Software. The subsequent problems can be solved using the software provided in *MATLAB-SMC.zip*.

Problem 1. Consider the following stylized state-space model (see Chapters 4.3 and 5.1 of Herbst and Schorfheide (2015)):

$$y_t = [1\ 1]s_t, \quad s_t = \begin{bmatrix} \phi_1 & 0 \\ \phi_3 & \phi_2 \end{bmatrix} s_{t-1} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \epsilon_t, \quad \epsilon_t \sim iidN(0,1). \tag{1}$$

The mapping between some structural parameters $\theta = [\theta_1, \theta_2]'$ and the reduced-form parameters $\phi = [\phi_1, \phi_2, \phi_3]'$ is assumed to be

$$\phi_1 = \theta_1^2, \quad \phi_2 = (1 - \theta_1^2), \quad \phi_3 - \phi_2 = -\theta_1\theta_2. \tag{2}$$

The first state, $s_{1,t}$, looks like a typical exogenous driving force of a DSGE model, e.g., total factor productivity, while the second state $s_{2,t}$ evolves like an endogenous state variable, e.g., the capital stock, driven by the exogenous process and past realizations of itself. The mapping from structural to reduced form parameters is chosen to highlight the identification problems endemic to DSGE models. First, θ_2 is not identifiable when θ_1 is close to 0, since it enters the model only multiplicatively. Second, there is a global identification problem. Root cancellation in the AR and MA lag polynomials for y_t causes a bimodality in the likelihood function. To illustrate this, we simulate $T = 200$ observations given $\theta = [0.45, 0.45]'$. This parameterization is observationally equivalent to $\theta = [0.89, 0.22]'$. Moreover, we use a prior distribution that is uniform on the square $0 \leq \theta_1 \leq 1$ and $0 \leq \theta_2 \leq 1$.

1. Choose a vector of "true" parameters and simulate a sample of observations Y.

2. Write a MATLAB program that implements the SMC sampler.

3. Plot the sequence of tempered posteriors $\pi_n(\theta_1)$.

4. Create a contour plot of the posterior distribution that shows the two modes and overlay draws from the posterior.

5. Plot the acceptance rates in the mutation step and the scaling constant c for the covariance matrix as a function of n.

6. Plot ESS as a function of n.

7. Change the tuning of the algorithm and explore its performance.

Problem 2. After exploring the SMC sampler for the stylized state-space model, you are ready to try it on the small-scale DSGE model (see Chapters 1, 2, and 5.3 of Herbst and Schorfheide (2015)):

1. Take a careful look at the MATLAB codes that are provided.
2. Run the codes and change the tuning parameters to explore the effect on the performance of the algorithm.

Reference: Herbst, Edward and Frank Schorfheide (2015): *Bayesian Estimation of DSGE Models*, Princeton University Press.