember banks as a temporary measure

This restriction, however, had limited impact on nonmember banks' ability to access the discount window. This was because nonmembers were still able to borrow from the Federal Reserve indirectly. When member banks could not rediscount the collateral which they received from their nonmember respondents, they would use their own eligible paper to borrow from the Fed and lend to their nonmember respondents (CQ Researcher (1923)). Hence, nonmember banks continued to enjoy the benefit of having indirect access to the Fed's discount window through interbank system even after 1923 (Virginia State Banking Division (1928) and Gruchy (1937)).

The ability of nonmember banks to gain indirect access to the discount window had major implications for the nature of the interbank system and the stability of the financial system. It allowed nonmembers to rely less on cash and interbank deposits in insuring against liquidity shocks and more on borrowing short-term funds from members. In addition, this indirect access increased nonmember banks' dependence on local correspondence relationships, thereby making the overall banking system more vulnerable to regional liquidity shocks. In Section 3, we propose a model and show how public liquidity provision affects systemic risk in the banking system.

### 3 Model

The goal of our model is to understand how nonmember banks' indirect access to the central bank's liquidity affects (1) the liquidity of the banking system, (2) the nature of interbank exposures, and (3) the structure of the interbank network. The use of the model enables us to compare the operations of the interbank system with and without a central bank, and derive implications for financial stability.

Our model not only helps us to understand the role of the Federal Reserve Bank creation on reshaping the interactions between member and nonmember banks, but its extrapolation is also informative about the behavior of modern unregulated financial intermediaries that are not considered banks in the traditional sense (the so-called shadow banks, such as money market funds, investment banks, etc.) and their interaction with regulated banks. Our model highlights the importance of understanding banking networks as a requisite to understand the effects of shadow banking in financial markets.

We begin with an environment with just two banks, a member and a nonmember, and analyze how the introduction of public liquidity affects aggregate liquidity and interbank relations. We then add more banks to study the structure of the interbank network.

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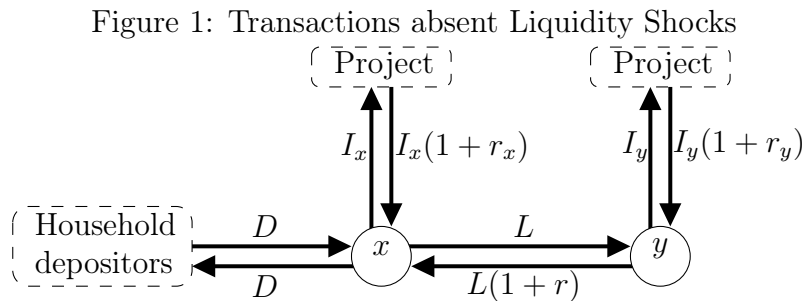
only during emergencies (Federal Reserve Board (1928)).

### 3.1 Environment

The economy is composed by two banks,  $x$  (nonmember bank) and  $y$  (member bank in a reserve city). Bank  $x$  accepts  $D$  household deposits and has access to a project that pays a net rate of return  $r_x > 0$ . Bank  $y$  does not have deposits and has a project that pays a net rate of return  $r_y > 0$ . Projects can be liquidated at any time to recover the original investment, but projects can only be liquidated in full (*no partial liquidation*).

**Reserves and investments** After investments, some depositors may need funds and withdraw from  $x$  before projects reach maturity –*liquidity shocks*. As projects can only be liquidated in full,  $x$  wants to maintain reserves to cover withdrawals, and may do so by holding cash or by depositing at bank  $y$ , earning net interest  $r$ , which we assume is low relative to the projects' returns.<sup>15</sup>

Denoting  $\Phi_x$  the reserves that  $x$  keeps as cash, and  $L$  the amount that  $x$  deposits at  $y$ , bank  $x$  invests  $I_x = D - \Phi_x - L$ . Assuming bank  $y$  is subject to reserve requirements in the form of holding a fraction  $\phi$  of liabilities in cash, and denoting  $\Phi_y$  the reserves that  $y$  keeps in cash,  $\Phi_y \geq \phi L$ . This implies that  $y$  invests  $I_y = L - \Phi_y$ . We call  $I_x$  and  $I_y$  *investments*,  $\Phi_x$  and  $\Phi_y$  *cash reserves*, and  $L$  the *interbank deposits*. The transactions and obligations described thus far, absent liquidity shocks, are shown in Figure 1.



**Liquidity shocks** Liquidity shocks caused by depositors withdrawing early can disrupt the previous flow of funds. We assume that full liquidation of projects always covers original investments, but projects can only be liquidated in full. This last assumption allows us to focus on liquidity crises and not solvency crises, as depositors can always recover  $D$  regardless of shocks, still maintaining the inefficiency of liquidations.

<sup>15</sup>During the National Banking Era, state regulators allowed state banks to keep reserves at reserve cities to meet reserve requirements, and reserve city banks paid 2% (and no more than 2%) interest on those deposits, which justify our assumption that  $r$  is exogenous (See James (1978)). We further discuss the rationale for these assumptions in Online Appendix C

We denote early withdrawals by  $\zeta \in [0, Z]$ , where  $Z$  is the upper bound on possible withdrawals and  $\zeta$  is drawn randomly from a distribution with CDF denoted by  $S$ . We call  $\zeta$  the *liquidity shock*. Depending on the size of the liquidity shock and the size of investments, there are various scenarios that can materialize regarding liquidations. Next, we describe these scenarios for the case in which, facing a withdrawal that forces liquidation, bank  $x$  always withdraws its deposits from  $y$  before liquidating the own project. Formally, this happens when  $I_x + \Phi_y > L$  and  $I_x r_x > Lr$ , which are conditions on endogenous variables that we prove later (in Lemma 1) that always occur on the path of play.<sup>16</sup>

1. If  $\zeta \leq \Phi_x + \Phi_y$ , the combined cash reserves from  $x$  and  $y$  are sufficient to meet the liquidity shock.
  - (a) If  $\zeta \leq \Phi_x$ , withdrawals are met by  $x$ 's cash in vault.
  - (b) If  $\Phi_x < \zeta \leq \Phi_x + \Phi_y$ ,  $x$ 's cash reserves are not enough and  $x$  borrows  $\zeta - \Phi_x$  short-term from  $y$  to cover the withdrawals.<sup>17</sup>
2. If  $\zeta > \Phi_x + \Phi_y$ , the combined cash reserves from  $x$  and  $y$  are not enough to cover the liquidity shocks, in which case  $x$  must either liquidate its own project or withdraw its deposits from  $y$  to an extent that  $y$  has to liquidate its project project. These are three possibilities:
  - (a) If  $\Phi_x + \Phi_y < \zeta \leq \Phi_x + L$ , the deposits of  $x$  at  $y$  are enough to cover the liquidity needs, together with  $x$ 's cash. Then  $x$  withdraws  $L$  from  $y$ , who has to liquidate its project.<sup>18</sup>
  - (b) If  $\Phi_x + L < \zeta \leq \Phi_x + \Phi_y + I_x$ ,  $x$  must liquidate its own project, as deposits at  $y$  are insufficient. In this case,  $x$  can keep its deposits at  $y$ . As the proceedings of liquidation may still be insufficient,  $x$  may need some of  $y$ 's cash.
    - i. If  $\Phi_x + L < \zeta \leq \Phi_x + I_x$ ,  $x$  does not borrow short-term from  $y$ .
    - ii.  $\Phi_x + I_x < \zeta \leq \Phi_x + \Phi_y + I_x$ , then  $x$  borrows  $\zeta - \Phi_x - I_x$  short-term from  $y$ .
  - (c) If  $\Phi_x + \Phi_y + I_x < \zeta$ , neither  $I_x$  from the liquidation of the project, nor deposits  $L$  at  $y$  suffice by themselves, hence  $x$  liquidates its project and withdraws its deposits from  $y$  to cover the large withdrawal.

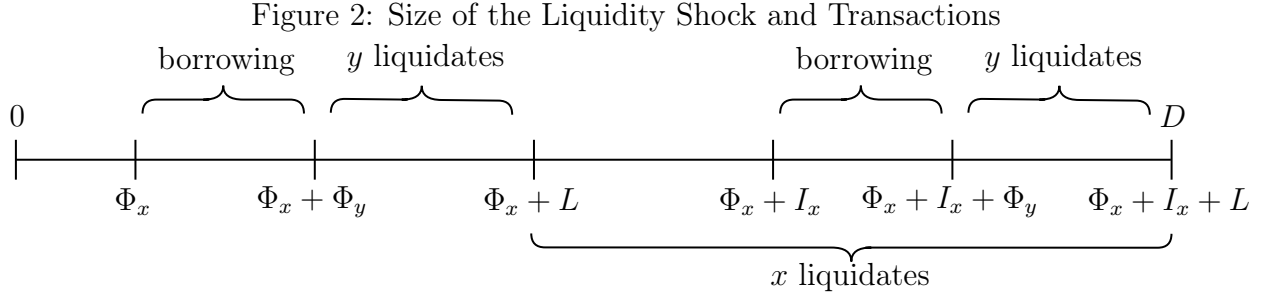
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<sup>16</sup>For expositional simplicity we focus on on-path scenarios, and we deal with off-path scenarios in the proof of Lemma 1.

<sup>17</sup>Such lending is risk-free so we assume that  $y$  does not charge an interest. Given this, whether  $x$  borrows  $\zeta - \Phi_x$  or  $\Phi_y$  is inconsequential. In what follows, we assume that  $x$  borrows the smallest amount that suffices for it to ride out the shock, which is robust to the existence of small borrowing costs.

<sup>18</sup>Since bank  $y$  is forced to liquidate the whole project upon withdrawal, we assume  $x$  withdraws the full amount  $L$  from  $y$ .

Figure 2 shows schematically all these scenarios.



These possible states determine ex-post short-term borrowing by bank  $x$  from bank  $y$ ,

$$b = \begin{cases} \zeta - \Phi_x & \text{if } \Phi_x < \zeta \leq \Phi_x + \Phi_y \\ \zeta - \Phi_x - I_x & \text{if } \Phi_x + I_x < \zeta \leq \Phi_x + I_x + \Phi_y \\ 0 & \text{otherwise} \end{cases}$$

and bank  $x$  ex-post profits,

$$\pi_x = \begin{cases} I_x r_x + Lr & \text{if } \zeta \leq \Phi_x + \Phi_y \\ I_x r_x & \text{if } \Phi_x + \Phi_y < \zeta \leq \Phi_x + L \\ Lr & \text{if } \Phi_x + L < \zeta \leq \Phi_x + \Phi_y + I_x \\ 0 & \text{if } \Phi_x + \Phi_y + I_x < \zeta \end{cases}$$

To obtain ex-ante short-term borrowing and ex-ante profits, we define

$$\Gamma \equiv S[\Phi_x + L]$$

the probability that  $x$ 's project is not liquidated and

$$\Delta \equiv S[\Phi_x + \Phi_y] + (S[\Phi_x + \Phi_y + I_x] - S[\Phi_x + L])$$

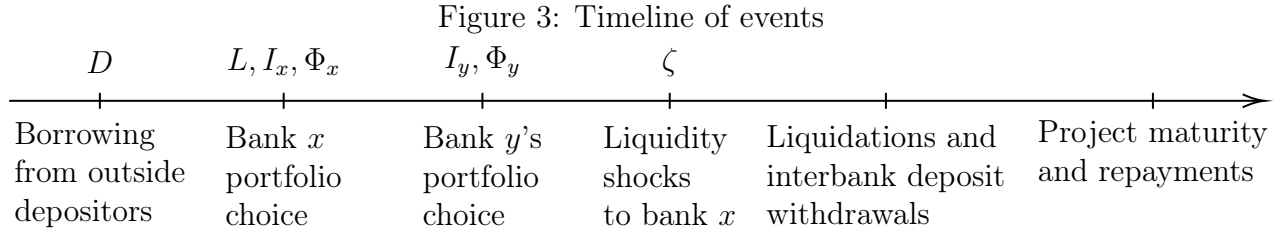
the probability that  $y$ 's project is not liquidated. Bank  $x$ 's expected profits are then

$$\Pi_x = \mathbb{E}[\pi_x] = \Gamma I_x r_x + \Delta Lr. \quad (1)$$

Following similar arguments, bank  $y$ 's expected profits are

$$\Pi_y = \Delta (I_y r_y - Lr). \quad (2)$$

**Timing** Given the expected profits, bank  $x$  chooses investment  $I_x$  and deposits  $L$ , which determines its cash reserves  $\Phi_x$ . Then  $y$  chooses investment  $I_y$ , which determines its cash reserves, which are subject to reserve requirements  $\Phi_y \geq \phi L$ .<sup>19</sup> Then, liquidity shocks materialize. This timeline is summarized in Figure 3.



In this setting we can define upstream contagion as follows: Consider a realized shock  $\zeta$ . If  $\zeta \leq \Phi_x + \Phi_y$ , there is no spillover from  $x$  to  $y$ . If  $\Phi_x + L < \zeta \leq \Phi_x + I_x + \Phi_y$ ,  $x$  liquidates its own project. In these two cases, there is no contagion from  $x$  to  $y$  in terms of forcing  $y$ 's project liquidation. If  $\Phi_x + \Phi_y < \zeta \leq L + \Phi_x$ , then  $x$  withdraws its deposits  $L$  from  $y$ . If  $\Phi_x + \Phi_y + I_x < \zeta$ , then both projects get liquidated. In both of these cases,  $y$ 's project gets liquidated. We call this situation *upstream contagion from  $x$  to  $y$* . The probability of upstream contagion is then  $1 - \Delta$ .

### 3.2 Equilibrium Without Public Liquidity

To obtain clean implications from the model by closed-form solutions, we assume that with probability  $\alpha \in [0, 1]$ ,  $\zeta$  is drawn from  $U[0, Z]$  with  $Z \geq D$ . With probability  $1 - \alpha$  there is no liquidity withdrawal and  $\zeta = 0$ .<sup>20</sup> The expected profits of  $x$  and  $y$  from equations (1) and (2) can be rewritten as

$$\Pi_x = \underbrace{\left(1 - \alpha \frac{I_x}{Z}\right)}_{\Gamma} I_x r_x + \underbrace{\left(1 - \alpha \frac{2L - 2\Phi_y}{Z}\right)}_{\Delta} Lr$$

<sup>19</sup>Bank  $y$  always accept the deposit since its outside option is 0. We will assume that  $(1 - \phi)r_y > r$  and so  $y$  strictly prefers to accept the deposit.

<sup>20</sup>That a bank faces more withdrawals than deposits implies additional legacy liabilities by an amount  $Z - D \geq 0$ . This extension avoids kinks in the solution once we introduce public liquidity, but  $Z > D$  is irrelevant in this part of the paper, and one can simply assume  $Z = D$  for now. Please see Online Appendix C for a detailed discussion of such  $Z \geq D$ .

$$\Pi_y = \left(1 - \alpha \frac{2L - 2\Phi_y}{Z}\right) (L(r_y - r) - \Phi_y r_y).$$

While expected short-term borrowing is

$$B = \mathbb{E}[b] = \frac{\alpha \Phi_y^2}{Z}.$$

We solve for subgame perfect Nash equilibrium, in which bank  $x$  chooses  $I_x$  and  $L$  to maximize  $\Pi_x$  subject to  $I_x, L \geq 0$  and  $I_x + L \leq D$ , given that bank  $y$  chooses  $I_y \in [0, (1 - \phi)L]$  to maximize  $\Pi_y$ .

In what follows we focus on a relatively low probability of early withdrawals, more specifically  $\alpha \leq \bar{\alpha} = \frac{Z}{Z + \rho D}$ , with  $\rho = \max \left\{ 0, \frac{2}{2 - \phi} \left( 2(1 - \phi) - \frac{r}{r_y} \right) - 1 \right\}$ . When  $\alpha$  is not too large, equilibrium reserve requirements bind for  $y$  and  $\Phi_y = \phi L$ . Intuitively, banks are less prone to hold cash buffers in order to prevent the liquidation of projects.

**Lemma 1.** *For relatively low rate on deposits (this is,  $2r < (1 - \phi)r_x$  and  $r < (1 - \phi)r_y$ , where  $\phi < 0.5$ ), reserve requirements bind on the path of play:  $\Phi_y = \phi L$ . Moreover,  $I_x + \Phi_y > L$  and  $I_x r_x \geq Lr$ .*

In order to save on otherwise cumbersome notation, we restrict attention to binding reserve requirements as described in Lemma 1. The next proposition shows how the allocation of funds changes in response to the probability of a liquidity shock  $\alpha$  (taking into consideration the expectation of the withdrawal size in case it happens).

**Proposition 1.** *Equilibrium Portfolios Without Public Liquidity Provision.*

*If  $\bar{\alpha} \geq \alpha \geq \alpha_2$ , the equilibrium quantities are*

$$L = \frac{D + Z_\alpha}{4(1 - \phi)}, \quad I_x = \frac{D + Z_\alpha}{2}, \quad \Phi_x = D - I_x - L, \quad \Phi_y = \phi L.$$

*where  $Z_\alpha \equiv \frac{Z(1 - \alpha)}{\alpha}$  and  $\alpha_2 \equiv \left(1 + \frac{D}{Z} \frac{1 - 2\phi}{3 - 2\phi}\right)^{-1}$ .*

*If  $\alpha_2 > \alpha > \alpha_1$ , the equilibrium quantities are instead*

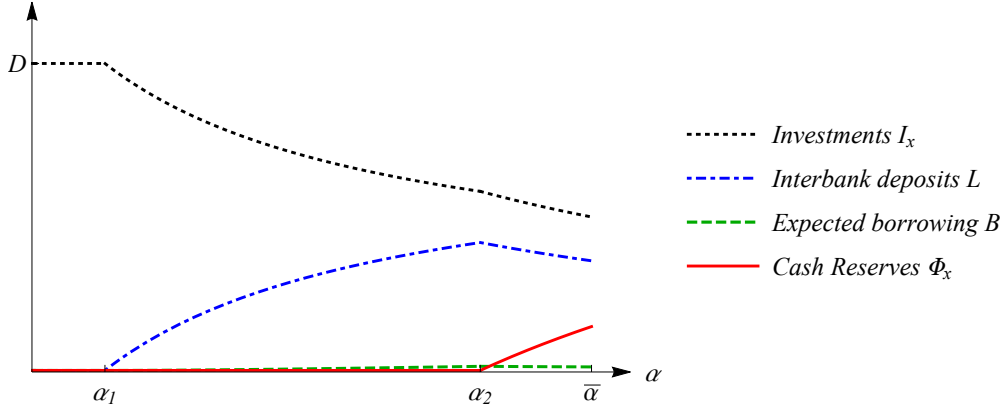
$$L = \frac{D(r_x + r) - Z_\alpha(r_x - r)}{2(r_x + 2(1 - \phi)r)}, \quad I_x = D - L, \quad \Phi_x = 0, \quad \Phi_y = \phi L.$$

*where  $\alpha_1 \equiv \left(1 + \frac{D}{Z} \frac{r_x + r}{r_x - r}\right)^{-1} < \alpha_2$ .*

*Finally, if  $\alpha_1 > \alpha$ , bank  $x$  does not deposit or hold cash, and  $I_x = D$ .*

Figure 4 illustrates the Proposition 1 as a function of the probability of a liquidity shock  $\alpha$ . As  $\alpha$  increases (that is, liquidity shocks become more likely), all instruments for dealing with these shocks increase (more cash reserve, more expected borrowing, and more interbank deposits). An increase in liquid assets is offset by a decline in illiquid investments.

Figure 4: Investments, Interbank Deposits, Short-Term Borrowing, and Cash Reserves



There is also a clear pecking order on holding liquid assets. When the risk of withdrawals is very low ( $\alpha < \alpha_1$ ), the return of an additional unit of investment for bank  $x$  is larger than the risk of liquidating the whole project, and then  $x$  would rather invest fully in the project, without holding any cash or depositing in  $y$ . Once the risk of withdrawals increase enough ( $\alpha_1 < \alpha < \alpha_2$ ), bank  $x$  reduces investment in the own project and places some deposits in  $y$ . The reason is intuitive: by depositing in  $y$  bank  $x$  *diversifies its portfolio* such that, in case of withdrawals that are not too large there is no need to liquidate a single large project but instead a smaller one (either  $x$ 's or  $y$ 's). Thus, as banks cannot liquidate a fraction of projects, diversification works through investing in smaller ones. Finally, once the probability of withdrawals is large enough ( $\alpha_2 < \alpha < \bar{\alpha}$ ), bank  $x$  also holds some cash and resorts more heavily on borrowing from  $y$ .

### 3.3 Equilibrium with Public Liquidity

In this section, we show a public liquidity provision reduces aggregate private liquidity in the banking system, including the private liquidity of banks that do not have direct access to public liquidity. In addition, a public liquidity provision can make the banking system more vulnerable to regional shocks because banks reduce their connectivity to core banks, and such connectivity provides a private tool to smooth out cross-regional liquidity shocks.



Suppose there is a *central bank* that provides short-term liquidity only to  $y$  (a member bank), for a maximum amount  $m$ , which we refer to as the public liquidity provision ( $m = 0$  is the baseline case of no liquidity provision of the previous section). Although bank  $x$  is not a member of the Federal Reserve System, it can indirectly access the Federal Reserve's liquidity facilities through its interbank relationship with  $y$ . We are interested in how the ability of  $x$  to indirectly access the central bank's liquidity affects  $x$ 's reserve holdings, and in turn affects contagion and systemic risk.

This extension reflects the Fed's operation at the time. First, we assume there is a maximum available amount  $m$  of public liquidity, motivated by the fact that the Fed operated under the gold standard when it was created. When studying the system's fragility in the next section we will relax the deterministic nature of  $m$ , allowing for shocks to the supply of gold. We also ignore members' costs to borrow from the Fed (possibly stigma) or the nonmembers' costs to borrow from members (due to the quality of collateral). These omissions are interesting features of the Fed's initial operations, but irrelevant for our qualitative results.

Regardless of the amount of public liquidity  $m$ , bank  $y$  does not want to keep reserves and  $\Phi_y = \phi L$ . For bank  $x$ , using idle reserves  $\Phi_x$  or borrowing at most  $m$  from the central bank via  $y$  are substitutes. For bank  $x$ , therefore, any shock  $\zeta$  below  $m$  can be met at no cost just by borrowing short-term from the member bank. In contrast, a shock above  $m$  will require banks to use their own reserves or to liquidate projects, as above.

Formally, from the viewpoint of bank  $x$ , future shocks become  $\zeta' = \max\{0, \zeta - m\}$ , with  $\zeta'$  equal to 0 with probability  $1 - \alpha + \alpha \frac{m}{Z}$ , and drawn from  $U[0, Z - m]$  with probability  $\alpha \frac{Z - m}{Z}$ . We focus on the values of  $m < Z - D$  so that public liquidity does not eliminate liquidity risk in the financial sector when liquidity shocks are large.

We can rewrite the ex-post profit of bank  $x$  as

$$\Pi_{x,m} = \underbrace{\left(1 - \alpha \frac{I_x - m}{Z}\right)}_{\Gamma_m} I_x r_x + \underbrace{\left(1 - \alpha \frac{2L - 2\Phi_y - m}{Z}\right)}_{\Delta_m} Lr$$

and the following proposition extends Proposition 1 with public liquidity provision.

**Proposition 2.** *Equilibrium Portfolios With Public Liquidity Provision.*

If  $\bar{\alpha} \geq \alpha \geq \hat{\alpha}_2$ , the equilibrium quantities are

$$L = \frac{D + Z_\alpha + m}{4(1 - \phi)}, \quad I_x = \frac{D + Z_\alpha + m}{2}, \quad \Phi_x = D - I_x - L, \quad \Phi_y = \phi L.$$

where  $Z_\alpha \equiv \frac{Z(1-\alpha)}{\alpha}$  and  $\hat{\alpha}_2 \equiv \left(1 + \frac{D}{Z} \frac{1-2\phi}{3-2\phi} - \frac{m}{Z}\right)^{-1}$ .

If  $\hat{\alpha}_2 > \alpha > \hat{\alpha}_1$ , the equilibrium quantities are instead

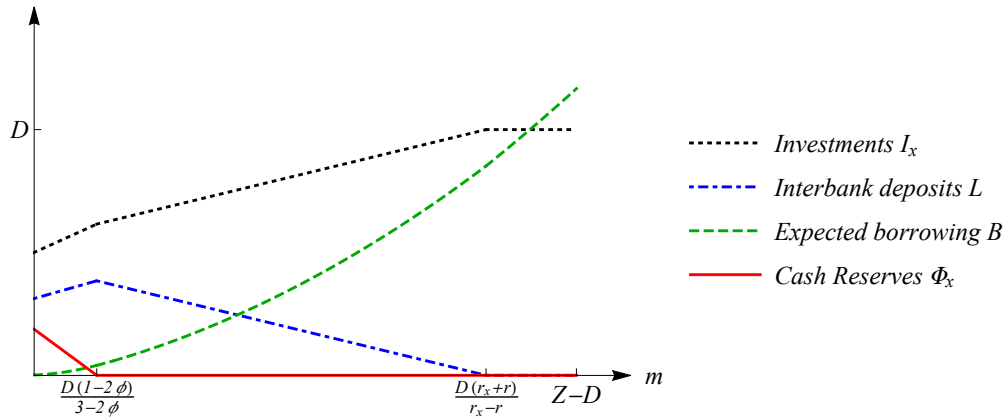
$$L = \frac{D(r_x + r) - (Z_\alpha + m)(r_x - r)}{2(r_x + 2(1 - \phi)r)}, \quad I_x = D - L, \quad \Phi_x = 0, \quad \Phi_y = \phi L.$$

where  $\hat{\alpha}_1 \equiv \left(1 + \frac{D}{Z} \frac{r_x + r}{r_x - r} - \frac{m}{Z}\right)^{-1} < \hat{\alpha}_2$ .

Finally, if  $\hat{\alpha}_1 > \alpha$ , bank  $x$  does not deposit or hold cash, and  $I_x = D$ .

Figure 5 shows how bank  $x$ 's choices change with public liquidity  $m$  for a level of  $\alpha$  that justified  $x$  holding cash with  $m = 0$  in Figure 4. For low levels of  $m$  (first parametric case in the previous proposition),  $I_x$  and  $L$  increase with  $m$  because both are treated as investments. This leads to a steep reduction in cash reserves. When  $m$  becomes large enough (second parametric case),  $x$  will not keep any cash reserves and will keep only interbank deposits. Then, as  $m$  goes up,  $x$  starts reducing interbank deposits  $L$  as it shifts its asset portfolio from low paying investment  $L$  to high paying investment  $I_x$ . All in all, the combined reserves of bank  $x$ ,  $\Phi_x + L$ , decrease in  $m$ . Intuitively, indirect access to public liquidity reduces the need for holding reserves privately and diversifying its portfolio.

Figure 5: Investments, Interbank Deposits, Short-Term Borrowing, and Cash Reserves



Equilibrium allocation as a function of central bank liquidity  $m$  in line with Propositions 2 and 3.

Next we describe how short-term borrowing reacts to  $m$ . The ex-post amount of  $x$ 's short-

term borrowing from  $y$  is

$$b = \begin{cases} \zeta - \Phi_x & \text{if } \Phi_x < \zeta \leq \Phi_x + \Phi_y + m \\ \zeta - L - \Phi_x & \text{if } \Phi_x + \max\{L, \Phi_y + m\} < \zeta \leq \Phi_x + L + m \\ \zeta - I_x - \Phi_x & \text{if } \Phi_x + \max\{I_x, L + m\} < \zeta \leq \Phi_x + I_x + \Phi_y + m \\ 0 & \text{otherwise} \end{cases}$$

These cases lead to the following proposition.

**Proposition 3.** *Equilibrium Short-Term Borrowing*

*Expected short term borrowing is*

$$B = \frac{\alpha}{2Z} (2(m + \Phi_y)^2 + m^2 - \max\{0, m + \Phi_y - L\}^2 - \max\{0, m + L - I_x\}^2)$$

*which is strictly increasing in  $m$  in equilibrium.*

Figure 5 also illustrates this proposition, with short-term borrowing increasing in  $m$ , simply because bank  $x$  relies more on public liquidity provided through bank  $y$ . Bank  $x$  holds more illiquid assets and a less diversified portfolio to meet withdrawals.

This simple analysis highlights the effect of public liquidity provision on the investments and private reserves of shadow banks. Compared with the case of no public provision of liquidity ( $m = 0$ ), shadow banks always invest more in illiquid assets and hold less in cash reserves, relying more on member banks, not to diversify projects but instead as a simple conduit to access public liquidity, disintermediating the system.

### 3.4 Systemic Risk: Fragility and Vulnerability

We have shown how banks adjust their portfolios in response to public liquidity  $m$ . Even though they reduce private liquidity as  $m$  increases, its potential negative effects is offset by higher public liquidity. Hence, they will not need to liquidate projects when they face liquidity shocks as often as they would in the absence of public liquidity. If public liquidity is costless, central banks providing an unlimited amount of public liquidity would be desirable, as implies investing all in the most productive project without the need to liquidate it.

To allow for the possibility of liquidations on path, we assume that, although banks expect public provision of liquidity by the central banks, they do not know the exact amount of such public liquidity. This is consistent with the Fed's operation at its creation as it was not able to

set its liquidity on demand as the United States was still under the gold standard. If banks overestimate the availability of public liquidity, they will hold too much in illiquid assets and may have to liquidate their investments. We model this uncertainty with stochastic  $m$ . Suppose that  $m$  is random between 0 and  $Z - D$ . Then regardless of the level of public liquidity, there is always a shock high enough to require the liquidation of both projects. Therefore, all of our earlier analyses go through simply by replacing  $m$  with  $\mathbb{E}[m]$  in the equilibrium quantities.

There are different ways to categorize risks in the financial system. A first category involves the identity of projects that need liquidation. *Direct risk* refers to the probability that the project of  $x$  gets liquidated as a consequence of the (direct) liquidity shock to  $x$ . *Contagion risk* refers to the probability that the project of  $y$  gets liquidated as a consequence of  $x$  withdrawing its interbank deposits from  $y$ . *Systemic risk* refers to the probability that all projects get liquidated.

A second category is based on banks' demand for public liquidity and their use of it. On the one hand *Fragility* refers to the liquidation risk of portfolios, where these portfolios are chosen based on expected liquidity shocks and expected public liquidity. Fragility takes into account all sources of liquidity. A fragile economy, then, is an economy that is more likely to have less than expected public liquidity (for political or macroeconomic shocks that imply less than expected  $m$ ) that forces project liquidation. On the other hand, *Vulnerability* refers to the liquidation risk of portfolios if there were no public liquidity available ex-post, where these portfolios are chosen based on expected liquidity shocks and expected public liquidity. Vulnerability takes into account only private liquidity. A vulnerable economy, for instance, would be one with very large projects and very few private reserves. A large provision of public liquidity would make an economy highly vulnerable but not fragile.

To obtain closed-form results that clarify comparisons, suppose that  $m$  is 0 with probability  $\beta$  and  $U[0, \frac{2m^*}{1-\beta}]$  with probability  $1 - \beta$ , where  $m^* < \frac{1-\beta}{2}(Z - D)$ . This distribution implies that  $m$  has mean  $m^*$  and mass  $\beta$  at no public liquidity. The next Proposition characterizes the effect of expected public liquidity  $m^*$  on the different categories of risk defined above,

**Proposition 4.** *Systemic Risk*

*Direct vulnerability is increasing in  $m^*$ . Systemic vulnerability and contagion vulnerability are increasing in  $m^*$  under  $m^* < D\frac{1-2\phi}{3-2\phi} - Z_\alpha$  and decreasing in  $m^*$  under  $m^* > D\frac{1-2\phi}{3-2\phi} - Z_\alpha$ . All notions of fragility are decreasing in  $m^*$ .*

When there is no expectation that central banks will provide liquidity support (this is  $m^* = 0$ ), fragility and vulnerability are the same. In that situation, projects that are vulnerable

*because* they may be liquidated without public liquidity support will indeed be liquidated. The larger the expected injection of public liquidity, the lower is the fragility given a level of system vulnerability.

The effect of expected public liquidity on fragility then has two components. An *injection effect* – more expected public liquidity always reduces the need for liquidation – and an *equilibrium effect* – more expected public liquidity reduces private liquidity and increases the need for liquidation. Notice that the equilibrium effect in the evaluation of fragility is, in fact, what we referred to as vulnerability.

$$\text{Fragility} = \text{Vulnerability} - \text{Injection Effect}$$

Intuitively, this explains why all measures of fragility decline given a level of vulnerability in Proposition 4 (all projects are less likely to be liquidated when there are large amounts of public liquidity in the system). Vulnerability, however, measures the exposure of the system to the need for liquidation. Direct vulnerability is increasing in  $m^*$  because bank  $x$  reduces the buffer  $L + \Phi_x$  that protects  $I_x$  when it expects large public liquidity support. As the project of bank  $x$  becomes more reliant on public liquidity, its direct vulnerability increases.

**Remark on the Costs of Systemic Risk:** The provision of public liquidity can be costly: distortionary taxation, inflationary costs, redistributive concerns, etc. Even though we do not introduce the cost of public liquidity explicitly, it is clear that the welfare effects of the systemic fragility and vulnerability we uncover will critically depend on it. If it is costly to provide public liquidity, the increase in vulnerability implies that the Fed will have to bailout a large size project  $x$  at a cost that is not internalized by banks when the system suffers withdrawal shocks. In absence of public liquidity banks would react by diversifying and investing in both projects  $x$  and  $y$ , which is also costly in terms of total output. The welfare implication of public liquidity provision depends on this comparison of these costs.

### 3.5 Networks

In this section, we extend our framework to study how the structure of the interbank network changes in response to the provision of public liquidity. We show that banks move their interbank relations towards counterparts that are less costly to maintain. If it is less costly to maintain relationships with correspondents close in geographic proximity, banks choose to connect less to central reserve cities and more to regional reserve cities. Hence, public insurance crowds out the private insurance that smooths out cross-regional liquidity shocks.

We extend our analysis to several banks. As a first step we focus on four banks in two pairs.

More specifically, banks  $x_1$  and  $y_1$  are linked as described in the baseline, and the same is true for banks  $x_2$  and  $y_2$ . We assume that banks  $x_1$  and  $x_2$  have household deposits and projects. In contrast, banks  $y_1$  and  $y_2$  have interbank deposits received from  $x_1$  and  $x_2$ , and projects. We call  $\{x_1, x_2\}$  *the periphery* and  $\{y_1, y_2\}$  *the core*. As a next step we generalize the functioning of banks in the core. We introduce these generalizations in Section 3.5.1. We study how, in the absence of public liquidity, core banks coinsure each other through forming a sort of clearinghouse, as large New York banks historically did before the Federal Reserve Act. Finally, in Section 3.5.2, we allow periphery banks located in different regions to choose their correspondents from among two groups of banks: those that have greater coinsurance possibilities but may be farther away (say, banks in New York) and those that have fewer coinsurance possibilities but may be closer (say, banks located in regional reserve cities). This allows us to study the effect of the central bank's liquidity provision  $m$  on the network structure. We show that central bank liquidity induces a shift of links from the far core (New York City) to the close core (regional reserve cities), thereby crowding out the private insurance that the system is able to provide.

### 3.5.1 Liquidity coinsurance

We assume that each of the core banks  $y_1$  and  $y_2$  has access to central bank liquidity, capped at (deterministic)  $m$  in total. We also assume that the shocks faced by  $x_1$  and  $x_2$  are negatively correlated, so we rule out competition over central bank liquidity.

There is  $\theta = \frac{\alpha}{2} \leq 0.5$  probability that the shock  $\zeta_1$  is drawn from  $U[0, Z]$  and the shock  $\zeta_2 = 0$ . The parameter  $\theta$  is also the symmetric probability that the shock  $\zeta_2$  is drawn from  $U[0, Z]$  and the shock  $\zeta_1 = 0$ . There is, then a probability  $1 - 2\theta = 1 - \alpha$  that there is no shock, and  $\zeta_1 = \zeta_2 = 0$ . This specification implies that only one bank needs liquidity at a time and that we do not need to model the priorities of the central bank over which bank to provide liquidity to, and how much. In other words, we abstract from aggregate liquidity shocks in the system such that the central bank has to rescue both pairs of banks.

We further allow core banks  $y_1$  and  $y_2$  to insure each other against liquidity shocks coming from bank  $x$  by reallocating liquidity between the two. When  $x_i$  faces a liquidity shock, it can borrow from  $y_i$ , which can borrow from  $y_j$  as well as from the central bank.

Without the liquidity coinsurance possibility, the ex-ante profit of  $x_i$  is

$$\Pi_{x_i} = \left(1 - \theta \frac{I_{x_i} - m}{Z}\right) I_{x_i} r_x + \left(1 - \theta \frac{2(1 - \phi)L_i - m}{Z}\right) L_i r$$

whereas with the liquidity coinsurance, the ex-ante profit is given by

$$\Pi_{x_i} = \left(1 - \theta \frac{I_{x_i} - m - \phi L_j}{Z}\right) I_{x_i} r_x + \left(1 - \theta \frac{2(1 - \phi)L_i - m - \phi L_j}{Z}\right) L_i r$$

**Proposition 5.** *Equilibrium Portfolios and Liquidity Co-Insurance*

Suppose that  $m < D \frac{r_x + r}{r_x - r} - Z \frac{1 - \theta}{\theta}$ . The equilibrium level of interbank deposits with and without liquidity co-insurance is given by

$$L_i^{no\ ins} = \frac{D(r_x + r) - (Z_\theta + m)(r_x - r)}{2(r_x + 2(1 - \phi)r)} > L_i^{ins} = \frac{D(r_x + r) - (Z_\theta + m)(r_x - r)}{2(r_x + 2(1 - \phi)r) + \phi(r_x - r)}$$

whereas  $I_{x_i} = D - L_i$ ,  $\Phi_{x_i} = 0$ , and  $\Phi_{y_i} = \phi L_i$  for both cases.

If  $m > D \frac{r_x + r}{r_x - r} - Z \frac{1 - \theta}{\theta}$ ,  $I_{x_i} = D$  for both cases.

This proposition shows that in the presence of coinsurance possibilities, interbank lending allows banks to hedge against liquidity shocks and allows them to increase its own investments. Hence, banking network at the core acts as an additional source of private liquidity on top of a public liquidity provision.

**Remark on the Costs of Crowding-Out Coinsurance:** Notice that coinsuring withdrawal shocks from respondent banks in the other region is costless because there are no other projects left after initial investments and banks do not suffer from losses due to holding cash. If public liquidity is also costless, then trivially crowding out private coinsurance is irrelevant. As soon as public liquidity, however, has some positive cost (such as coordinating regional Federal Reserve Banks), crowding out private insurance is welfare reducing.

### 3.5.2 Endogenous network

Here we extend the framework to show that the provision of public liquidity insurance crowds out the provision of private liquidity insurance. This happens when the periphery banks' choices of correspondents changes under the central bank liquidity provision and leads to the formation of a new network structure.

Let  $x_i$  represent a bank in region  $i$  which can place deposits in a local reserve-city bank  $y_i^C$  or a New York City bank  $y_i^N$ . Similarly, let  $x_j$  represent a bank in region  $j$ , which can place deposits in a local reserve-city bank  $y_j^C$  or a New York City bank  $y_j^N$ . For both banks  $x_i$  and  $x_j$ , placing deposits in New York City banks incurs a higher cost than placing deposits in regional reserve-city banks because of the geographical distance between respondents and

correspondents. As discussed above, two New York City banks  $y_i^N$  and  $y_j^N$  insure each other against liquidity shocks by reallocating liquidity in the system. In the absence of the central bank  $x_i$  and  $x_j$  will choose  $y_i^N$  and  $y_j^N$  in order to reduce their exposure to local liquidity shocks. Since liquidity shocks are not perfectly correlated between regions  $i$  and  $j$ ,  $x_i$  and  $x_j$  can smooth local liquidity shocks by adjusting their interbank deposits in New York City.

Now, we introduce central bank liquidity,  $m$ . Since  $x_i$  can mitigate local liquidity shocks by borrowing from a regional correspondent  $y_i^C$  directly, we study conditions under which it will choose to connect to  $y_i^C$  rather than  $y_i^N$ , which is more expensive. Similarly,  $x_j$  will choose to connect to  $y_j^C$  rather than  $y_j^N$ . There are two options for equilibria. Banks can either connect to New York City banks for private insurance but pay higher costs, or they can connect to regional reserve city banks.<sup>21</sup>

From the analysis in Section 3.5.1, if both banks connect to their regional correspondents,

$$\Pi_{x_i}^C = \left(1 - \theta \frac{(D - L_C) - m}{D}\right) (D - L_C) r_x + \left(1 - \theta \frac{2(1 - \phi) L_C - m}{D}\right) Lr$$

where  $L_C$  is given by  $L^{no\ ins}$  in Proposition 5. If both banks connect to New York City banks,

$$\Pi_{x_i}^N = \left(1 - \theta \frac{D - L_N - \phi L_N - m}{D}\right) (D - L_N) r_x + \left(1 - \theta \frac{2(1 - \phi) L_N - \phi L_N - m}{D}\right) L_N r - c$$

where  $L_N$  is given by  $L^{ins}$  in Proposition 5.

The next Lemma shows that the relative gain to connect with core banks decline with the volume of public liquidity offered by the Federal Reserve System.

**Lemma 2.** *If  $0 \leq m < D \frac{r_x + r}{r_x - r} - Z \frac{1 - \theta}{\theta}$*

$$\frac{d(\Pi_{x_i}^C - \Pi_{x_i}^N)}{dm} > \frac{r(r_x - r)\phi}{2(r_x + 2(1 - \phi)r) + \phi(r_x - r)} \frac{\theta D}{Z} > 0.$$

This characterization leads to the following proposition.

**Proposition 6.** *Network Geographic Concentration*

*There exists  $m_c$  such that, for  $m < m_c$ , banks in both regions deposit their reserves at New York City banks, and for  $m > m_c$  banks in both regions deposit their reserves in their corresponding reserve cities.<sup>22</sup>*

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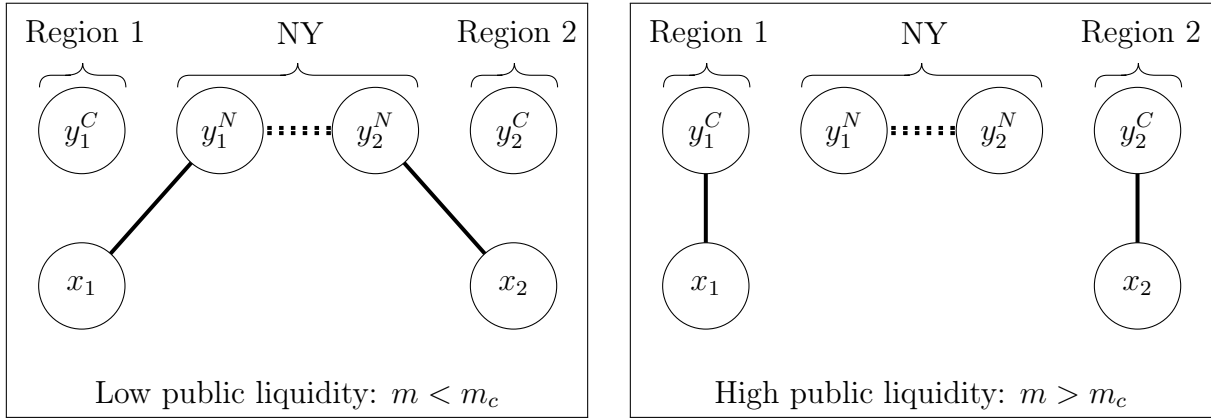
<sup>21</sup>Odell and Weiman (1998) and Jaremski and Wheelock (2020) present evidence that after the founding of the Fed, banks increased their correspondent links to nearby cities with Federal Reserve offices, suggesting that banks did find some advantages in connecting to nearby correspondents.

<sup>22</sup>We use stability as our equilibrium concept, which allows for  $x_1$  and  $x_2$  to deviate together.



Under high enough public liquidity (more specifically, when  $m > m_c$ ), there are no deposits placed in New York City banks, as periphery banks do not rely explicitly on the cross-regional insurance services they provide. With lower levels of public liquidity, however (more specifically,  $m < m_c$ ), the extensive margin of lending changes. Even after accounting for endogenous deposit levels, the marginal benefit of connecting to New York City banks decreases as the amount of central bank liquidity increases. Because public liquidity increases the ability of banks to absorb local liquidity shocks,  $x_i$  and  $x_j$  reduce their reliance on New York City banks and rely on banks in regional reserve cities. A new network structure emerges as the concentration of links decreases. These changes are illustrated in Figure 6.

Figure 6: Network Reactions to Public Liquidity Provision



Change in the structure of the regional interbank network.

**Remark on payment network vs. funding network:** The previous discussion shows that banks tend to form links close in geographical proximity as public liquidity increases. Is this result independent of the nature of the link? Does public liquidity provision lead to a concentration of the *payment network* (i.e., interbank deposits relations) in the same extent as a concentration in the *funding network* (i.e., short-term borrowing relations)? The answer is generally no because the payment network is formed ex-ante shocks (before withdrawal shocks arise) whereas the funding network is formed ex-post (after the shock arise). Hence, the payment network is “more strategic,” and the funding link is “more random.” This means that the previous analysis reflects the payment network better than the funding network.

Still, there is correlation (even though imperfect) between payment and funding networks. Suppose a bank  $x$  holding deposits in a correspondent bank  $y$  (a payment link). When a withdrawal shock happens, the bank can either appeal to the correspondent or search for any other random bank to obtain short-term funding (to form a funding link). The benefit of going directly to the correspondent is that, by threatening to withdraw the deposits from  $y$ ,

bank  $x$  induces  $y$  to also search for liquidity among all its counterparties. Then, even though the funding network has a larger random component, it is still correlated with the payment network. In Section 4, we present this evidence.

### 3.6 Summary

Our simple model generates three testable predictions of public liquidity provision,  $m$ , for the allocation of funds and the shape of interbank linkages. These are:

1. *An increase in public liquidity provision ( $m$ ) intensifies interbank borrowing.* High and concentrated exposures to interbank borrowing expose the banking system to wholesale funding runs and increase the possibility of contagion.
2. *An increase in public liquidity provision ( $m$ ) reduces aggregate private liquidity.* The ability of nonmember banks to manage liquidity pressures through interbank borrowing enables them to hold less liquid assets (cash and interbank deposits).
3. *An increase in public liquidity provision ( $m$ ) dissipates the overall interbank network.* As the interbank system becomes less concentrated, the banking system becomes more vulnerable to regional shocks. This effect is stronger for the payment network than for the funding network.

## 4 Empirical Evidence

In this section, we introduce a novel dataset with detailed information about bank balance sheets and interbank relationships before and after the Fed creation. The amounts of interbank deposits and short-term loans were listed along with the names of correspondents, enabling us to examine interbank relationships on the intensive and extensive margins. We use this rich dataset to test the main predictions of the model, all of which point towards an increase in systemic risk as an unintended consequence of the Federal Reserve System.

### 4.1 Data Sources

We collected state bank examination reports for all state-chartered banks in Virginia in 1911 and 1922. We chose 1911 because this was the year bank examinations were first introduced as part of bank regulators' efforts to improve supervision and examination of a banking system

that expanded rapidly to accommodate the rising needs of industrialization and commercial activities.<sup>23</sup> These reports were filled by regulators once or twice a year.

We chose 1922 for several reasons. First, in 1917 the Federal Reserve provided a three-year phase-in period allowing member banks to adjust to new reserve requirements. Second, in 1917 Congress amended the 1913 legislation and lowered reserve requirements in order to attract more state banks. Third, after the nation's entrance into World War I (April 1917), the Federal Reserve offered a preferential discount rate on loans secured by government debt to support the war effort, but between 1920 and 1921 it removed this preferential rate, raising its discount rate and tightening banks' access to the discount window. Lastly, there was a severe recession between 1920 and 1921. Hence, 1922 was the first tranquil period in which the Fed was already in place and running without major adjustments.

National and state banks that were members of the Federal Reserve System were regulated and supervised by the Federal Reserve Bank of Richmond. The Richmond Fed was known to be a "conservative" district during the 1920s. It followed the real bills doctrine and took a strict stand on rediscounting, prohibiting nonmember banks to rediscount their commercial paper at the Fed directly (Wheelock (2004)). Still nonmembers were able to borrow short-term funds using "bills payable" (relatively worse collateral) through their correspondents in Virginia. In addition, they rediscounted their paper through correspondents in other Federal Reserve districts. (Virginia State Banking Division (1922)). Since only 11 state banks joined the Federal Reserve System, in what follows we focus on nonmember banks.

For a given bank, the dataset reports details of the balance sheet components and three types of interbank connections: deposits *due from* other banks, deposits *due to* other banks, and *borrowed money* from other banks, each with dollar amount and corresponding name and location. Examiners recorded detailed information on interbank deposits for regulatory purposes. In Virginia, nonmember banks could hold up to  $7/12$  of required reserves in the form of interbank deposits with approved reserve agents.<sup>24</sup> Hence, examiners verified whether state banks were holding enough interbank deposits to meet regulatory reserve requirements.

Examination reports also provide details on whether a bank borrowed on a collateralized basis from its correspondents, the amount of the loan, and the identity of the lender. These short-term borrowings took the form of rediscounts and bills payable. "Rediscounts" were loans sold with recourse. "Bills payable" consisted of either promissory notes of the borrowing

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<sup>23</sup>In 1903, the Virginia banking law was amended to impose a more strict supervision of state banks. In 1910, the banking law was amended again to include provisions for the examinations of all state banks and other financial institutions. Bank examinations were held annually in the 1910s, but were made semiannually in the 1920s (Gruchy (1937)).

<sup>24</sup>Virginia state bank regulators did not make differentiated reserve requirements for Richmond and country banks so we can analyze them jointly. Richmond was not a reserve city in 1911 but was one by 1922.

bank or borrowing from Federal Reserve Banks.<sup>25</sup> Examiners paid close attention to this information because it was a good indicator of a bank’s credit position. Bills payable was the last resort for banks to increase funding after rediscounting all eligible commercial paper. An increase in the use of borrowings from other banks using bills payable was taken by examiners as indication of an unhealthy financial position.<sup>26</sup>

Table 1 summarizes our sample. While all Virginia state banks placed interbank deposits, not all of them borrowed short-term funds. “Banks” indicates the number of banks in our sample, and “respondents” indicates those that either placed deposit and/or borrowed short-term funds. For instance, there were 200 state banks in 1911. All 200 banks placed interbank deposits, but only 59 banks placed deposits and borrowed short-term funds. Panel A shows all banks in our dataset, while Panel B focus on banks that operated in both years. As can be seen, in 1922 there was 146 *incumbent* (those operating also in 1911) and 169 *new* banks.

Table 1: Correspondent Relationships, 1911 and 1922

|                                      | Banks | Respondents | Total Links | Mean | SD  |
|--------------------------------------|-------|-------------|-------------|------|-----|
| Panel A: All Banks                   |       |             |             |      |     |
| Year 1911                            |       |             |             |      |     |
| Due-from                             | 200   | 200         | 933         | 4.7  | 3.9 |
| Borrowing                            | 200   | 59          | 87          | 1.5  | 0.9 |
| Year 1922                            |       |             |             |      |     |
| Due-from                             | 315   | 315         | 1025        | 3.3  | 2.3 |
| Borrowing                            | 315   | 160         | 252         | 1.6  | 0.9 |
| Panel B: Banks both in 1911 and 1922 |       |             |             |      |     |
| Year 1911                            |       |             |             |      |     |
| Due-from                             | 146   | 146         | 635         | 4.3  | 3.4 |
| Borrowing                            | 146   | 37          | 55          | 1.5  | 0.8 |
| Year 1922                            |       |             |             |      |     |
| Due-from                             | 146   | 146         | 581         | 4.0  | 2.6 |
| Borrowing                            | 146   | 82          | 133         | 1.6  | 0.9 |

*Notes:* “Due-from” indicates deposits in other banks. “Borrowing” indicates short-term borrowing from other banks. “Banks” indicate the total number of Virginia banks in the sample. “Respondent” indicates banks that either deposit or borrow. “Total links” indicate the total number of linkages of a respondent bank. *Source:* *Virginia State Bank Examination Reports*.

These summary statics before and after the Fed are already consistent with our model’s testable predictions. First, banks reduced the diversification through interbank deposits, as the number of correspondents receiving deposits in average declined from 4.7 in 1911 to 3.3 in 1922. This reduction was driven, however, by entrants and not incumbents (who

<sup>25</sup>Bills payable include certificates of deposits representing borrowed money, amounts due to other banks in the form of overdrafts, and notes and bills re-discounted with correspondent banks.

<sup>26</sup>Borrowing had to be less than the bank’s capital and surplus. In addition, assets pledged as collateral had to be less than 150 percent of the amount borrowed (Virginia State Banking Division (1928)).

declined only from 4.3 to 4 corresponding banks). Second, banks increased their dependence on short-term borrowing: a third of all banks borrowed in 1911 and more than half in 1922.<sup>27</sup>

The respondent (corresponding) banks that *only* placed (received) deposits are in blue, while banks that *both* placed (received) deposits and borrowed (lent) short-term funds are in red. Panel A shows that more Virginia state banks borrowed. Panel B shows that the number of correspondent banks decreased from 1911 to 1922. The map also shows that more banks became reliance on interbank borrowing. In the following sections, we investigate the changes in the banking system after the creation of the Federal Reserve System.

## 4.2 Balance sheet analysis

We examine here how banks' portfolios changed after the creation of the Fed. We examine the balance sheet components that the model generates testable predictions: *loans*, *cash*, *due froms* (deposits due from other banks), and *borrowings* (short-term borrowing from other banks). The model predicts that banks would increase short-term borrowing to manage liquidity problems, then holding more loans and less liquid assets (cash and interbank deposits).

Table 2 shows the balance sheet ratios for these banks for the years 1911 and 1922. First, nonmember banks in Virginia reduced liquid asset holdings. The share of vault cash (specie and legal tender notes) significantly declined by roughly 33% (from 4.8% of total assets in 1911 to 3.2% in 1922). Second, the share of deposits in other banks also declined by roughly 30% (from around 13% of total assets to around 8%). Third, banks increased short-term borrowing significantly by almost 67% (from 3.3% of total liabilities in 1911 to 5.6% in 1922). While reducing liquid assets and increasing short-term wholesale funding, nonmember banks increased their holdings of illiquid assets (bonds and loans). Additional noticeable changes related to how banks raised funds (their liability structure), which our model has not explored, is less equity funding (which declined by about 5 percentage points, from 24% to 19%) and more deposit funding (which increased by about 3 percentage points of liabilities). All these changes are consistent with the theoretical predictions of our model.

The mechanism for these changes is that nonmember banks that increase short-term borrowing by exploiting indirect access to public liquidity through member banks would reduce liquidity. To test this mechanism, we compare the portfolio behavior of banks based on their borrowing status. We divide the 146 incumbent banks into four groups: (1) banks that did not borrow at all, (2) banks that borrowed in both years, (3) banks that borrowed only in

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<sup>27</sup>Figure B4 maps respondent banks (Virginia state banks) and their correspondent banks for the years 1911 and 1922, only for banks that existed in both years.

Table 2: Balance Sheet Ratios, Virginia State Banks, 1911 and 1922

|                          | 1911             | 1922              | Difference           |
|--------------------------|------------------|-------------------|----------------------|
| Cash to assets           | 0.048<br>(0.029) | 0.032<br>(0.034)  | -0.016***<br>(0.004) |
| Duefroms to assets       | 0.129<br>(0.077) | 0.077<br>(0.0509) | -0.052***<br>(0.007) |
| Bonds to assets          | 0.035<br>(0.075) | 0.084<br>(0.116)  | 0.050***<br>(0.012)  |
| Loans to assets          | 0.726<br>(0.131) | 0.759<br>(0.141)  | 0.033**<br>(0.016)   |
| Equity to liabilities    | 0.243<br>(0.088) | 0.191<br>(0.077)  | -0.052***<br>(0.009) |
| Deposits to liabilities  | 0.704<br>(0.133) | 0.736<br>(0.132)  | 0.032**<br>(0.016)   |
| Duetos to liabilities    | 0.017<br>(0.083) | 0.014<br>(0.070)  | -0.004<br>(0.009)    |
| Borrowing to liabilities | 0.033<br>(0.062) | 0.056<br>(0.078)  | 0.022**<br>(0.008)   |
| Obs.                     | 146              | 146               |                      |

1911, and (4) banks that borrowed only in 1922. In Table 3, we report balance sheet ratios for each group based on their borrowing status in 1922 and compute mean differences among groups using the Dunnett’s method. On the asset side, banks that borrowed (either in both years or in 1922) held less liquid assets (cash and deposits) and increased lending. On the liability side, banks became less reliant on deposits and more dependent on short-term borrowing. This evidence shows that the banks that relied more on short-term borrowing also run a riskier business model, with less liquid assets and more loans.

To formally test whether the ability of banks to borrow changed banks’ portfolios after the introduction of the Federal Reserve, we implement a difference-in-differences strategy. Motivated by the previous table, we define the introduction of the Federal Reserve System as the treatment event. The treated groups are three groups of banks that used short-term funds: *always a borrower* (those that borrowed in 1911 and 1922), *old borrower* (those that borrowed only in 1911), and *new borrower* (those that borrowed only in 1922). The

Table 3: Balance Sheet Ratios by Borrowing Status, 1922

|                          | none             | 1911 and 1922    | 1911 only         | 1922 only        | Difference           |                   |                      |
|--------------------------|------------------|------------------|-------------------|------------------|----------------------|-------------------|----------------------|
|                          | (1)              | (2)              | (3)               | (4)              | (2) - (1)            | (3) - (1)         | (4) - (1)            |
| Cash to assets           | 0.045<br>(0.055) | 0.025<br>(0.014) | 0.029<br>(0.012)  | 0.025<br>(0.011) | -0.019**<br>(0.007)  | -0.016<br>(0.010) | -0.020**<br>(0.007)  |
| Duefroms to assets       | 0.101<br>(0.053) | 0.077<br>(0.049) | 0.084<br>(0.048)  | 0.053<br>(0.039) | -0.024*<br>(0.011)   | -0.017<br>(0.014) | -0.048**<br>(0.010)  |
| Bonds to assets          | 0.122<br>(0.158) | 0.051<br>(0.073) | 0.082<br>(0.087)  | 0.073<br>(0.091) | -0.071**<br>(0.026)  | -0.039<br>(0.034) | -0.049*<br>(0.023)   |
| Loans to assets          | 0.695<br>(0.161) | 0.806<br>(0.107) | 0.760<br>(0.082)  | 0.787<br>(0.135) | 0.110***<br>(0.030)  | 0.065<br>(0.040)  | 0.092***<br>(0.027)  |
| Equity to liabilities    | 0.184<br>(0.064) | 0.191<br>(0.107) | 0.195<br>(0.072)  | 0.195<br>(0.068) | 0.007<br>(0.017)     | 0.011<br>(0.023)  | 0.011<br>(0.016)     |
| Deposits to liabilities  | 0.802<br>(0.065) | 0.683<br>(0.156) | 0.798<br>(0.0731) | 0.692<br>(0.142) | -0.119***<br>(0.027) | -0.004<br>(0.036) | -0.111***<br>(0.024) |
| Duetos to liabilities    | 0.025<br>(0.108) | 0.002<br>(0.004) | 0.029<br>(0.102)  | 0.006<br>(0.017) | -0.023<br>(0.016)    | 0.005<br>(0.021)  | -0.018<br>(0.014)    |
| Borrowing to liabilities | 0<br>(0)         | 0.110<br>(0.087) | 0<br>(0)          | 0.088<br>(0.066) | 0.110***<br>(0.013)  | 0.000<br>(0.017)  | 0.088***<br>(0.011)  |
| Obs.                     | 47               | 35               | 15                | 49               |                      |                   |                      |

control group is *never a borrower* (those banks that did not borrow in either one of the two years). Letting  $i$  denote one of the 146 banks that were present both in 1911 and 1922, and  $t \in \{1911, 1922\}$ , our specification is:

$$Y_{i,t} = \alpha + \beta_1 * \mathbb{I}_j + \beta_2 * \mathbb{I}_j * \mathbb{I}_{1922} + \varepsilon_{i,t} \quad (3)$$

where  $Y_{i,t}$  is the balance sheet ratios (such as the ration of cash to assets, or the ratio of equity to liabilities),  $\mathbb{I}_j$  is an indicator function that takes the value 1 if bank  $i$  corresponds to group  $j \in \{\text{always a borrower, never a borrower, old borrower, new borrower}\}$ , and 0 otherwise, and  $\mathbb{I}_{1922}$  is an indicator function that takes the value 1 if the year is 1922 (after the Fed's founding). The variable  $\varepsilon_{i,t}$  is a mean-zero and possibly heteroskedastic error term. We cluster error terms at the bank level in order to account for the serial correlation of error terms. The coefficient  $\beta_2$  captures captures the effect of the Fed's founding for for the banks that borrowed and the banks that did not. We examine the effects of the Fed's founding on both asset and liability sides of balance sheets.

Table 4 presents the results for asset ratios. It shows that the creation of the Fed reduced

the liquidity of the banks that borrowed. These banks reduced both cash and interbank deposits. A reduction in liquid assets is offset by an increase in loans. These changes are most pronounced in banks that borrowed in 1922.

Table 4: The Effect of the Fed's Founding on Bank Assets

|   | Cash to assets       | Duefrom to assets    | Bonds to assets   | Loans to assets     |
|---|----------------------|----------------------|-------------------|---------------------|
| <i>Always a borrower</i> $\times \mathbb{I}_{1922}$ | -0.029***<br>(0.005) | -0.051***<br>(0.014) | 0.010<br>(0.018)  | 0.089***<br>(0.027) |
| <i>New borrower</i> $\times \mathbb{I}_{1922}$      | -0.030***<br>(0.005) | -0.075***<br>(0.012) | 0.032*<br>(0.019) | 0.070**<br>(0.027)  |
| <i>Old borrower</i> $\times \mathbb{I}_{1922}$      | -0.026***<br>(0.006) | -0.044***<br>(0.016) | 0.042<br>(0.026)  | 0.043<br>(0.028)    |
| Observations  | 292                  | 292                  | 292               | 292                 |
| R2  | 0.12                 | 0.29                 | 0.11              | 0.10                |

Table 5 presents the results for liability ratios. The creation of the Fed did not have an effect on the banks that borrowed in both years, but it changed the liability structure of new borrowers. These banks changed their funding structure significantly. They reduced deposit financing, but increased equity financing and wholesale funding. While not explored in the model, banks liability structures would have affected a run risk. When banks reduce their dependence on household deposits and increase their reliance on short-term borrowing, they become less exposed to withdrawals by household depositors, but become more exposed to runs by institutional investors. This is because correspondent banks that provided short-term funding may not extend these short-term loans when they face liquidity shocks themselves. To sum, these results suggest that the availability of short-term funding changed the funding structures of banks, altering also the nature of bank runs.

Table 5: The Effect of the Fed's Founding on Bank Liabilities

|   | Equity to liab     | Deposits to liab     | Duetos to liab     | Borrowing to liab    |
|---|--------------------|----------------------|--------------------|----------------------|
| <i>Always a borrower</i> $\times \mathbb{I}_{1922}$ | -0.008<br>(0.020)  | -0.008<br>(0.025)    | -0.028*<br>(0.017) | 0.017<br>(0.015)     |
| <i>New borrower</i> $\times \mathbb{I}_{1922}$      | 0.036**<br>(0.015) | -0.107***<br>(0.031) | -0.046*<br>(0.026) | 0.088***<br>(0.009)  |
| <i>Old borrower</i> $\times \mathbb{I}_{1922}$      | -0.017<br>(0.034)  | 0.146***<br>(0.035)  | -0.007<br>(0.031)  | -0.112***<br>(0.018) |
| Observations  | 292                | 292                  | 292                | 292                  |
| R2  | 0.11               | 0.20                 | 0.22               | 0.51                 |



### 4.3 Network Analysis

The changes in bank balance sheets do not provide insights on how banks changed their interbank relationships. In this section, we study how the creation of the Federal Reserve System affected the nature and structure of the interbank system in Virginia, both at extensive (number of counterparties) and intensive (the exposure and strength of relations with each counterparty) margins.

We begin our analysis by presenting a specific example that illustrates our data. In Figure 7, we show the interbank relationships of the Bank of Warm Springs in Warm Spring, Virginia. The correspondent banks that received *only* deposits from the Bank of Warm Springs are in blue and the ones that *both* received deposits and lent the Bank of Warm Springs short-term loans are in red. In the tabular component of the map, we provide detailed information about these correspondent relationships. Columns (1) and (2) provide the names and locations of the correspondent banks of the Bank of Warm Springs. Columns (3) and (4) show the amount of interbank deposits due from these banks and the amount of short-term funds borrowed from them in each year.

Figure 7: Bank Network for Bank of Warm Springs

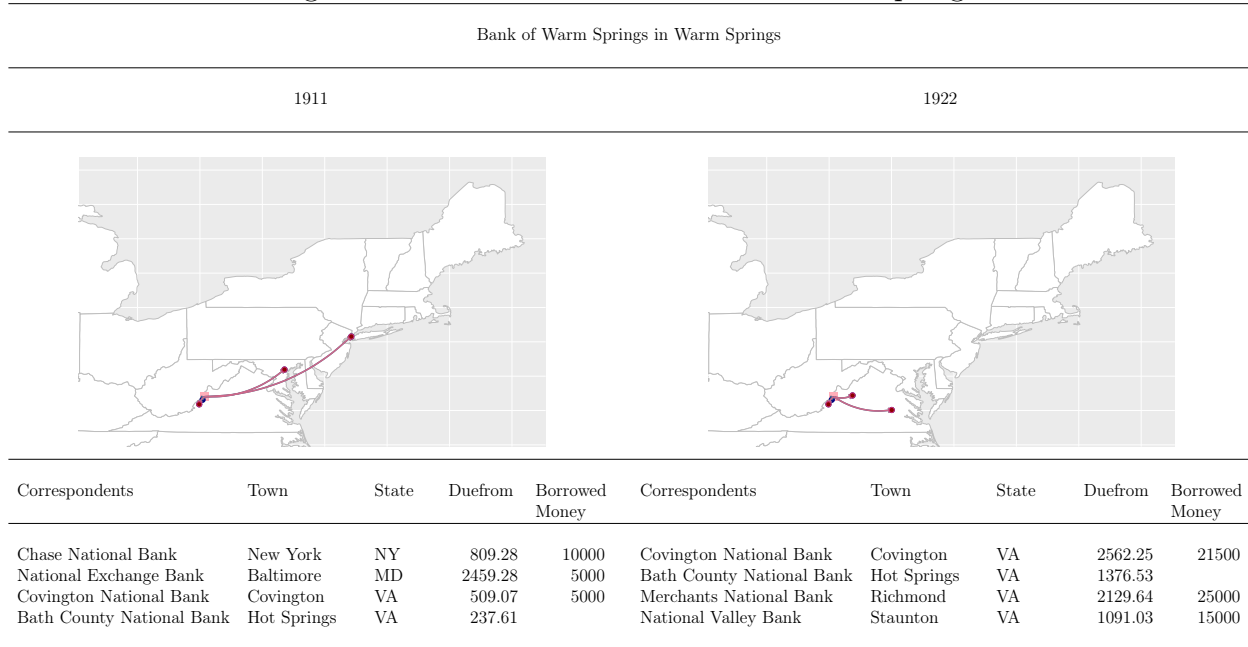


Figure 7 shows that the interbank relationships of this particular bank changed in two major ways following the introduction of the Fed. First, short-term borrowing became more important than interbank deposits. While the volume of exposures to counterparties through interbank deposits increased modestly, the size of exposures to counterparties through in-

terbank borrowing increased significantly. Second, correspondent relationships became more local. The Bank of Warm Springs maintained correspondent banking relationships in New York and Baltimore in 1911, but it had dissolved these relationships and opened new ones with closer banks in Richmond and Staunton in 1922. Next we show these changes were not an exemption, but quite ubiquitous.

Table 6 shows an increase in the concentration of interbank deposits. While the proportion of deposits at the largest counterparty against total interbank deposits did not change in average, the proportion of deposits at the largest counterparty against total assets did declined. New banks, however, increased deposit exposure to the largest counterparty compared to existing banks (almost a 20% increase in concentration out of total deposits and an increase of more than 50% out of total assets). This is consistent with our assumption that new banks are more flexible when they create new relationships. Table 7 shows the changes in a bank's exposure to correspondents that took deposits and extended short-term loans simultaneously. Here again new banks behaved quite differently than existing banks, increasing their deposit concentration to banks where they borrow more heavily.

Table 6: Due-from Exposure to the Correspondent with the Most interbank Deposits

| Duefrom in largest<br>counterparty | Existing - Across years |                  |                      | Across banks     |                  |                     |
|------------------------------------|-------------------------|------------------|----------------------|------------------|------------------|---------------------|
|                                    | 1911                    | 1922             | Difference           | Existing         | New              | Difference          |
| to total duefroms                  | 0.663<br>(0.232)        | 0.655<br>(0.217) | -0.007<br>(0.026)    | 0.655<br>(0.217) | 0.776<br>(0.208) | 0.121***<br>(0.024) |
| to total assets                    | 0.084<br>(0.062)        | 0.050<br>(0.036) | -0.034***<br>(0.006) | 0.050<br>(0.036) | 0.078<br>(0.069) | 0.028***<br>(0.007) |
| Respondent Bank                    | 146                     | 146              |                      | 146              | 169              |                     |
| Correspondent Bank                 | 65                      | 56               |                      | 56               | 74               |                     |
| Obs.                               | 146                     | 146              |                      | 146              | 169              |                     |

Using the geographical distribution of correspondent relationships, we examine whether the Fed's founding changed the concentration of nonmembers correspondent relationships, both at the extensive and intensive margins. The Federal Reserve Act had a direct impact on the network structure of its member banks because it disallowed them to use interbank deposits to satisfy reserve requirements. In contrast, state regulators continued to allow state nonmember banks to do so. Tables 8 and 9 show the geographical distribution of correspondent links for the payment and funding networks, respectively.

Table 8 shows the geographic distribution of correspondent deposits due from other banks, known as the *geographic payment network*. We find that the creation of the Fed reduced the concentration of interbank deposits in financial centers. Before the Fed's founding, banks

Table 7: Degree of Due-from and Borrowing Exposure to Lending Correspondents

| Duefrom in largest<br>counterparty | Existing - Across years |                  |                   | Across banks     |                  |                  |
|------------------------------------|-------------------------|------------------|-------------------|------------------|------------------|------------------|
|                                    | 1911                    | 1922             | Difference        | Existing         | New              | Difference       |
| Duefroms to total assets           | 0.037<br>(0.042)        | 0.026<br>(0.030) | -0.011**<br>0.005 | 0.026<br>(0.030) | 0.038<br>(0.046) | 0.012**<br>0.005 |
| Borrowing to total liabilities     | 0.057<br>(0.056)        | 0.055<br>(0.044) | -0.002<br>0.007   | 0.055<br>(0.044) | 0.068<br>(0.051) | 0.014**<br>0.006 |
| Respondent Bank                    | 37                      | 82               |                   | 82               | 78               |                  |
| Corresponent Bank                  | 32                      | 60               |                   | 60               | 68               |                  |
| Obs.                               | 55                      | 133              |                   | 133              | 119              |                  |

held deposits with banks outside Virginia, such as New York City and Baltimore. After the Fed’s founding, they shifted their deposits away from banks in other regions and toward banks in Virginia. These changes are consistent at the both extensive and intensive margins. In addition, we analyze the concentration of interbank deposits separately for incumbent and new banks and report the results in tables B1 and B2 in the Appendix. The results show that these changes were driven predominantly by the 169 new banks in our sample, not the 146 incumbent banks. In other words, new banks chose local correspondents when they opened new correspondent relationships while incumbent banks maintained their existing correspondent relationships.

Table 9 shows the geographic distribution of short-term loans from correspondents, known as the *geographic funding network*. We find that the creation of the Fed reduced the concentration of short-term loans from correspondents in financial centers as well, but to a lesser extent than interbank deposits. These results are also consistent with the predictions of our model. Before the Fed’s founding, 40% of country banks borrowed short-term funds from their correspondents in Richmond banks. After the Fed’s founding, banks borrowed more heavily from rural country banks in Virginia instead of banks in Richmond, reducing the amount of borrowing by more than 20%. These changes are consistent at the both extensive and intensive margins. Tables B3 and B4 in the Appendix report the concentration of short-term funding separately for incumbent and new banks. Much like the payment network, the changes were driven predominantly by the 169 new banks, not the 146 incumbent banks.

Based on this detailed information about the location of respondent and correspondent banks we can identify more clearly the change in the geographical concentration of the interbank system. We compute the distances in miles between respondent and correspondent banks. Table 10 shows the longest and average distance between each banks and the correspondent

Table 8: Geographic Payment Network, All Banks

| Due from Deposits in:          | Extensive Margin (Links) |                  |                      | Intensive Margin (Amount) |                  |                      |
|--------------------------------|--------------------------|------------------|----------------------|---------------------------|------------------|----------------------|
|                                | 1911                     | 1922             | Difference           | 1911                      | 1922             | Difference           |
| New York City                  | 0.195<br>(0.184)         | 0.127<br>(0.167) | -0.068***<br>(0.016) | 0.108<br>(0.163)          | 0.069<br>(0.140) | -0.038***<br>(0.014) |
| Baltimore                      | 0.094<br>(0.180)         | 0.069<br>(0.166) | -0.025<br>(0.016)    | 0.110<br>(0.242)          | 0.070<br>(0.198) | -0.040**<br>(0.020)  |
| Washington, DC                 | 0.022<br>(0.079)         | 0.017<br>(0.099) | -0.005<br>(0.008)    | 0.018<br>(0.075)          | 0.016<br>(0.108) | -0.002<br>(0.009)    |
| Richmond                       | 0.212<br>(0.200)         | 0.223<br>(0.277) | 0.011<br>(0.022)     | 0.289<br>(0.326)          | 0.272<br>(0.346) | -0.017<br>(0.030)    |
| Reserve Cities in Other States | 0.024<br>(0.072)         | 0.034<br>(0.136) | 0.010<br>(0.011)     | 0.026<br>(0.085)          | 0.040<br>(0.168) | 0.015<br>(0.013)     |
| Country Banks in VA            | 0.423<br>(0.279)         | 0.500<br>(0.345) | 0.077**<br>(0.029)   | 0.408<br>(0.368)          | 0.511<br>(0.406) | 0.104***<br>(0.036)  |
| Country Banks in Other States  | 0.027<br>(0.109)         | 0.028<br>(0.104) | 0.001<br>(0.009)     | 0.031<br>(0.145)          | 0.017<br>(0.095) | -0.014<br>(0.011)    |
| Obs.                           | 200                      | 315              |                      | 200                       | 315              |                      |

banks for the payments and funding relationships. The distance between correspondents and respondents for the payments network was longer than the distance between correspondents and respondents for the funding network. It was because banks held deposits with correspondents outside the state whereas they tended to borrow more locally. Following the Fed's founding, the distance between correspondents and respondents declined for the payment network. In contrast, the distance between correspondents and respondents remained unchanged for the funding network. These results indicate the need to hold interbank deposits in other regions to meet local liquidity shocks because banks became less dependent on interbank deposits as a liquidity source.

A reduction in the concentration of correspondence linkages in financial centers suggests that the ability to borrow from local correspondents reduced the need for banks to depend on financial center banks to manage liquidity shocks. As documented in Carlson and Wheelock (2018a), New York City banks provided private liquidity arrangement against regional liquidity shocks. New York City banks accommodated liquidity transfers between regions and smoothed interregional flows by pooling reserves from different regions (Gilbert (1983) and James and Weiman (2010)). However, the provision of liquidity by the Fed crowded out the

Table 9: Geographic Funding Network, All Banks

| Short-term Borrowing from:     | Extensive Margin (Links) |                  |                      | Intensive Margin (Amount) |                  |                          |
|--------------------------------|--------------------------|------------------|----------------------|---------------------------|------------------|--------------------------|
|                                | 1911                     | 1922             | Difference           | 1911                      | 1922             | Difference               |
| New York City                  | 0.083<br>(0.225)         | 0.088<br>(0.225) | 0.005<br>(0.032)     | 0.075<br>(0.216)          | 0.085<br>(0.227) | 0.011<br>(0.030)         |
| Baltimore                      | 0.128<br>(0.303)         | 0.074<br>(0.235) | -0.054<br>(0.036)    | 0.132<br>(0.312)          | 0.071<br>(0.234) | -0.062*<br>(0.036)       |
| Washington, DC                 | 0.020<br>(0.122)         | 0.017<br>(0.118) | -0.003<br>(0.017)    | 0.019<br>(0.119)          | 0.016<br>(0.117) | -0.003<br>(0.016)        |
| Richmond                       | 0.363<br>(0.428)         | 0.213<br>(0.351) | -0.151***<br>(0.052) | 0.367<br>(0.438)          | 0.211<br>(0.355) | -<br>0.156***<br>(0.052) |
| Reserve Cities in Other States | 0.030<br>(0.137)         | 0.042<br>(0.177) | 0.013<br>(0.023)     | 0.029<br>(0.137)          | 0.042<br>(0.175) | 0.014<br>(0.022)         |
| Country Banks in VA            | 0.341<br>(0.422)         | 0.536<br>(0.436) | 0.195***<br>(0.060)  | 0.320<br>(0.419)          | 0.505<br>(0.446) | 0.185***<br>(0.060)      |
| Country Banks in Other States  | 0.034<br>(0.146)         | 0.031<br>(0.150) | -0.004<br>(0.021)    | 0.019<br>(0.116)          | 0.029<br>(0.147) | 0.009<br>(0.019)         |
| Obs.                           | 59                       | 160              |                      | 59                        | 160              |                          |

provision of liquidity by New York City banks. These results are consistent with the findings of Jaremski and Wheelock (2020) who documented the concentration of correspondent linkages (extensive margin) at cities with regional Federal Reserve Banks.<sup>28</sup>

To summarize, the introduction of liquidity provided by a central bank changed the nature and structure of the interbank system for the banking system that operated outside the Federal Reserve System. The founding of the Federal Reserve System increased banks' ability to borrow short-term funds. The ability of nonmember banks to indirectly access public liquidity through their local correspondents reduced the need for nonmember banks to rely on banks in different regions to obtain liquidity. The ability of nonmember banks to borrow locally eliminated the role of New York City banks as the ultimate liquidity provider. The shift of correspondent relationships away from New York and toward local banks transformed what had been a *national core-periphery structure* based in New York City into a *regional core-periphery structure* based in reserve cities.

<sup>28</sup>They focus on changes in correspondent linkages (not differentiating the type of networks) before and after the founding of the Fed and find that rural increased their correspondent links to nearby cities with Federal Reserve offices, suggesting that banks did find some advantages in connecting to nearby correspondents.

Table 10: Distance between Respondent and Correspondent Banks, All Banks

|                  | Due-froms        |                  |                      | Short-term borrowing |                  |                 |
|------------------|------------------|------------------|----------------------|----------------------|------------------|-----------------|
|                  | 1911             | 1922             | Difference           | 1911                 | 1922             | Difference      |
| Longest Distance | 293.5<br>(151.2) | 213.9<br>(422.6) | -79.7**<br>(31.1)    | 144.0<br>(146.1)     | 162.9<br>(573.9) | 19.0<br>(75.7)  |
| Mean Distance    | 131.6<br>(74.0)  | 114.7<br>(405.1) | -16.9<br>(29.0)      | 101.4<br>(101.3)     | 130.4<br>(568.5) | 29.0<br>(74.6)  |
| Total Distance   | 638.2<br>(686.1) | 366.9<br>(556.8) | -271.31***<br>(55.2) | 247.1<br>(292.2)     | 219.8<br>(597.3) | -27.3<br>(81.2) |
| Obs.             | 200              | 315              |                      | 59                   | 160              |                 |

## 5 Conclusion

Academics and policymakers have long debated how the provision of public liquidity affects the financial system. Yet, it is difficult to answer this question due to the complexity of modern banking and the ubiquitous presence of central banking. We have examined this question by exploring how the creation of the Federal Reserve System changed the banking system. In 1913 the Federal Reserve Act was passed to provide liquidity to member banks. While this public insurance came at the social cost of taxation, it brought the benefits of regulating and supervising members. Using unique data on individual payments and funding relationships of Virginia banks, we show the introduction of the Federal Reserve System changed the nature and anatomy of the interbank system.

It changed the nature because nonmember banks borrowed from member banks to indirectly access the discount window. Hence, to manage liquidity, banks became more dependent on short-term loans and less dependent on correspondent deposits, transforming a payment network into a funding network. This change makes the system more vulnerable to wholesale funding runs and financial contagion. This change also modified individual banks portfolios, which become less liquid, with less cash and interbank deposits. This change increased also the overall vulnerability of the banking system. It changes its anatomy by reducing the concentration of the interbank network, as banks were not as dependent of financial centers (with New York at the core) to manage liquidity risk. This change crowded out private inter-regional insurance.

All these changes point towards an accumulation of systemic risk, which may have exacerbated the severity of banking crises through interbank borrowing during the Great Depres-

sion. Since the Federal Reserve System played a passive role during the crisis, at least in earlier stages, implementing restrictive monetary policy and not intervening to stabilize the banking system, member banks may have been forced to reduce short-term loans to exposed nonmember banks, which would have to respond by withdrawing their interbank balances at member banks in financial centers. More research should be conducted to examine the relationship between interbank borrowing and bank failures during the Great Depression.

Our study has important implications for policy today. Financial regulations following the financial crisis of 2007-2009 attempt to prevent non-bank financial institutions from accessing public liquidity. As our results show, restricting “official” access to public liquidity does not prevent “real” access to public liquidity, creating a landscape favorable to the flourishing of shadow banks that operate with illiquid assets and connect in ways that induce systemic risk.

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# A Aggregate Balance Sheet Information

In the main text, we provide summary statistics of the balance sheet data aggregated at the state level. In Figure A1, we plot the movement of balance sheet ratios from 1910 to 1929. Figure A1 shows that in the 1920s, short-term borrowing increased and liquid assets declined.

Figure A1: Aggregate Balance Sheet Ratios, 1910-1929

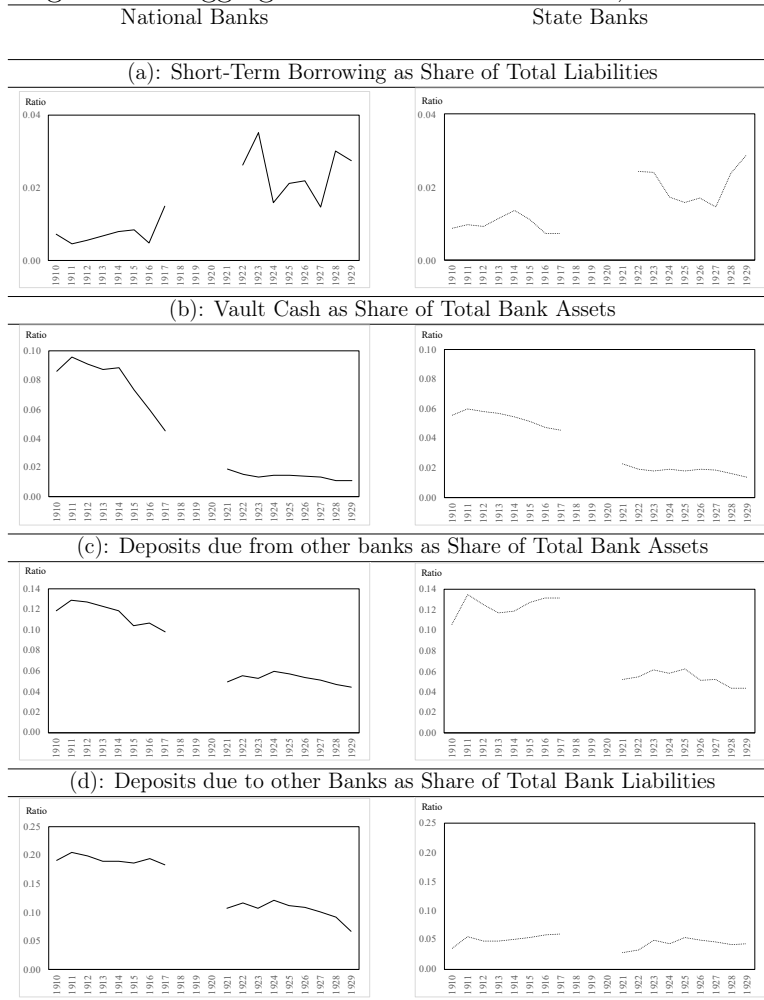


Figure A1 plots the ratio of short-term borrowing to total liabilities for national and state banks. All data are aggregated by the OCC: data for national banks are aggregated across all states, 17 reserve cities, and 3 central-reserve cities; data for state banks are aggregated across all states, all reserve cities, and reserve cities.

Source: *Annual Report of the Comptroller of the Currency*.

In addition, we check the robustness of our findings by restricting the data in two dimensions. First, we restrict our sample using state bank participation rate. As shown in Figure ??, states with financial and manufacturing sectors displayed a higher proportion of state bank membership than agricultural states. Given the irregular geographic distribution of membership, one might be concerned that the described changes were generated by state

member banks and that therefore our classifying all state banks as nonmembers clutters the analysis. To alleviate this concern, we restrict our sample and compare the asset composition of member and nonmember banks only in states where the membership ratio of state banks was under 10% in 1920.

Second, we restrict our sample using state-level reserve requirements. Changes in the liquidity of the state banking system might be driven by changes in reserve requirements by state regulators rather than by voluntary liquidity changes. To rule out this possibility, we divided states into three groups: (1) states that decreased their reserve requirements, (2) states that increased their reserve requirements, and (3) states that did not change their reserve requirements. Between 1910 and 1929, 22 states reduced reserve requirements, 10 states increased reserve requirements, and 16 states kept reserve requirements unchanged.<sup>29</sup>

For states where the state bank participation rate was below 10%, Figure A2 plots the fraction of total assets that state banks in those states held in borrowing, cash, and interbank deposits. In all cases, and regardless of the change in reserve requirements, nonmember banks reduced cash and interbank deposits and increased borrowing after the Federal Reserve came into existence (in 1914).

To summarize, we find that the existence of the Federal Reserve reduced liquidity (in the form of cash and interbank deposits) and intensified interbank relations (in the form of higher short-term borrowing) for both member and nonmember banks. Furthermore, member banks significantly reduced their relations with other member banks, but not their relations with nonmember banks. These factors suggest less private cross-insurance but still exposure to withdrawals, which contributed to the possibility of more contagion and greater vulnerability of the financial system.

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<sup>29</sup>See White (2014) for information on state reserve requirements. We classify CA, DE, GA, IN, KS, KY, LA, MI, MN, MT, NM, NY, OK, OR, PA, SD, TX, VA, WA, WI, WV as states with decreasing reserve requirements. In addition, we classify AR, CO, IA, MD, MS, NH, SC, TN, VT, WY as states with increasing reserve requirements. Last, we classify AL, CT, FL, ID, IL, MA, ME, MO, NC, ND, NE, NJ, NV, OH, OK, UT as states that did not change reserve requirements.

Figure A2: Bank Liquidity and Changes in State-Level Reserve Requirements, 1910-1929

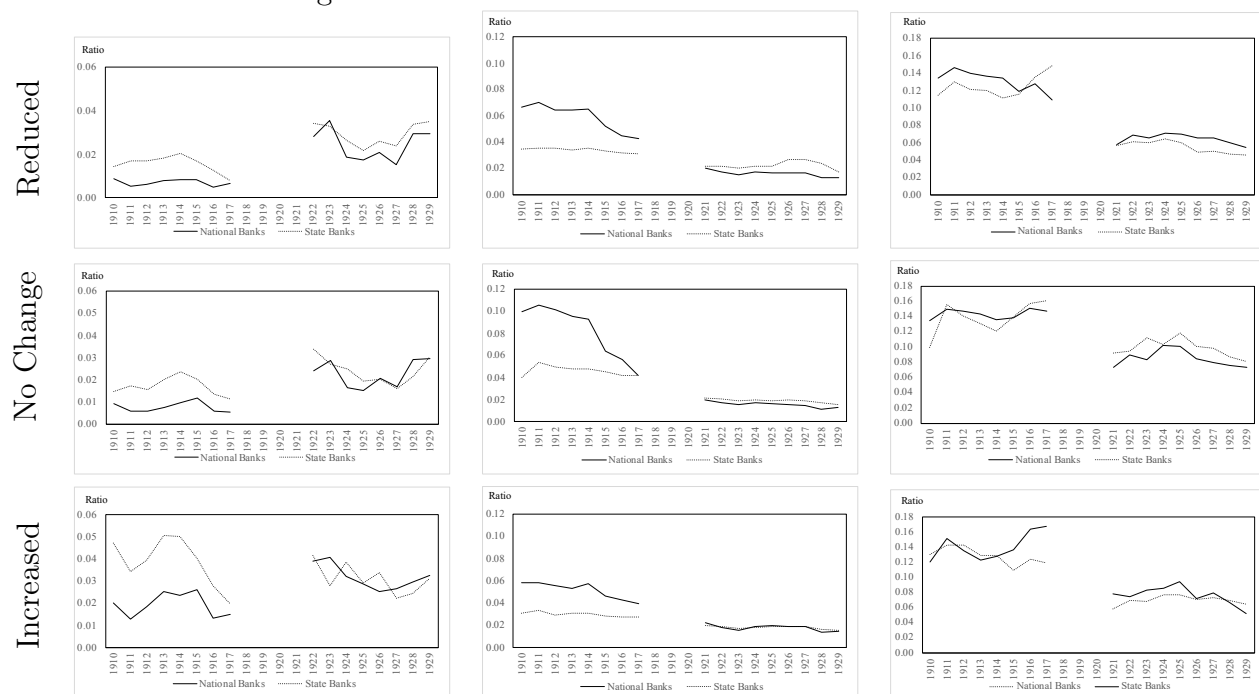


Figure A2 the share of short-term borrowing against total liabilities, the share of vault cash against total assets, and the share of deposits due from other banks against total assets for states with different reserve requirements. Data are further restricted for states where the Federal Reserve membership ratio of state banks was under 10% in 1920. All data are aggregated by the OCC: data for national banks are aggregated across all states, 17 reserve cities, and 3 central-reserve cities; data for state banks are aggregated across all states, all reserve cities, and reserve cities.

Source: *Annual Report of the Comptroller of the Currency*.

## B Virginia State Bank Examination Reports

In Figure B3 we present images of representative pages in the state bank examination reports used for this study. The reports provide information on three types of interbank relationships: on the asset side of the balance sheet, the amounts due from other banks by individual debtor banks; on the liability side of the balance sheet, the amounts due to other banks by individual creditor banks; and the amounts of borrowed money and the provider of these short-term loans. In some cases, the reports provide information on collateral used for securing short-term funds.

Figure B3: Virginia State Bank Examination Reports

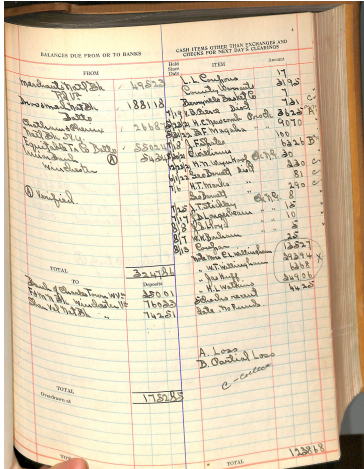
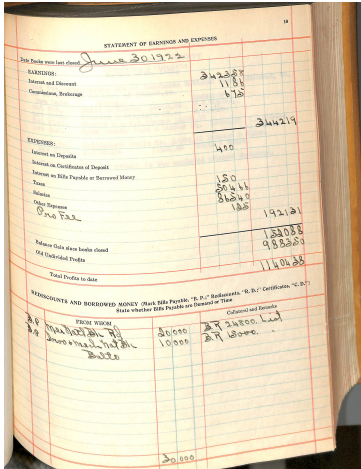
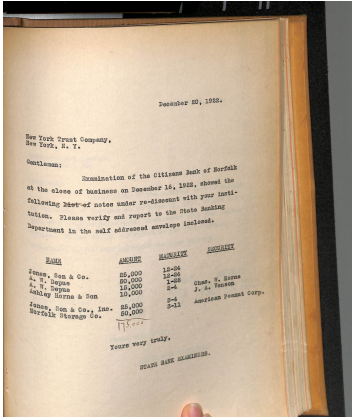
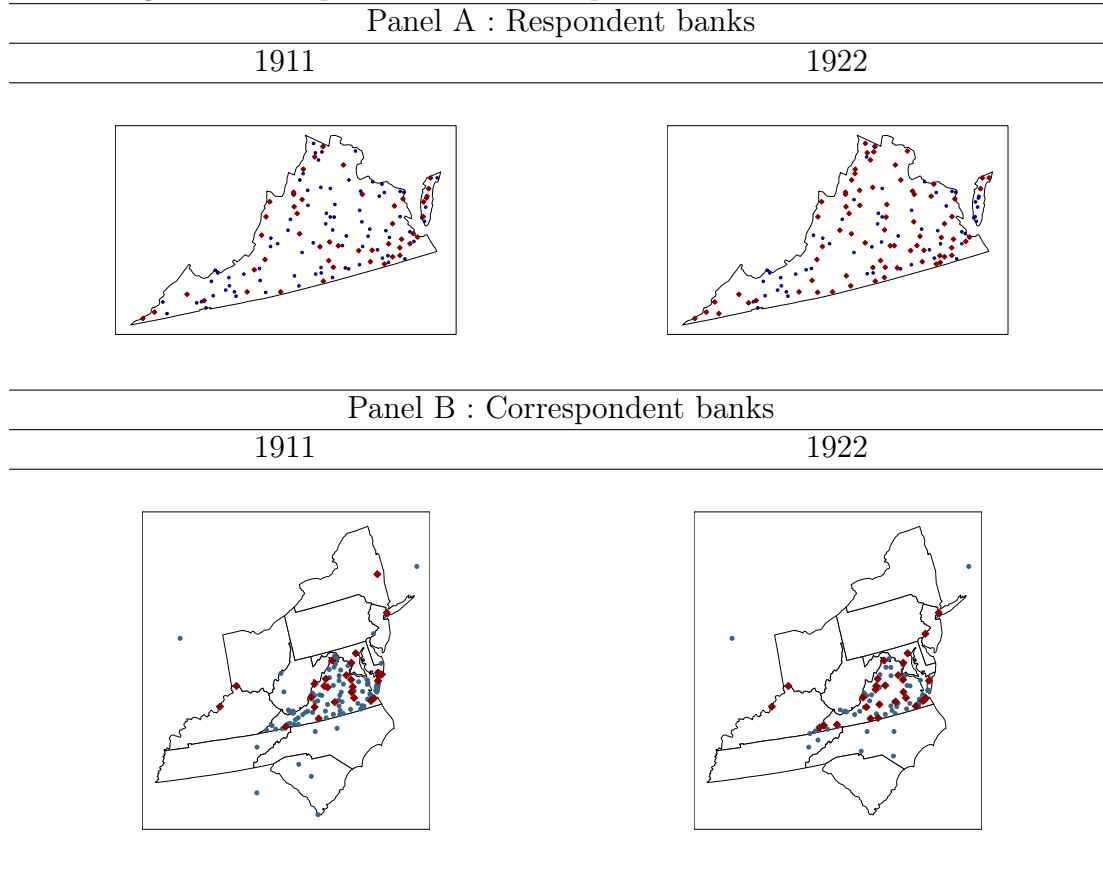
| Interbank Deposits   | Short-term Borrowing   | Collateral for Borrowing   |
|--|--|--|
|  |  |  |

Figure B4: Respondent and Correspondent Banks, 1911 and 1922



*Notes:* Figure B4 maps all respondent banks (Virginia state banks) and correspondent banks for the years 1911 and 1922. The respondent (corresponding) banks that *only* placed (received) deposits are in blue, while banks that *both* placed (received) deposits and borrowed (lent) short-term funds are in red.

*Source:* *Virginia State Bank Examination Reports*.

Table B1: Distribution of "Due from" Deposits, Banks in Both 1911 and 1922

|                                | Extensive Margin (Links) |                       |                 | Intensive Margin (Amount) |                       |                 |
|--------------------------------|--------------------------|-----------------------|-----------------|---------------------------|-----------------------|-----------------|
|                                | (1)                      | (2)                   | (3)             | (4)                       | (5)                   | (6)             |
|                                | 1911                     | 1922                  | Difference      | 1911                      | 1922                  | Difference      |
| New York City                  | 0.195<br>(0.189)         | 0.176<br>(0.171)      | 0.02<br>0.021   | 0.102<br>(0.153)          | 0.0980<br>(0.151)     | 0.004<br>0.018  |
| Chicago                        | 0.000298<br>(0.00360)    | 0.000360<br>(0.00436) | 0<br>0.001      | 0.000315<br>(0.00380)     | 0.000111<br>(0.00134) | 0<br>0.001      |
| Baltimore                      | 0.0964<br>(0.194)        | 0.0966<br>(0.193)     | -0.001<br>0.022 | 0.112<br>(0.246)          | 0.0957<br>(0.219)     | 0.016<br>0.027  |
| Washington, DC                 | 0.0228<br>(0.0864)       | 0.0135<br>(0.0611)    | 0.009<br>0.009  | 0.0130<br>(0.0631)        | 0.00597<br>(0.0472)   | 0.007<br>0.007  |
| Richmond                       | 0.212<br>(0.195)         | 0.234<br>(0.240)      | -0.022<br>0.026 | 0.285<br>(0.321)          | 0.325<br>(0.339)      | -0.041<br>0.039 |
| Reserve Cities in Other States | 0.0191<br>(0.0666)       | 0.0279<br>(0.106)     | -0.009<br>0.011 | 0.0221<br>(0.0845)        | 0.0372<br>(0.151)     | -0.015<br>0.015 |
| Country Banks in VA            | 0.425<br>(0.289)         | 0.426<br>(0.301)      | -0.002<br>0.035 | 0.430<br>(0.370)          | 0.422<br>(0.372)      | 0.009<br>0.044  |
| Country Banks in Other States  | 0.0295<br>(0.118)        | 0.0256<br>(0.0780)    | 0.004<br>0.012  | 0.0334<br>(0.158)         | 0.00838<br>(0.0453)   | 0.025*<br>0.014 |
| Obs.                           | 146                      | 146                   |                 | 146                       | 146                   |                 |

*Notes:* Rows indicate the location of correspondent banks. New York was a central reserve city. Baltimore and Washington, DC, were reserve cities. Richmond was not a reserve city in 1911 but was one by 1922. Columns indicate the location of respondent banks. Extensive margins are the proportions of links in each location against total links. Intensive margins are proportions of correspondent deposits held at different locations against total due-from deposits. Standard deviations in parentheses.

*Source:* Virginia State Bank Examination Reports.

Table B2: Distribution of "Due from" Deposits, Incumbents vs. New Entrants, 1922

|                                | Extensive Margin (Links) |                   |                    | Intensive Margin (Amount) |                   |                    |
|--------------------------------|--------------------------|-------------------|--------------------|---------------------------|-------------------|--------------------|
|                                | (1)                      | (2)               | (3)                | (4)                       | (5)               | (6)                |
|                                | Existing Bank            | New Bank          | Difference         | Existing Bank             | New Bank          | Difference         |
| New York City                  | 0.176<br>(0.171)         | 0.0848<br>(0.151) | 0.091***<br>0.018  | 0.0980<br>(0.151)         | 0.0451<br>(0.124) | 0.053***<br>0.016  |
| Chicago                        | 0.000360<br>(0.00436)    | 0<br>(0)          | 0.001<br>0.001     | 0.000111<br>(0.00134)     | 0<br>(0)          | 0<br>0             |
| Baltimore                      | 0.0966<br>(0.193)        | 0.0465<br>(0.135) | 0.05***<br>0.018   | 0.0957<br>(0.219)         | 0.0483<br>(0.176) | 0.048**<br>0.022   |
| Washington, DC                 | 0.0135<br>(0.0611)       | 0.0210<br>(0.122) | -0.007<br>0.011    | 0.00597<br>(0.0472)       | 0.0243<br>(0.141) | -0.018<br>0.012    |
| Richmond                       | 0.234<br>(0.240)         | 0.213<br>(0.307)  | 0.021<br>0.032     | 0.325<br>(0.339)          | 0.226<br>(0.347)  | 0.1**<br>0.039     |
| Reserve Cities in Other States | 0.0279<br>(0.106)        | 0.0400<br>(0.158) | -0.012<br>0.016    | 0.0372<br>(0.151)         | 0.0431<br>(0.182) | -0.006<br>0.019    |
| Country Banks in VA            | 0.426<br>(0.301)         | 0.564<br>(0.368)  | -0.138***<br>0.039 | 0.422<br>(0.372)          | 0.590<br>(0.420)  | -0.168***<br>0.045 |
| Country Banks in Other States  | 0.0256<br>(0.0780)       | 0.0304<br>(0.122) | -0.005<br>0.012    | 0.00838<br>(0.0453)       | 0.0250<br>(0.122) | -0.017<br>0.011    |
| Obs.                           | 146                      | 168               |                    | 146                       | 168               |                    |

*Notes:* Rows indicate the location of correspondent banks. New York was a central reserve city. Baltimore and Washington, DC, were reserve cities. Richmond was not a reserve city in 1911 but was one by 1922. Columns indicate the location of respondent banks. Extensive margins are the proportions of links in each location against total links. Intensive margins are proportions of correspondent deposits held at different locations against total due-from deposits. Standard deviations in parentheses.

*Source:* *Virginia State Bank Examination Reports.*



Table B3: Distribution of Borrowed Money, Banks in Both 1911 and 1922

|                                | Extensive Margin (Links) |                   |                 | Intensive Margin (Amount) |                   |                 |
|--------------------------------|--------------------------|-------------------|-----------------|---------------------------|-------------------|-----------------|
|                                | (1)                      | (2)               | (3)             | (4)                       | (5)               | (6)             |
|                                | 1911                     | 1922              | Difference      | 1911                      | 1922              | Difference      |
| New York City                  | 0.0885<br>(0.226)        | 0.140<br>(0.278)  | -0.051<br>0.047 | 0.0848<br>(0.226)         | 0.135<br>(0.280)  | -0.051<br>0.047 |
| Baltimore                      | 0.172<br>(0.357)         | 0.0908<br>(0.258) | 0.081<br>0.054  | 0.173<br>(0.358)          | 0.0875<br>(0.260) | 0.086<br>0.053  |
| Washington, DC                 | 0.0208<br>(0.144)        | 0.0181<br>(0.122) | 0.003<br>0.024  | 0.0200<br>(0.141)         | 0.0137<br>(0.110) | 0.007<br>0.022  |
| Richmond                       | 0.304<br>(0.405)         | 0.289<br>(0.387)  | 0.015<br>0.072  | 0.305<br>(0.416)          | 0.295<br>(0.396)  | 0.009<br>0.072  |
| Reserve Cities in Other States | 0.0365<br>(0.163)        | 0.0251<br>(0.129) | 0.012<br>0.026  | 0.0334<br>(0.159)         | 0.0309<br>(0.150) | 0.003<br>0.028  |
| Country Banks in VA            | 0.352<br>(0.417)         | 0.415<br>(0.422)  | -0.063<br>0.076 | 0.321<br>(0.409)          | 0.394<br>(0.425)  | -0.073<br>0.075 |
| Country Banks in Other States  | 0.0260<br>(0.148)        | 0.0221<br>(0.127) | 0.004<br>0.025  | 0.0227<br>(0.142)         | 0.0235<br>(0.132) | -0.001<br>0.024 |
| Obs.                           | 50                       | 84                |                 | 50                        | 84                |                 |

*Notes:* Rows indicate the location of correspondent banks. Extensive margins provide information on the proportions of links in each location against total links. Intensive margins provide information on the proportions of borrowed money from correspondents at different locations against total borrowed money. Standard deviations in parentheses.

*Source:* *Virginia State Bank Examination Reports.*

Table B4: Distribution of Borrowed Money, Incumbents vs. New Entrants, 1922

|                                | Extensive Margin (Links) |                   |                    | Intensive Margin (Amount) |                   |                    |
|--------------------------------|--------------------------|-------------------|--------------------|---------------------------|-------------------|--------------------|
|                                | (1)                      | (2)               | (3)                | (4)                       | (5)               | (6)                |
|                                | Existing Bank            | New Bank          | Difference         | Existing Bank             | New Bank          | Difference         |
| New York City                  | 0.140<br>(0.278)         | 0.0344<br>(0.135) | 0.106***<br>0.035  | 0.135<br>(0.280)          | 0.0358<br>(0.144) | 0.1***<br>0.034    |
| Baltimore                      | 0.0908<br>(0.258)        | 0.0566<br>(0.208) | 0.034<br>0.036     | 0.0875<br>(0.260)         | 0.0539<br>(0.205) | 0.034<br>0.036     |
| Washington, DC                 | 0.0181<br>(0.122)        | 0.0154<br>(0.114) | 0.003<br>0.018     | 0.0137<br>(0.110)         | 0.0186<br>(0.125) | -0.005<br>0.018    |
| Richmond                       | 0.289<br>(0.387)         | 0.134<br>(0.292)  | 0.155***<br>0.053  | 0.295<br>(0.396)          | 0.128<br>(0.289)  | 0.167***<br>0.053  |
| Reserve Cities in Other States | 0.0251<br>(0.129)        | 0.0601<br>(0.215) | -0.035<br>0.028    | 0.0309<br>(0.150)         | 0.0537<br>(0.198) | -0.023<br>0.027    |
| Country Banks in VA            | 0.415<br>(0.422)         | 0.660<br>(0.418)  | -0.245***<br>0.066 | 0.394<br>(0.425)          | 0.615<br>(0.442)  | -0.221***<br>0.067 |
| Country Banks in Other States  | 0.0221<br>(0.127)        | 0.0391<br>(0.171) | -0.017<br>0.024    | 0.0235<br>(0.132)         | 0.0334<br>(0.162) | -0.01<br>0.022     |
| Obs.                           | 83                       | 81                |                    | 84                        | 85                |                    |

*Notes:* Rows indicate the location of correspondent banks. Extensive margins provide information on the proportions of links in each location against total links. Intensive margins provide information on the proportions of borrowed money from correspondents at different locations against total borrowed money. Standard deviations in parentheses.

*Source:* *Virginia State Bank Examination Reports.*

# ONLINE APPENDIX

## C Remarks on the model and assumptions

**The size of the liquidity shocks.** We assume that the liquidity shock can exceed  $D$  so we do not deal with the corner solutions. In particular, the liquidity shock  $\zeta$  is 0 w.p.  $1 - \alpha$  and  $U[0, Z]$  w.p.  $\alpha$  where  $Z > D$ . The story is as follows. There are legacy assets and liabilities.  $M$  captures the sum of legacy liabilities and  $K$  captures the sum of returns from illiquid legacy assets. These are safe but the return time for legacy assets and withdrawal time for legacy liabilities are random.  $K \geq M$  so there is no solvency issue. There can be an illiquidity issue. At the time of the liquidity shock, if the return so far from legacy assets is  $k$  and the amount of legacy liabilities realized so far is  $m$ , and the realized liquidity withdrawal from depositors (who have seniority) is  $d \in [0, D]$  then the actual liquidity need at the time of the liquidity shock is  $l = d + m - k$ . We assume that  $l$  has distribution  $U[-K, D + M]$ . Now denote  $\alpha = \frac{D+M}{D+M+K}$  and  $Z = D + M$ . Then  $l \leq 0$  w.p.  $1 - \alpha$  and  $l \sim U[0, Z]$  w.p.  $\alpha$ . Now let  $\zeta = l_+$  the private liquidity need. (We use the notation  $z_+ = \max\{z, 0\}$ .) Then  $\zeta = 0$  w.p.  $1 - \alpha$  and  $U[0, Z]$  w.p.  $\alpha$ . When there is central bank liquidity  $m$ , the effect of  $m$  will be to make the private liquidity need  $(\zeta - m)_+$ .

**Notation.** Going forward, the fundamentals of the model are  $r_x, r_y, r$  for the return rates,  $\alpha, Z, \zeta$ , for shocks,  $D, m$  for liquidity. Denote  $Z_{\alpha, m} = \frac{Z(1-\alpha)}{\alpha} + m$ . For a random variable  $X$ ,  $F_X$  denotes its CDF. Also,  $f \underset{z}{\propto} g$  means that  $f$  and  $g$  are monotone transformations of each other as functions of  $z$ .

**Discussion of parametric assumptions.** We will take  $Z$  to be large enough compared to  $D$  and  $m$  in order to avoid corner issues in the algebra. In particular,  $Z > m + D$  so that even the entire liquidity in the system may not suffice, although this event has small probability. This way, we do not need to worry about cumbersome corner solutions in the algebra. This, in a way, “convexifies” the problem.

**Assumption 1.**  $0 \leq m \leq Z - D$ .

Also, for technical reasons and for the simplicity of algebra, we will restrict attention to  $\alpha$  that is not too large.

**Assumption 2.**  $\alpha \leq \bar{\alpha} = \frac{Z}{Z + \rho D}$  where

$$\rho = \max \left\{ 0, \frac{2}{2 - \phi} \left( 2(1 - \phi) - \frac{r}{r_y} \right) - 1 \right\}$$

The major role of this assumption is to make sure that the reserve requirements bind and  $\Phi_y = \phi L$ . Finally, we assume that  $r_x$  and  $r_y$  are relatively large compared to  $r$ .

**Assumption 3.**  $(1 - \phi)r_x > 2r$  and  $(1 - \phi)r_y > r$ .

The condition on  $r_y$  is innocuous. If  $(1 - \phi)r_y$  were less than  $r$ ,  $y$  would not borrow. The condition on  $r_x$  deserves some discussion. One might think, at first, that by  $r_x > r$ , bank  $x$ 's own project is a better investment than the “interbank investment” of lending to  $y$ . Since each investments provide buffer against liquidation of the other, each investment would be non-zero under sufficiently high risk. But by  $r_x > r$ ,  $I_x$  would be larger than  $L$ . But this simple logic is missing a critical point. Bank  $y$  pays interest on the full loan  $L$ , not the investment size  $I_y$ . At least  $\phi L$  is kept by  $y$  as reserves, which is a source of short term liquidity for  $x$  at the time of shocks. That is, interbank investment has an extra benefit above and beyond its investment value and diversification value. This complicates proofs. For this reason we make a simplifying assumption  $(1 - \phi)r_x > 2r$  that makes sure there is a pecking order: first priority is the project of bank  $x$ , then the interbank investment.

## D Proofs

*Proof.* (**Proof of Lemma 1**)

Here we provide a general proof that allows for public liquidity  $m \geq 0$ . The proof of Lemma 1 can be obtained by replacing  $m$  with 0 below. It is easy to see that for a given portfolio profile  $(I_x, L, I_y)$  and a level of liquidity shortage  $\zeta' = (\zeta - m - \Phi_x - \Phi_y)_+$ , liquidations induced by the optimal behavior of  $x$  at the liquidation stage is given by

- If  $\zeta' = 0$ , nothing is liquidated.
- If  $0 < \zeta' \leq \min\{I_x, I_y\}$ , then
  - If  $Lr \leq I_x r_x$ , then  $I_y$  is liquidated.
  - If  $Lr > I_x r_x$ , then  $I_x$  is liquidated.
- If  $\min\{I_x, I_y\} < \zeta' \leq \max\{I_x, I_y\}$ , then  $\max\{I_x, I_y\}$  is liquidated.
- If  $\max\{I_x, I_y\} < \zeta'$ , then both  $I_x$  and  $I_y$  are liquidated.

Then  $I_y$  does not get liquidated iff one of the following hold:

- $\zeta' = 0$
- $0 < \zeta' \leq \min\{I_x, I_y\}$  and  $Lr > I_x r_x$
- $I_y < \zeta' \leq I_x$ .

Then the expected profit of  $y$  is

$$\begin{aligned}\Pi_y(I_y) &= (F_{\zeta'}(0) + 1_{Lr > I_x r_x} (F_{\zeta'}(\min\{I_x, I_y\}) - F(0)) + 1_{I_x > I_y} (F_{\zeta'}(I_x) - F_{\zeta'}(I_y))) (I_y r_y - Lr) \\ &= \frac{\alpha}{Z} r_y \times \begin{cases} u_1(I_y) := \left(I_y - \frac{Lr}{r_y}\right) (Z_{\alpha, m} + D - I_y) & \text{if } Lr > I_x r_x \\ u_2(I_y) := \left(I_y - \frac{Lr}{r_y}\right) (Z_{\alpha, m} + D - 2I_y) & \text{on } I_x \geq I_y \text{ if } Lr \leq I_x r_x \\ u_3(I_y) := \left(I_y - \frac{Lr}{r_y}\right) (Z_{\alpha, m} + D - I_x - I_y) & \text{on } I_x \leq I_y \text{ if } Lr \leq I_x r_x \wedge I_x \leq L(1 - \phi) \end{cases}\end{aligned}$$

All of  $u_1, u_2, u_3$  are concave quadratics. They are increasing up to their unique unconstrained arg max and decreasing afterwards. The unconstrained arg max of  $u_1, u_2, u_3$  are given by

$$\begin{aligned}I_1^* &= \frac{1}{2} \left( Z_{\alpha, m} + D + \frac{Lr}{r_y} \right) \\ I_2^* &= \frac{1}{2} \left( \frac{Z_{\alpha, m} + D}{2} + \frac{Lr}{r_y} \right) \\ I_3^* &= \frac{1}{2} \left( Z_{\alpha, m} + D - I_x + \frac{Lr}{r_y} \right)\end{aligned}$$

Then  $I_y^* = \arg \max \Pi_y(I_y)$  in these three regions are given by

$$\Pi_y(I_y^*) = \frac{\alpha}{Z} \begin{cases} u_1(\min\{L(1 - \phi), I_1^*\}) & \text{if } Lr > I_x r_x \\ u_2(\min\{L(1 - \phi), I_x, I_2^*\}) & \text{on } I_x \geq I_y \text{ if } Lr \leq I_x r_x \\ u_3(\max\{I_x, \min\{L(1 - \phi), I_3^*\}\}) & \text{on } I_x \leq I_y \text{ if } Lr \leq I_x r_x \wedge I_x \leq L(1 - \phi) \end{cases}$$

By Assumption 2, we have  $L(1 - \phi) \leq I_3^*$ . Also clearly  $I_3^* \leq I_1^*$ . Then

$$\min\{L(1 - \phi), I_1^*\} = \max\{I_x, \min\{L(1 - \phi), I_3^*\}\} = L(1 - \phi)$$

Then

$$\Pi_y(I_y^*) = \frac{\alpha}{Z} \begin{cases} u_1(L(1-\phi)) & \text{if } Lr > I_x r_x \\ u_2(\min\{L(1-\phi), I_2^*\}) & \text{on } I_x \geq I_y \text{ if } Lr \leq I_x r_x \wedge I_x \geq L(1-\phi) \\ u_2(\min\{I_x, I_2^*\}) & \text{on } I_x \geq I_y \text{ if } Lr \leq I_x r_x \wedge I_x \leq L(1-\phi) \\ u_3(L(1-\phi)) & \text{on } I_x \leq I_y \text{ if } Lr \leq I_x r_x \wedge I_x \leq L(1-\phi) \end{cases}$$

For the first case [if  $Lr > I_x r_x$ ],  $I_y^* = L(1-\phi)$ . For the second case [on  $I_x \geq I_y$  if  $Lr \leq I_x r_x \wedge I_x \geq L(1-\phi)$ ] note that  $I_x \geq L(1-\phi)$  implies both  $I_x \geq I_y$  and  $Lr \leq I_x r_x$ . So this case can be restated as simply  $[I_x \geq L(1-\phi)]$ . For the third and fourth cases jointly, we compare  $u_2(\min\{I_x, I_2^*\})$  and  $u_3(L(1-\phi))$  under  $[Lr \leq I_x r_x \wedge I_x \leq L(1-\phi)]$ . Note that

$$u_2(I_2^*) = \frac{1}{2} \left( \frac{Z_{\alpha,m} + D}{2} - \frac{Lr}{r_y} \right)^2$$

$$u_3(L(1-\phi)) = \left( L(1-\phi) - \frac{Lr}{r_y} \right) (Z_{\alpha,m} + D - I_x - L(1-\phi))$$

Suppose  $I_2^* < L(1-\phi)$ . Then we have  $\frac{1}{2} \left( \frac{Z_{\alpha,m} + D}{2} + \frac{Lr}{r_y} \right) < L(1-\phi)$ , and so  $\frac{1}{2} \left( \frac{Z_{\alpha,m} + D}{2} - \frac{Lr}{r_y} \right) < L(1-\phi) - \frac{Lr}{r_y}$ . Also  $\left( \frac{Z_{\alpha,m} + D}{2} - \frac{Lr}{r_y} \right) < (Z_{\alpha,m} + D - I_x - L(1-\phi))$ . Thus,  $u_2(\min\{I_x, I_2^*\}) \leq u_2(I_2^*) < u_3(L(1-\phi))$ . Now suppose  $I_2^* \geq L(1-\phi)$ . Then by  $I_x \leq L(1-\phi)$  we have  $I_2^* \geq I_x$ . Then  $u_2(\min\{I_x, I_2^*\}) = u_2(I_x)$ . Recall that  $u_2(I_x) = u_3(I_x)$ ,  $L(1-\phi) \leq I_3^*$ , and  $u_3$  is increasing up to  $I_3^*$ . Then we have  $I_x \leq L(1-\phi) \leq I_3^*$  and  $u_3(I_x) \leq u_3(L(1-\phi)) \leq u_3(I_3^*)$ . Combining these we have  $u_2(\min\{I_x, I_2^*\}) = u_2(I_x) = u_3(I_x) \leq u_3(L(1-\phi))$ . So in general,  $u_2(\min\{I_x, I_2^*\}) \leq u_3(L(1-\phi))$  and  $I_y^* = L(1-\phi)$  in the union of third and fourth cases, i.e.  $[Lr \leq I_x r_x \wedge I_x \leq L(1-\phi)]$ . Therefore,

$$I_y^* = \begin{cases} \min\{L(1-\phi), I_2^*\} & \text{if } I_x > L(1-\phi) \\ L(1-\phi) & \text{otherwise} \end{cases}$$

Under  $I_x > L(1-\phi)$  and Assumption 2, we have  $L(1-\phi) \leq I_2^*$  and so  $I_y^* = L(1-\phi)$ .

Next consider the optimal portfolio of  $x$ . Let  $(I_x, L)$  be optimal and suppose that  $I_x < L(1-\phi)$ . Then the expected profit of  $x$  is

$$\begin{aligned} \Pi_x = & F_{\zeta'}(0) (I_x r_x + Lr) \\ & + (F_{\zeta'}(I_x) - F_{\zeta'}(0)) \max\{Lr, I_x r_x\} \\ & + (F_{\zeta'}(L(1-\phi)) - F_{\zeta'}(I_x)) I_x r_x \end{aligned}$$

$$\begin{aligned} & \underset{(I_x, L)}{\infty} (Z_{\alpha, m} + D - I_x - L(1 - \phi)) (I_x r_x + Lr) \\ & + I_x \max \{Lr, I_x r_x\} + (L(1 - \phi) - I_x) I_x r_x \end{aligned}$$

By  $I_x < L(1 - \phi)$ , right partial derivative w.r.t.  $I_x$  must be negative and left partial derivative w.r.t.  $L$  must be positive. If  $I_x r_x \neq Lr$ , these derivatives are given by the following: The F.O.C. w.r.t.  $I_x$  is

$$\begin{aligned} 0 & \geq - (I_x r_x + Lr) + r_x (Z_{\alpha, m} + D - I_x - L(1 - \phi)) \\ & + \begin{cases} Lr & \text{if } Lr > I_x r_x \\ 2I_x r_x & \text{if } Lr < I_x r_x \end{cases} \\ & + (L(1 - \phi) - 2I_x) r_x \\ & = r_x (Z_{\alpha, m} + D - 2I_x) - \begin{cases} 2I_x r_x & \text{if } Lr > I_x r_x \\ Lr & \text{if } Lr < I_x r_x \end{cases} \\ & \implies Z_{\alpha, m} + D \leq 2I_x + \begin{cases} 2I_x & \text{if } Lr > I_x r_x \\ \frac{Lr}{r_x} & \text{if } Lr < I_x r_x \end{cases} \end{aligned}$$

The F.O.C. w.r.t. to  $L$  is

$$\begin{aligned} 0 & \leq - (1 - \phi) (I_x r_x + Lr) + r (Z_{\alpha, m} + D - I_x - L(1 - \phi)) \\ & + \begin{cases} I_x r & \text{if } Lr > I_x r_x \\ 0 & \text{if } Lr < I_x r_x \end{cases} \\ & + (1 - \phi) I_x r_x \\ & = r (Z_{\alpha, m} + D - I_x - 2L(1 - \phi)) \\ & + \begin{cases} I_x r & \text{if } Lr > I_x r_x \\ 0 & \text{if } Lr < I_x r_x \end{cases} \\ & \implies Z_{\alpha, m} + D \geq I_x + 2L(1 - \phi) - \begin{cases} I_x r & \text{if } Lr > I_x r_x \\ 0 & \text{if } Lr < I_x r_x \end{cases} \end{aligned}$$

Combining the two, we get

$$2I_x + \begin{cases} 2I_x & \text{if } Lr > I_x r_x \\ \frac{Lr}{r_x} & \text{if } Lr < I_x r_x \end{cases}$$

$$\begin{aligned} &\geq I_x + 2L(1 - \phi) - \begin{cases} I_x r & \text{if } Lr > I_x r_x \\ 0 & \text{if } Lr < I_x r_x \end{cases} \\ \implies 0 &\leq I_x - 2L(1 - \phi) + \begin{cases} 3I_x & \text{if } Lr > I_x r_x \\ \frac{Lr}{r_x} & \text{if } Lr < I_x r_x \end{cases} \end{aligned}$$

Under  $Lr < I_x r_x$ , we get

$$0 \leq I_x - 2L(1 - \phi) + \frac{Lr}{r_x} < I_x - 2L(1 - \phi) + I_x < 0$$

So we must have  $Lr > I_x r_x$ . Then  $2I_x \geq L(1 - \phi)$ . But then  $2I_x r_x \geq L(1 - \phi)r_x > 2Lr$ , by Assumption 3. Hence  $I_x r_x > Lr$ . This is a contradiction.

So we must have  $I_x r_x = Lr$ . This implies that  $I_x \neq 0$ . Then the right partial derivative of the profit w.r.t.  $I_x$  must be negative and left partial derivative of the profit w.r.t.  $I_x$  must be positive. In particular, the right derivative is

$$r_x (Z_{\alpha, m} + D - 2I_x) - Lr$$

and the left derivative is

$$r_x (Z_{\alpha, m} + D - 2I_x) - 2I_x r_x$$

Then the left derivative is smaller than the right derivative. Contradiction. So the optimal portfolio satisfies  $I_x \geq L(1 - \phi) = L - \Phi_y$ . By  $(1 - \phi)r_x > r$  this further implies that  $I_x r_x > Lr$ .  $\square$

*Proof. (Proof of Proposition 1 and Proposition 2)*

Proposition 1 is simply a corollary of Proposition 2, obtained by replacing  $m$  with 0, so we provide the proof for Proposition 2. By the proof of Lemma 1 above, which allows for  $m \geq 0$ , we have  $\Phi_y = L\phi$  and  $I_x > L(1 - \phi)$ . Then the ex-post profit for bank  $x$  is given by

$$\pi_x = \begin{cases} I_x r_x + Lr & \text{if } 0 \leq \zeta \leq m + \Phi_x + L\phi \\ I_x r_x & \text{if } m + \Phi_x + L\phi < \zeta \leq m + \Phi_x + L \\ Lr & \text{if } m + \Phi_x + L < \zeta \leq m + \Phi_x + L\phi + I_x \\ 0 & \text{if } m + \Phi_x + L\phi + I_x < \zeta \end{cases}$$

The expected profit is



$$\Pi_x \underset{(I_x, L)}{\propto} r_x (Z_{\alpha, m} + D - I_x) I_x + 2(1 - \phi)r \left( \frac{Z_{\alpha, m} + D}{2(1 - \phi)} - L \right) L$$

The unconstrained maximizer is

$$L = \frac{Z_{\alpha, m} + D}{4(1 - \phi)}, \quad I_x = \frac{Z_{\alpha, m} + D}{2}$$

At these values,  $L, I_x \geq 0$  and  $I_x \geq L(1 - \phi)$  hold. The remaining constraint is

$$D \geq L + I_x \iff \alpha \geq \frac{Z}{Z + D \left( \frac{1-2\phi}{3-2\phi} \right) - m}$$

(Note that this lower bound is less than  $\bar{\alpha}$  for  $m = 0$  if  $\frac{r}{r_y} \geq \frac{2(1-2\phi)(1-\phi)}{3-2\phi}$ , which makes this region of parameters is non-empty for  $m = 0$ . This guarantees that the following regions are also non-empty for  $m = 0$ . As  $m$  grows, it is natural that some regions become obsolete in the pecking order.)

Next consider  $\alpha < \frac{Z}{Z + D \left( \frac{1-2\phi}{3-2\phi} \right) - m}$  ( $D \frac{1-2\phi}{3-2\phi} < Z_{\alpha, m}$ ). The constraint  $I_x + L \leq D$  binds. Under constraint  $I_x = D - L \in [0, D]$ , the FOC gives

$$\frac{d\Pi_x}{dL} = 0 \implies I_x = \frac{D(4(1 - \phi)r + r_x - r) + Z_{\alpha, m}(r_x - r)}{2(r_x + 2(1 - \phi)r)}$$

As  $r_x > r$  we have  $L \leq D$  and  $I_x \geq 0$ . On the other hand

$$L \geq 0 \iff \alpha \geq \frac{Z}{Z + D \frac{r_x + r}{r_x - r} - m}$$

This also ensures  $I_x \leq D$ . The last constraint  $I_x \geq L(1 - \phi)$  holds trivially.

Finally, under  $\alpha < \frac{Z}{Z + D \left( \frac{r_x + r}{r_x - r} \right) - m}$  ( $D \frac{r_x + r}{r_x - r} < Z_{\alpha, m}$ ), we have  $L = 0$  and  $I_x = D$ .

Summarizing:

1. If  $\bar{\alpha} \geq \alpha > \frac{Z}{Z + D \left( \frac{1-2\phi}{3-2\phi} \right) - m}$  ( $D \frac{1-2\phi}{3-2\phi} \geq Z_{\alpha, m}$ ),

$$I_x = \frac{D + Z_{\alpha, m}}{2}, \quad L = \frac{D + Z_{\alpha, m}}{4(1 - \phi)}, \quad \Phi_x = D - I_x - L > 0$$

2. If  $\frac{Z}{Z+D\left(\frac{1-2\phi}{3-2\phi}\right)-m} > \alpha > \frac{Z}{Z+D\left(\frac{r_x+r}{r_x-r}\right)-m}$  ( $D\frac{r_x+r}{r_x-r} > Z_{\alpha,m} > D\frac{1-2\phi}{3-2\phi}$ ) then

$$I_x = \frac{D(4(1-\phi)r + r_x - r) + Z_{\alpha,m}(r_x - r)}{2(r_x + 2(1-\phi)r)}$$

$$L = \frac{D(r_x + r) - Z_{\alpha,m}(r_x - r)}{2(r_x + 2(1-\phi)r)}, \quad \Phi_x = 0$$

3. If  $\frac{Z}{Z+D\left(\frac{r_x+r}{r_x-r}\right)-m} > \alpha$  ( $Z_{\alpha,m} > D\frac{r_x+r}{r_x-r}$ ), then

$$I_x = D, \quad L = 0, \quad \Phi_x = 0.$$

□

*Proof. (Proof of Proposition 3)* Note that there is some inconsequential multiplicity in the amount of ex-post short term borrowing. As the short-term borrowing is risk-free in the model, for simplicity, we have assumed away interest on it. For robustness, we assume the smallest amount of short-term borrowing to meet the shock takes place. If  $\zeta < \Phi_x$ , there is no need for short-term borrowing. For  $\Phi_x < \zeta \leq \Phi_x + \Phi_y + m$ ,  $y$  can lend the shortage  $\zeta - \Phi_x$  to  $x$  to avoid liquidations. If  $\zeta > \Phi_x + \Phi_y + m$ , liquidation is inevitable. If  $\Phi_x + L + m > \zeta > \Phi_x + \Phi_y + m$ ,  $x$  liquidates  $L$ . This gives  $L$  extra liquidity to  $x$  on top of its reserves  $\Phi_x$ . Bank  $x$  can still borrow  $m$  from  $y$  in this case. But if  $\zeta < L + \Phi_x$ ,  $x$  does not need to borrow from  $y$ . Only when  $\zeta > L + \Phi_x$ , there is borrowing from  $y$  at the amount of shortage  $\zeta - L - \Phi_x$ . Therefore, when  $\Phi_x + L + m > \zeta > \max\{\Phi_x + \Phi_y + m, L + \Phi_x\}$ , there is  $\zeta - L - \Phi_x$  borrowing. Continuing with the same logic, we find that the ex-post amount of short-term borrowing by  $x$  from  $y$  under  $m$  is given by

$$b = \begin{cases} \zeta - \Phi_x & \text{if } \Phi_x < \zeta \leq \Phi_x + \Phi_y + m \\ \zeta - L - \Phi_x & \text{if } \Phi_x + \max\{L, \Phi_y + m\} < \zeta \leq \Phi_x + L + m \\ \zeta - I_x - \Phi_x & \text{if } \Phi_x + \max\{I_x, L + m\} < \zeta \leq \Phi_x + I_x + \Phi_y + m \\ 0 & \text{otherwise} \end{cases}$$

The expectation of this w.r.t.  $\zeta$  is

$$B = \frac{\alpha}{Z} (2(m + \Phi_y)^2 + m^2 - \max\{0, m + \Phi_y - L\}^2 - \max\{0, m + L - I_x\}^2)$$

Under  $D^{\frac{1-2\phi}{3-2\phi}} \geq Z_{\alpha,m}$ , this is

$$B = \frac{\alpha}{2Z} (2(m + \Phi_y)^2 + m^2 - \max\{0, m - (1 - \phi)L\}^2 - \max\{0, m - (1 - 2\phi)L\}^2)$$

Note that  $D^{\frac{1-2\phi}{3-2\phi}} \geq Z_{\alpha,m}$  implies  $L = \frac{D+Z_{\alpha,m}}{4(1-\phi)} > \frac{Z_{\alpha,m}}{1-2\phi} > \frac{m}{1-2\phi}$ . So  $B = \frac{\alpha}{2Z} (2(m + \Phi_y)^2 + m^2)$  which is increasing in  $m$ .

For the case of  $D^{\frac{1-2\phi}{3-2\phi}} < Z_{\alpha,m}$ , note that  $B$  is continuous in  $m$ . Also, the negative terms  $\max\{0, m + \Phi_y - L\}$  and  $\max\{0, m + L - I_x\}$  are increasing in  $m$ . So if

$$2(m + \Phi_y)^2 + m^2 - (m + \Phi_y - L)^2 - (m + L - I_x)^2$$

is increasing in  $m$ , then  $B$  is increasing in  $m$ . The derivative of this expression w.r.t.  $m$  is 2 times

$$2(m + \Phi_y) \left(1 + \phi \frac{dL}{dm}\right) + m - (m + \Phi_y - L) \left(1 - (1 - \phi) \frac{dL}{dm}\right) - (m + L - I_x) \left(1 + \frac{dL}{dm} - \frac{dI_x}{dm}\right)$$

Under  $Z_{\alpha,m} > D^{\frac{r_x+r}{r_x-r}}$  this is

$$2(m + \Phi_y) + m - (m + \Phi_y - L) - (m + L - I_x) = m + \Phi_y + I_x > 0$$

Under  $D^{\frac{r_x+r}{r_x-r}} > Z_{\alpha,m} > D^{\frac{1-2\phi}{3-2\phi}}$  this is

$$\begin{aligned} & 2(m + \Phi_y) \left(1 + \phi \frac{dL}{dm}\right) + m - (m + \Phi_y - L) \left(1 - (1 - \phi) \frac{dL}{dm}\right) - (m + 2L - D) \left(1 + 2 \frac{dL}{dm}\right) \\ & > D \left(1 - \frac{r_x - r}{r_x + 2(1 - \phi)r}\right) + L \left(\frac{(5 - 2\phi)(r_x - r)}{2(r_x + 2(1 - \phi)r)} - (1 - \phi)\right) > 0 \end{aligned}$$

Thus,  $B$  is continuous and increasing.  $\square$

**Proof. (Proof of Proposition 4)**

Now suppose that  $m$  is independently drawn from distribution  $F_m$  with support  $[0, \bar{m}]$  and mean  $m^*$ . Assume  $\bar{m} < Z - D$ . In principle, stochastic  $m$  could complicate the algebra dramatically. But, as the shocks can always be larger than the shocks the all results regarding portfolios still hold with  $m^*$  instead of  $m$ . In order to formalize this, go back to the liquidations induced by the optimal behavior of  $x$  after the shock, as outlined in the proof of Lemma 1. The last region of the shock where both project are liquidated is given by

$\max\{I_x, I_y\} < \zeta' = \zeta - m - \Phi_x - \Phi_y$ . This is,  $\zeta > \max\{I_x, I_y\} + m + \Phi_x + \Phi_y$ . By  $\bar{m} < Z - D$ ,

$$\max\{I_x, I_y\} + m + \Phi_x + \Phi_y < \max\{I_x, I_y\} + Z - D + \Phi_x + \Phi_y < Z$$

Therefore, there is positive probability that both project get liquidated regardless of the portfolio. So all regions of shocks in the cases for liquidations have positive probability. Then the expected payoffs are given by

$$\begin{aligned} \frac{Z}{\alpha} \mathbb{E}_m [\Pi_x] &= (Z_{\alpha,0} + m^* + D - I_x - I_y) (I_x r_x + Lr) + \min\{I_x, I_y\} \max\{Lr, I_x r_x\} \\ &\quad + (\max\{I_x, I_y\} - \min\{I_x, I_y\}) \min\{I_x, I_y\} r_{\arg \min_z I_z} \\ \frac{Z}{\alpha} \mathbb{E}_m [\Pi_x] &= (I_y r_y - Lr) \left[ (Z_{\alpha,0} + m^* + D - I_x - I_y) + \mathbf{1}_{I_x r_x < Lr} \min\{I_x, I_y\} \right. \\ &\quad \left. + \mathbf{1}_{I_x < L} (\max\{I_x, I_y\} - \min\{I_x, I_y\}) \right] \end{aligned}$$

So the solution is identical, just by replacing  $m$  with  $m^*$  now. For closed form results we suppose that  $m$  is 0 w.p.  $\beta$  and  $U[0, \frac{2m^*}{1-\beta}]$  w.p.  $1 - \beta$  where  $m^* < \frac{1-\beta}{2} (Z - D)$ . Note that this has mean  $m^*$ .

We first consider the event that all funded projects get liquidated, which we call systemic risk. This is,  $\zeta' > I_x$ . (Under  $D \frac{r_x+r}{r_x-r} \geq Z_{\alpha,m^*}$   $y$ 's project is indeed funded. Otherwise, the only funded project is  $x$ 's.) Systemic risk is

$$\begin{aligned} &\frac{\alpha}{Z} (Z - D + (1 - \phi)L - m^*) \\ &\underset{(m^*, \beta)}{\propto} - m^* + (1 - \phi) \begin{cases} \frac{D + Z_{\alpha,m^*}}{4(1-\phi)} & \text{if } D \frac{1-2\phi}{3-2\phi} \geq Z_{\alpha,m^*} \\ \frac{D(r_x+r) - (r_x-r)Z_{\alpha,m^*}}{2(r_x+2(1-\phi)r)} & \text{if } D \frac{r_x+r}{r_x-r} > Z_{\alpha,m^*} > D \frac{1-2\phi}{3-2\phi} \\ 0 & \text{if } Z_{\alpha,m^*} > D \frac{r_x+r}{r_x-r} \end{cases} \end{aligned}$$

The first term  $-m^*$  is the direct effect of the availability of public liquidity. This has a natural effect of reducing the risk of liquidations. The second term after the bracket is the equilibrium effect of public liquidity. The availability of public liquidity influences the availability of private liquidity in the system through the portfolio choices, in particular, through  $L$ . The equilibrium effect increases in  $m^*$  up to  $D \frac{1-2\phi}{3-2\phi} - Z_{\alpha,0}$  and decreases afterwards. The net effect is always to reduce systemic risk.

Next consider contagion risk, the probability that the project of  $y$  gets liquidated. This event is the union of  $\zeta' > I_x$  (systemic risk) and  $0 < \zeta' \leq L(1 - \phi)$ , “only-contagion.” The probability of only-contagion is  $\frac{\alpha}{Z} L(1 - \phi)$ . This is increasing in  $m^*$  for  $m^* < D \frac{1-2\phi}{3-2\phi} - Z_{\alpha,0}$

and decreasing afterwards  $m^*$ . We have calculated systemic risk, and contagion risk is

$$\frac{\alpha}{Z} (Z - D + 2(1 - \phi)L - m^*) \underset{(m^*, \beta)}{\propto} 2(1 - \phi)L - m^*$$

This is always decreasing in  $m^*$ .

Now consider direct risk, the probability that the project of  $x$  gets liquidated. This event is given by  $L(1 - \phi) < \zeta'$ . The part  $I_x < \zeta'$  is the systemic risk. The part of  $L(1 - \phi) < \zeta' \leq I_x$  is “only-direct-risk.” Only-direct-risk is given by

$$\frac{\alpha}{Z} (I_x - L(1 - \phi)) \underset{(m^*, \beta)}{\propto} \begin{cases} \frac{D + Z_{\alpha, m^*}}{4} & \text{if } D \frac{1-2\phi}{3-2\phi} \geq Z_{\alpha, m^*} \\ \frac{D(3(1-\phi)r - r + \phi r_x) + (2-\phi)(r_x - r)Z_{\alpha, m^*}}{2(r_x + 2(1-\phi)r)} & \text{if } D \frac{r_x + r}{r_x - r} > Z_{\alpha, m^*} > D \frac{1-2\phi}{3-2\phi} \\ D & \text{if } Z_{\alpha, m^*} > D \frac{r_x + r}{r_x - r} \end{cases}$$

This is always increasing in  $m^*$ . The public liquidity always increases the only-direct-risk. This is perhaps particularly relevant for the Great Depression. The combined direct-risk to  $x$  is

$$\frac{\alpha}{Z} (Z - D + (1 - \phi)L - m^* + I_x - L(1 - \phi)) \underset{(m^*, \beta)}{\propto} -m^* + \begin{cases} \frac{D + Z_{\alpha, m^*}}{2} & \text{if } D \frac{1-2\phi}{3-2\phi} \geq m^* + Z_{\alpha, 0} \\ \frac{D(4(1-\phi)r + r_x - r) + Z_{\alpha, m^*}(r_x - r)}{2(r_x + 2(1-\phi)r)} & \text{if } D \frac{r_x + r}{r_x - r} > m^* + Z_{\alpha, 0} > D \frac{1-2\phi}{3-2\phi} \\ D & \text{if } m^* + Z_{\alpha, 0} > D \frac{r_x + r}{r_x - r} \end{cases}$$

This is always decreasing in  $m^*$ . The public liquidity always reduces the direct-risk to  $x$ .

Finally, we consider vulnerability, that is, the risks conditional on  $m = 0$ . Systemic vulnerability is given by

$$\frac{\alpha}{Z} (Z - D + (1 - \phi)L)$$

This is increasing in  $m^*$  for small  $m^*$  and decreasing for large  $m^*$ . Contagion vulnerability is

$$\frac{\alpha}{Z} (Z - D + 2(1 - \phi)L)$$

also increasing in  $m^*$  for small  $m^*$  and decreasing for large  $m^*$ . Direct vulnerability is

$$\frac{\alpha}{Z} (I_x - L(1 - \phi))$$

always increasing in  $m^*$ . □

*Proof. (Proof of Proposition 5)*

Now there is  $\theta$  probability that  $x_i$  gets a shock. Then Proposition 1 goes through by replacing  $\alpha$  with  $\theta$ . Note that  $\frac{Z}{Z+D(\frac{1-2\phi}{3-2\phi})-m} > \frac{1}{2} > \theta$  so we do not have the region in which  $\Phi_{x_i} > 0$ .

Now suppose that the the core banks can borrow each others reserves. We assume  $Z > 2D+m$  so that the shock can always be larger than the total cash in the system and we can avoid corner cases. For the pair  $i$ , the cash reserves of  $y_i$  act as an addition to  $m$ . Also note that  $x_i$  and  $x_j$  do not keep reserves and so we do not need to worry about  $x_i$  short-term lending to  $y_i$  and  $y_i$  intermediating this to  $y_j$ . Thus, for  $x_i$ , the best response is given by

$$L_i = \left( \frac{D(r_x + r) - (Z_{\theta,m} + L_j\phi)(r_x - r)}{2(r_x + 2(1-\phi)r)} \right)_+$$

The symmetric equilibrium is given by

$$L = \left( \frac{D(r_x + r) - (Z_{\theta,m} + L\phi)(r_x - r)}{2(r_x + 2(1-\phi)r)} \right)_+$$

1. If  $\frac{1}{2} > \theta > \frac{Z}{Z+D(\frac{r_x+r}{r_x-r})-m}$  ( $D\frac{r_x+r}{r_x-r} > Z_{\theta,m}$ ) then

$$L_{x_i} = \frac{D(r_x + r) - Z_{\theta,m}(r_x - r)}{2(r_x + 2(1-\phi)r) + \phi(r_x - r)}, \quad I_x = D - L$$

2. If  $\frac{Z}{Z+D(\frac{r_x+r}{r_x-r})-m} > \theta$ , ( $D\frac{r_x+r}{r_x-r} < Z_{\theta,m}$ ) then

$$L_{x_i} = 0, \quad I_{x_i} = D$$

□

*Proof. (Proof of Lemma 2)*

For  $D\frac{r_x+r}{r_x-r} < Z_{\theta,m}$ ,  $L_C = L_D = 0$ . There is no network. So consider the region  $D\frac{r_x+r}{r_x-r} > Z_{\theta,m}$ . From the earlier analysis we know that if both banks connect to their regional correspondents, in equilibrium,

$$\frac{Z}{\alpha} \Pi_{x_i}^C = (Z_{\theta,m} + D - I_x) I_x r_x + (Z_{\theta,m} + D - 2(1-\phi)L) L r$$

where

$$L_C = \frac{D(r_x + r) - Z_{\theta,m}(r_x - r)}{2(r_x + 2(1 - \phi)r)}$$

If both regions connect to NY, in equilibrium,

$$\frac{Z}{\alpha} \Pi_{x_i}^N = (Z_{\theta,m} + \phi L + D - I_x) I_x r_x + (Z_{\theta,m} + \phi L + D - 2(1 - \phi)L) Lr - c(L)$$

where

$$L_N = \frac{D(r_x + r) - Z_{\theta,m}(r_x - r)}{2(r_x + 2(1 - \phi)r) + \phi(r_x - r)}$$

Note

$$\frac{d\left(\frac{Z}{\alpha} \Pi_{x_i}^C\right)}{dm} = I_{x,C} r_x + L_C r$$

and

$$\frac{d\left(\frac{Z}{\alpha} \Pi_{x_i}^N\right)}{dm} = (I_{x,N} r_x + L_N r) \left(1 + \phi \frac{dL_N}{dm}\right)$$

Denote  $A = 2(r_x + 2(1 - \phi)r)$  and  $B = D(r_x + r) - Z_{\theta,m}(r_x - r)$ . Then

$$\begin{aligned} \frac{Z}{\alpha} \frac{d\left(\Pi_{x_i}^C\right)}{dm} &= Dr_x - (r_x - r) \frac{B}{A} \\ \frac{Z}{\alpha} \frac{d\left(\Pi_{x_i}^N\right)}{dm} &= \left(Dr_x - (r_x - r) \frac{B}{A + \phi(r_x - r)}\right) \left(\frac{A}{A + \phi(r_x - r)}\right) \\ \frac{Z}{\alpha} \left(\frac{d\left(\Pi_{x_i}^C\right)}{dm} - \frac{d\left(\Pi_{x_i}^N\right)}{dm}\right) &= Dr_x - (r_x - r) \frac{B}{A} \\ &> \frac{Dr(r_x - r)\phi}{A + \phi(r_x - r)} > 0 \end{aligned}$$

□

*Proof. (Proof of Proposition 6)*

Since the difference in the derivative is bounded away from zero, as  $m$  grows,  $\Pi_{x_i}^C$  exceeds  $\Pi_{x_i}^N$  eventually. The switching point  $m_c$  depends on the fixed cost  $c$  as well. If the cost  $c$  is very large, the stable network is regional for all  $m$ . In this case,  $m_c = D \frac{r_x + r}{r_x - r} - Z \frac{1 - \theta}{\theta}$ . If  $c$  is very small, the stable network is central for all  $m$ . Then  $m_c = 0$ . In between as  $c$  grows,  $m_c$  grows from 0 to  $D \frac{r_x + r}{r_x - r} - Z \frac{1 - \theta}{\theta}$ .

□