On-line Appendix for Sticky Prices Versus Monetary Frictions: An Estimation of Policy Trade-offs *

S. Borağan Aruoba — *University of Maryland*

Frank Schorfheide — *University of Pennsylvania*

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*Correspondence: B. Aruoba: Department of Economics, University of Maryland, College Park, MD 20742. Email: aruoba@econ.umd.edu. F. Schorfheide: Department of Economics, 3718 Locust Walk, University of Pennsylvania, Philadelphia, PA 19104. Email: schorf@ssc.upenn.edu.

Gauss Programs that implement the empirical analysis are available at *http://www.ssc.upenn.edu/~schorf*.

1 Introduction

This Appendix provides detailed derivations of the equilibrium conditions presented in the main text. Moreover, we are presenting further empirical results that are not reported in the published paper. Additional details can found in Aruoba and Schorfheide (2009).

2 Solving the Search-Based Model

2.1 The Households' Problem

The households' CM problem takes the form

$$V_t^{CM}(\hat{m}_t, k_t, i_{t-1}, b_t, \mathcal{S}_t) = \max_{x_t, h_t, m_{t+1}, i_t, k_{t+1}, b_{t+1}} \left\{ U(x_t) - Ah_t + \beta E_t[V_{t+1}^{DM}(m_{t+1}, k_{t+1}, i_t, b_{t+1}, \mathcal{S}_{t+1})] \right\}$$

subject to the constraints

$$\begin{aligned} P_t x_t + P_t i_t + b_{t+1} + m_{t+1} &\leq P_t W_t h_t + P_t R_t^k k_t + \Pi_t + R_{t-1} b_t + \hat{m}_t - T_t + \Omega_t \quad (1) \\ k_{t+1} &= (1-\delta)k_t + \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right] i_t. \quad (2) \end{aligned}$$

Using Υ_t to denote the Lagrange multiplier for (2) and after eliminating h using (1), the FOC are

$$\begin{aligned} x_t &: U'(x_t) = \frac{A}{W_t} \quad (3) \\ m_{t+1} &: \frac{U'(x_t)}{P_t} = \beta E[V_{t+1,m}^{DM}(m_{t+1}, k_{t+1}, i_t, b_{t+1}, \mathcal{S}_{t+1})] \quad (4) \\ i_t &: U'(x_t) = \Upsilon_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right) + \frac{i_t}{i_{t-1}} S'\left(\frac{i_t}{i_{t-1}}\right)\right] \quad (5) \\ &\quad + \beta E[V_{t+1,i}^{DM}(m_{t+1}, k_{t+1}, i_t, b_{t+1}, \mathcal{S}_{t+1})] \\ k_{t+1} &: \Upsilon_t = \beta E[V_{t+1,k}^{DM}(m_{t+1}, k_{t+1}, i_t, b_{t+1}, \mathcal{S}_{t+1})] \quad (6) \\ b_{t+1} &: \frac{U'(x_t)}{P_t} = \beta E[V_{t+1,b}^{DM}(m_{t+1}, k_{t+1}, i_t, b_{t+1}, \mathcal{S}_{t+1})] \quad (7) \end{aligned}$$

assuming that an interior solution exists. Second, we have the following envelope conditions,

$$
\begin{aligned}
V^{CM}_{t,m}(\hat{m}_t, k_t, i_{t-1}, b_t, \mathcal{S}_t) &= \frac{A}{P_t W_t} \\
V^{CM}_{t,k}(\hat{m}_t, k_t, i_{t-1}, b_t, \mathcal{S}_t) &= \frac{A R^k_t}{W_t} + (1-\delta)\Upsilon_t \\
V^{CM}_{t,i}(\hat{m}_t, k_t, i_{t-1}, b_t, \mathcal{S}_t) &= \Upsilon_t \left(\frac{i_t}{i_{t-1}}\right)^2 S'\left(\frac{i_t}{i_{t-1}}\right) \\
V^{CM}_{t,b}(\hat{m}_t, k_t, i_{t-1}, b_t, \mathcal{S}_t) &= \frac{A R_{t-1}}{W_t}
\end{aligned}
$$

which show that $V^{CM}_t(.)$ is linear in $\hat{m}_t$.

We now turn to the analysis of household activity in the decentralized market. To solve (4)-(7), we need:

$$
\begin{aligned}
V^{DM}_{t,m}(m_t, k_t, i_{t-1}, b_t, \mathcal{S}_t) &= \frac{A}{P_t W_t} + \sigma\left[\chi_t u'\left(q^b_t\right)\frac{\partial q^b_t}{\partial m_t} - \frac{A}{P_t W_t}\frac{\partial d^b_t}{\partial m_t}\right] \\
&\quad + \sigma\left[\frac{A}{P_t W_t}\frac{\partial d^s_t}{\partial m_t} - c_q(q^s_t, k_t, Z_t)\frac{\partial q^s_t}{\partial m_t}\right] && (8) \\
V^{DM}_{t,k}(m_t, k_t, i_{t-1}, b_t, \mathcal{S}_t) &= \frac{A R^k_t}{W_t} + (1-\delta)\Upsilon_t + \sigma\left[\chi_t u'\left(q^b_t\right)\frac{\partial q^b_t}{\partial k_t} - \frac{A}{P_t W_t}\frac{\partial d^b_t}{\partial k_t}\right] \\
&\quad + \sigma\left[\frac{A}{P_t W_t}\frac{\partial d^s_t}{\partial k_t} - c_q(q^s_t, k_t, Z_t)\frac{\partial q^s_t}{\partial k_t} - c_k(q^s_t, k_t, Z_t)\right] && (9) \\
V^{DM}_{t,i}(m_t, k_t, i_{t-1}, b_t, \mathcal{S}_t) &= V^{CM}_{t,i}(m_t, k_t, i_{t-1}, b_t, \mathcal{S}_t) && (10) \\
V^{DM}_{t,b}(m_t, k_t, i_{t-1}, b_t, \mathcal{S}_t) &= \frac{A}{P_t W_t} R_{t-1} && (11)
\end{aligned}
$$

It remains to specify how the terms of trade (q, d) are determined, so that we can substitute for their derivatives in (8) and (9) which we turn to next. We consider two alternatives: bilateral bargaining via generalized Nash bargaining and price-taking.

Bargaining: The bargaining problem takes the form

$$
\max_{q,d}\left[\chi u(q) - \frac{Ad}{PW}\right]^{\theta}\left[\frac{Ad}{PW} - c(q, k^s, Z)\right]^{1-\theta} \quad \text{s.t. } d \le m^b.
$$

Inserting $d = m^b$ and taking the FOC with respect to q, we obtain:

$$
\frac{m^b A}{PW} = g(q, k^s, \chi, Z), \tag{12}
$$

where

$$
g(\cdot) = \frac{\theta\chi c(q, k^s, Z)u'(q) + (1-\theta)\chi c_q(q, k^s, Z)u(q)}{\theta\chi u'(q) + (1-\theta)c_q(q, k^s, Z)}.
$$

Turning to the partial derivatives, we obtain

$$\frac{\partial d}{\partial m^b} = 1, \quad \frac{\partial q}{\partial m^b} = \frac{A}{PWg_q(q,k,\chi,Z)} > 0, \quad \text{and} \quad \frac{\partial q}{\partial k^s} = -\frac{g_k(q,k,\chi,Z)}{g_q(q,k,\chi,Z)} > 0,$$

while the other derivatives in (8) and (9) are 0. Now reintroducing the time subscripts and inserting these results, (8) and (9) reduce to

$$V_{t,m}(m_t, k_t, i_{t-1}, b_t, \mathcal{S}_t) = \frac{(1-\sigma)A}{P_t W_t} + \frac{\sigma A \chi_t u'(q_t)}{P_t W_t g_q(q_t, k_t, \chi_t, Z_t)} \tag{13}$$

$$V_{t,k}(m_t, k_t, i_{t-1}, b_t, \mathcal{S}_t) = \frac{AR_t^k}{W_t} + (1-\delta)\Upsilon_t - \sigma\Gamma(q_t, k_t, \chi_t, Z_t), \tag{14}$$

where

$$\Gamma(\cdot) = \frac{c_k(\cdot)g_q(\cdot) - c_q(\cdot)g_k(\cdot)}{g_q(\cdot)}.$$

determines the the marginal return of having capital in the DM when the household is a seller.

Price-Taking: Recall that $V_m^{CM}(\cdot) = \frac{A}{PW}$ and does not depend on m. The first-order conditions for buyer and seller are

$$\chi u'(q) = \tilde{p} V_m^{CM}(m - \tilde{p}q, \cdot) + \lambda\tilde{p}, \quad c_q(\cdot) = \tilde{p} V_m^{CM}(m + \tilde{p}q, \cdot),$$

where λ here denotes the Lagrange multiplier associated with the constraint $\tilde{p}q \leq m$. Assuming that the constraint is binding $\tilde{p}q = m$ and the FOC of the seller yields:

$$\frac{m}{P} = \frac{qc_q(\cdot)W}{A}.$$

Turning to the partial derivatives, we obtain:

$$\frac{\partial d}{\partial m^b} = 1, \quad \frac{\partial q}{\partial m^b} = \frac{1}{\tilde{p}} = \frac{A}{PWc_q(q,k,Z)} > 0, \quad \frac{\partial q}{\partial k^s} = -\frac{c_{qk}(q,k,\chi,Z)}{c_{qq}(q,k,\chi,Z)} > 0 \quad \text{and} \quad \frac{\partial d}{\partial k^s} = \tilde{p}\frac{\partial q}{\partial k^s},$$

while the other derivatives in (8) and (9) are 0. Finally, reintroducing time subscripts and using these results we get the envelope conditions

$$V_{t,m}(m_t, k_t, i_{t-1}, b_t, \mathcal{S}_t) = \frac{(1-\sigma)A}{P_t W_t} + \frac{\sigma A \chi_t u'(q_t)}{P_t W_t q_t c_q(q_t, k_t, Z_t)} \tag{15}$$

$$V_{t,k}(m_t, k_t, i_{t-1}, b_t, \mathcal{S}_t) = \frac{AR_t^k}{W_t} + (1-\delta)\Upsilon_t - \sigma c_k(q_t, k_t, Z_t). \tag{16}$$

We obtain the optimality conditions for the household under bargaining by simply substituting (10), (11), (13) and (14) into the household's FOC. For the price-taking model we replace (13) and (14) by (15) and (16).

2.2 Firms in the Centralized Market

The setup of the centralized market resembles that of a New Keynesian DSGE model. Production is carried out by two types of firms in the CM: final good producers combine differentiated intermediate goods. Intermediate goods producing firms hire labor and capital services from the households to produce the inputs for the final good producers. To introduce nominal rigidity we follow Calvo (1983) by assuming that only a constant fraction of the intermediate goods producers is able to re-optimize prices.

Final Good Producers solve the problem

$$\max_{Y_t, Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \quad \text{s.t.} \quad Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda}} di \right]^{1+\lambda} \tag{17}$$

taking $P_t(i)$ as given. The first-order condition is:

$$P_t(i) = P_t Y_t^{\frac{\lambda}{1+\lambda}} Y_t(i)^{-\frac{\lambda}{1+\lambda}}. \tag{18}$$

A free entry condition ensures that profits are zero in equilibrium.

Intermediate Goods Producers: Intermediate goods producers, indexed by i, face the demand function

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\frac{1+\lambda}{\lambda}} Y_t \tag{19}$$

and use a Cobb-Douglas technology with fixed costs $\mathcal{F}$:

$$Y_t(i) = \max \left\{ Z_t K_t(i)^{\alpha} H_t(i)^{1-\alpha} - \mathcal{F}, 0 \right\}. \tag{20}$$

Cost minimization subject to (20) yields the conditions:

$$P_t W_t = \mu_t(i) P_t(i) (1-\alpha) Z_t K_t(i)^{\alpha} H_t(i)^{-\alpha} \tag{21}$$

$$P_t R_t^k = \mu_t(i) P_t(i) \alpha Z_t K_t(i)^{\alpha-1} H_t(i)^{1-\alpha}, \tag{22}$$

where $\mu_t(i)$ is the Lagrange multiplier associated with (20). In turn, these conditions imply:

$$K_t(i) = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k} H_t(i).$$

If we integrate both sides of the equation with respect to di and define $K_t = \int K_t(i)di$ and $H_t = \int H_t(i)di$ we obtain a relationship between aggregate labor and capital:

$$K_t = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t^k}H_t. \tag{23}$$

Thus, the aggregate capital labor ratio is a linear function of the ratio of factor prices. Total variable cost (VC_t) is given by

$$VC_t(i) = \left(W_t + R_t^k \frac{K_t(i)}{H_t(i)}\right) H_t(i) = \left(W_t + R_t^k \frac{K_t(i)}{H_t(i)}\right) Z_t^{-1} \left(\frac{K_t(i)}{H_t(i)}\right)^{-\alpha} Y_t^v(i),$$

where $Y_t^v(i) = Z_t K_t(i)^\alpha H_t(i)^{1-\alpha}$ is the "variable" part of output $Y_t(i)$. The real marginal cost MC_t is the same for all firms and equal to:

$$MC_t = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} W_t^{1-\alpha} (R_t^k)^\alpha Z_t^{-1}. \tag{24}$$

The first-order condition for an intermediate good producing firm is:

$$I\!E_t\left\{\sum_{s=0}^{\infty} \zeta^s \beta^s \Xi^p_{t+s|t} \frac{1}{P_t^o(i)} \left(\frac{P_t^o(i)\pi^{adj}_{t+s|t}}{P_{t+s}}\right)^{-\frac{1+\lambda}{\lambda}} Y_{t+s}\left[P_t^o(i)\pi^{adj}_{t+s|t} - (1+\lambda)P_{t+s}MC_{t+s}\right]\right\} = 0. \tag{25}$$

Define and rewrite

$$\begin{aligned}
\mathcal{F}_t^{(1)} &= I\!E_t\left[\sum_{s=0}^{\infty} \zeta^s \beta^s \Xi^p_{t+s|t} \left(\frac{P_t^o(i)\pi^{adj}_{t+s|t}}{P_{t+s}}\right)^{-\frac{1+\lambda}{\lambda}} Y_{t+s}\pi^{adj}_{t+s|t}\right] \qquad (26)\\
&= \left(\frac{P_t^o(i)}{P_t}\right)^{-\frac{1+\lambda}{\lambda}} Y_t + \zeta\beta I\!E_t\left[\sum_{s=0}^{\infty} \zeta^s \beta^s \Xi^p_{t+1+s|t} \left(\frac{P_t^o(i)\pi^{adj}_{t+1+s|t}}{P_{t+1+s}}\right)^{-\frac{1+\lambda}{\lambda}} Y_{t+1+s}\pi^{adj}_{t+1+s|t}\right] \\
&= \left(\frac{P_t^o(i)}{P_t}\right)^{-\frac{1+\lambda}{\lambda}} Y_t + \zeta\beta\left(\pi_t^\iota \pi_{**}^{(1-\iota)}\right)^{-1/\lambda} \\
&\quad \times I\!E_t\left[\left(\frac{P_t^o(i)}{P_{t+1}^o(i)}\right)^{-\frac{1+\lambda}{\lambda}} \Xi^p_{t+1|t} \sum_{s=0}^{\infty} \zeta^s \beta^s \Xi^p_{t+1+s|t+1} \left(\frac{P_{t+1}^o(i)\pi^{adj}_{t+1+s|t+1}}{P_{t+1+s}}\right)^{-\frac{1+\lambda}{\lambda}} Y_{t+1+s}\pi^{adj}_{t+1+s|t+1}\right] \\
&= \left(\frac{P_t^o(i)}{P_t}\right)^{-\frac{1+\lambda}{\lambda}} Y_t + \zeta\beta\left(\pi_t^\iota \pi_{**}^{(1-\iota)}\right)^{-1/\lambda} I\!E_t\left[\left(\frac{P_t^o(i)}{P_{t+1}^o(i)}\right)^{-\frac{1+\lambda}{\lambda}} \Xi^p_{t+1|t}\mathcal{F}^{(1)}_{t+1}\right].
\end{aligned}$$

Similarly,

$$\begin{aligned}
\mathcal{F}_t^{(2)} &= I\!E_t\left[\sum_{s=0}^{\infty} \zeta^s \beta^s \Xi_{t+s} \left(\frac{P_t^o(i)\pi^{adj}_{t+s|t}}{P_{t+s}}\right)^{-\frac{1+\lambda}{\lambda}} Y_{t+s}\frac{P_{t+s}MC_{t+s}}{P_t^o(i)}\right] \qquad (27)\\
&= \left(\frac{P_t^o(i)}{P_t}\right)^{-\frac{1+\lambda}{\lambda}} Y_t\frac{P_t MC_t}{P_t^o(i)} + \zeta\beta\left(\pi_t^\iota \pi_{**}^{(1-\iota)}\right)^{-\frac{1+\lambda}{\lambda}} I\!E_t\left[\left(\frac{P_t^o(i)}{P_{t+1}^o(i)}\right)^{-\frac{1+\lambda}{\lambda}-1} \Xi^p_{t+1|t}\mathcal{F}^{(2)}_{t+1}\right].
\end{aligned}$$

and the first-order condition becomes

$$\mathcal{F}_t^{(1)} = (1+\lambda)\mathcal{F}_t^{(2)}. \tag{28}$$

We are considering only the symmetric equilibrium in which all firms that can readjust prices will choose the same $P_t^o(i)$ and hence will drop the i index. Moreover, let $p_t^o = P_t^o/P_t$ and $\pi_t = P_t/P_{t-1}$. Then we can write the first-order conditions as:

$$\mathcal{F}_t^{(1)} = (p_t^o)^{-\frac{1+\lambda}{\lambda}} Y_t + \zeta\beta \left(\pi_t^{\iota}\pi_{**}^{(1-\iota)}\right)^{-1/\lambda} \mathbb{E}_t \left[\left(\frac{p_t^o}{\pi_{t+1} p_{t+1}^o} \right)^{-\frac{1+\lambda}{\lambda}} \Xi_{t+1|t}^p \mathcal{F}_{t+1}^{(1)} \right] \tag{29}$$

$$\mathcal{F}_t^{(2)} = \zeta\beta \left(\pi_t^{\iota}\pi_{**}^{(1-\iota)}\right)^{-\frac{1+\lambda}{\lambda}} \mathbb{E}_t \left[\left(\frac{p_t^o}{\pi_{t+1} p_{t+1}^o} \right)^{-\frac{1+\lambda}{\lambda}-1} \Xi_{t+1|t}^p \mathcal{F}_{t+1}^{(2)} \right] + (p_t^o)^{-\frac{1+\lambda}{\lambda}-1} Y_t MC_t \tag{30}$$

$$\mathcal{F}_t^{(1)} = (1+\lambda)\mathcal{F}_t^{(2)} \tag{31}$$

To capture the evolution of the price distribution we introduce the variable

$$D_t = \int \left(\frac{P_t(i)}{P_t} \right)^{-\frac{(1+\lambda)}{\lambda}} di$$

Its law of motion can be derived as follows:

$$\begin{aligned} D_t &= (1-\zeta) \sum_{j=0}^{\infty} \zeta^j \left(\frac{(\pi_{t-1}\pi_{t-2}\cdots\pi_{t-j})^{\iota}}{\pi_t \pi_{t-1} \cdots \pi_{t-j+1}} \frac{P_{t-j}^o}{P_{t-j}} \right)^{-\frac{1+\lambda}{\lambda}} \\ &= (1-\zeta) \left[\frac{P_t^o}{P_t} \right]^{-\frac{1+\lambda}{\lambda}} \\ &\quad + (1-\zeta)\zeta \left[\left(\frac{\pi_{t-1}}{\pi_t} \right)^{\iota} \left(\frac{1}{\pi_t} \right)^{(1-\iota)} \frac{P_{t-1}^o}{P_{t-1}} \right]^{-\frac{1+\lambda}{\lambda}} \\ &\quad + (1-\zeta)\zeta^2 \left[\left(\frac{\pi_{t-2}}{\pi_t} \right)^{\iota} \left(\frac{1}{\pi_t \pi_{t-1}} \right)^{(1-\iota)} \frac{P_{t-2}^o}{P_{t-2}} \right]^{-\frac{1+\lambda}{\lambda}} \quad \cdots. \end{aligned}$$

Lagging D_t by one period yields

$$\begin{aligned} D_{t-1} &= (1-\zeta) \left[\frac{P_{t-1}^o}{P_{t-1}} \right]^{-\frac{1+\lambda}{\lambda}} \\ &\quad + (1-\zeta)\zeta \left[\left(\frac{\pi_{t-2}}{\pi_{t-1}} \right)^{\iota} \left(\frac{1}{\pi_{t-1}} \right)^{(1-\iota)} \frac{P_{t-2}^o}{P_{t-2}} \right]^{-\frac{1+\lambda}{\lambda}} \\ &\quad + (1-\zeta)\zeta^2 \left[\left(\frac{\pi_{t-3}}{\pi_{t-1}} \right)^{\iota} \left(\frac{1}{\pi_{t-1} \pi_{t-2}} \right)^{(1-\iota)} \frac{P_{t-3}^o}{P_{t-3}} \right]^{-\frac{1+\lambda}{\lambda}} \quad \cdots. \end{aligned}$$

Therefore, we obtain the following law of motion for the price dispersion:

$$D_t = \zeta \left[\left(\frac{\pi_{t-1}}{\pi_t} \right)^{\iota} \left(\frac{1}{\pi_t} \right)^{(1-\iota)} \right]^{-\frac{1+\lambda}{\lambda}} D_{t-1} + (1-\zeta) \left[\frac{P_t^o}{P_t} \right]^{-\frac{1+\lambda}{\lambda}}. \tag{32}$$

2.3 Aggregate Resource Constraint and National Accounting

To take the model to the data we will now construct a GDP deflator and a measure of real output that is consistent with this GDP deflator. Following NIPA conventions, we use a Fisher price index. However, to simplify the analysis we replace time-varying nominal shares by steady state shares. The DM share of nominal output in the steady state is

$$s_* = \frac{\sigma \mathcal{M}_*}{Y_* \pi_* + \sigma \mathcal{M}_*}. \tag{33}$$

Define $\pi_t^{DM} = P_t^{DM}/P_{t-1}^{DM}$ and let

$$\ln \pi_t^{GDP} = \ln \frac{P_t^{GDP}}{P_{t-1}^{GDP}} = (1 - s_*) \ln \pi_t + s_* \ln \pi_t^{DM}. \tag{34}$$

Thus,

$$P_t^{GDP} = P_0^{GDP} \prod_{\tau=1}^{t} \pi_\tau^{1-s_*} (\pi_\tau^{DM})^{s_*}. \tag{35}$$

We now define real GDP as

$$\mathcal{Y}_t^{GDP} = \frac{\mathcal{Y}_t^{(n)}}{P_t^{GDP}} = \mathcal{Y}_t \frac{P_t}{P_t^{GDP}}. \tag{36}$$

It can be verified that up to a first-order approximation changes in real GDP evolve according to a Fisher quantity index with fixed (steady state) weights. Let X_* denote the steady state of a variable X_t and $\tilde{X}_t = \ln X_t/X_*$. Recall that real output in terms of the CM good was given by

$$\mathcal{Y}_t = Y_t + \sigma M_t/P_t. \tag{37}$$

Log-linearizing and differencing (37) yields

$$\Delta \tilde{\mathcal{Y}}_t = (1 - s_*) \Delta \tilde{Y}_t + s_* [\Delta \tilde{M}_t - \tilde{\pi}_t]. \tag{38}$$

Here Δ denotes the temporal difference operator. According to the definition of prices in the DM

$$\tilde{\pi}_t^{DM} = \Delta \tilde{M}_t - \Delta \tilde{q}_t. \tag{39}$$

Combining (38) and (39) leads to:

$$\Delta\tilde{\mathcal{Y}}_t = (1-s_*)\Delta\tilde{Y}_t + s_*[\Delta\tilde{q}_t + \tilde{\pi}_t^{DM} - \tilde{\pi}_t].$$

Thus,

$$\Delta\tilde{\mathcal{Y}}_t^{GDP} = \Delta\tilde{\mathcal{Y}}_t + \tilde{\pi}_t - (1-s_*)\tilde{\pi}_t - s_*\tilde{\pi}_t^{DM} = (1-s_*)\Delta\tilde{Y}_t + s_*\Delta\tilde{q}_t. \tag{40}$$

Hence, the level of GDP in period t is given by

$$\tilde{\mathcal{Y}}_t^{GDP} = (1-s_*)\tilde{Y}_t + s_*\tilde{q}_t + [\tilde{\mathcal{Y}}_0^{GDP} - (1-s_*)\tilde{Y}_0 - s_*\tilde{q}_0].$$

Under the normalizations $P_0^{GDP} = 1$ and $P_0 = 1$ we obtain

$$\tilde{\mathcal{Y}}_0^{GDP} = (1-s_*)\tilde{Y}_0 + s_*(\mathcal{M}_0 - \pi_0).$$

We can therefore further simplify our expression for GDP to

$$\tilde{\mathcal{Y}}_t^{GDP} = (1-s_*)\tilde{Y}_t + s_*\tilde{q}_t + s_*(\tilde{\mathcal{M}}_0 - \tilde{\pi}_0 - \tilde{q}_0). \tag{41}$$

2.4 Functional Forms

We use a slightly more general specification of the utility and production functions in the subsequent exposition:

$$U(x) = B\frac{x^{1-\gamma}}{1-\gamma}, \quad u(q) = \frac{(q+\kappa)^{1-\eta} - \kappa^{1-\eta}}{1-\eta}.$$

Moreover, we let $f(e,k) = e^{\Phi}k^{1-\Phi}$.

2.5 Equilibrium Conditions

We now summarize the equilibrium conditions for the search-based model. The timing is such that all t shocks are realized at the beginning of t and $\bar{S}_t = (Z_t, g_t, \chi_t)$ and R_t are observed. $\bar{S}_t$ summarizes the exogenous state variables. We define $\mathcal{S}_t = \left(\bar{S}_t, R_t\right)$ which will be the aggregate state variables of the household's problem. In the following definitions, we do not track h_t (individual labor supply) and B_t (the bond supply of the government). We also do not track nominal money balances but instead track $\mathcal{M}_t = M_t/P_{t-1}$. Recall that M_t is determined based on $t-1$ information and so is $\mathcal{M}_t$. Finally, we use $\pi_t \equiv P_t/P_{t-1}$ and do not track the level of prices. Given exogenous states $\left\{\bar{S}_t\right\}_{t=0}^{\infty}$, a monetary equilibrium is defined as allocations $\{q_t, X_t, H_t, K_t, I_t, \mu_t, Y_t, \mathcal{M}_t, \mathcal{Y}_t\}_{t=0}^{\infty}$, policy $\{R_t\}_{t=0}^{\infty}$ and prices $\left\{W_t, R_t^k, p_t^0, \pi_t, D_t\right\}_{t=0}^{\infty}$ such that :

Household's Problem: Given exogenous states, policy and prices, $\left\{q_t, X_t, H_t, K_t, I_t, \mu_t, \mathcal{M}_t, \Xi_{t+1|t}^p\right\}_{t=0}^{\infty}$ satisfy

$$W_t = \frac{A}{U'(X_t)} \tag{42}$$

$$1 = \beta E_t\left[\frac{U'(X_{t+1})}{U'(X_t)}\frac{R_t}{\pi_{t+1}}\right] \tag{43}$$

$$\begin{aligned} 1 &= \mu_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}}S'\left(\frac{I_t}{I_{t-1}}\right)\right] \\ &\quad + \beta E_t\left\{\mu_{t+1}\frac{U'(X_{t+1})}{U'(X_t)}\left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{I_t}\right)\right\} \end{aligned} \tag{44}$$

$$K_{t+1} = (1-\delta)K_t + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right]I_t \tag{45}$$

$$\mu_t U'(X_t) = \beta E_t\left\{U'(X_{t+1})\left[R_{t+1}^k + (1-\delta)\mu_{t+1}\right] - \sigma\gamma(q_{t+1}, K_{t+1}, \chi_{t+1}, Z_{t+1})\right\} \tag{46}$$

$$\mathcal{M}_t = \frac{g\left(q_t, K_t, \chi_t, Z_t\right)W_t\pi_t}{A} \tag{47}$$

$$U'(X_t) = \beta E_t\left\{\frac{U'(X_{t+1})}{\pi_{t+1}}\left[\frac{\sigma\chi_{t+1}u'(q_{t+1})}{g_q(q_{t+1}, K_{t+1}, \chi_{t+1}, Z_{t+1})} + (1-\sigma)\right]\right\} \tag{48}$$

$$\Xi_{t+1|t}^p = \frac{U'(X_{t+1})}{U'(X_t)\pi_{t+1}} \tag{49}$$

In the price-taking version we replace (46), (47) and (48) with

$$\mu_t U'(X_t) = \beta E_t\left\{U'(X_{t+1})\left[R_{t+1}^k + (1-\delta)\mu_{t+1}\right] - \sigma c_k(q_{t+1}, K_{t+1}, Z_{t+1})\right\} \tag{50}$$

$$\mathcal{M}_t = \frac{q_t c_q\left(q_t, K_t, Z_t\right)W_t\pi_t}{A} \tag{51}$$

$$U'(X_t) = \beta E_t\left\{\frac{U'(X_{t+1})}{\pi_{t+1}}\left[\frac{\sigma\chi_{t+1}u'(q_{t+1})}{c_q(q_{t+1}, K_{t+1}, Z_{t+1})} + (1-\sigma)\right]\right\} \tag{52}$$

Intermediate Goods Producing Firms' Problem: Intermediate goods firms choose their capital labor ratio as a function of the factor prices to minimize costs:

$$K_t = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t^k}H_t. \tag{53}$$

Firms that are allowed to change prices are choosing a relative price $p_t^o(i)$ (relative to the aggregate price level) to maximize expected profits subject to the demand curve for their differentiated product, taking the aggregate price level P_t as well as the prices charged by other firms as given, which leads to

$$
\begin{aligned}
MC_t &= \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} W_t^{1-\alpha} (R_t^k)^\alpha Z_t^{-1} && (54)\\
\mathcal{F}_t^{(1)} &= (p_t^o)^{-\frac{1+\lambda}{\lambda}} Y_t + \zeta\beta \left(\pi_t^\iota \pi_{**}^{(1-\iota)}\right)^{-1/\lambda} \mathbb{E}_t \left[\left(\frac{p_t^o}{\pi_{t+1} p_{t+1}^o} \right)^{-\frac{1+\lambda}{\lambda}} \Xi_{t+1|t}^p \mathcal{F}_{t+1}^{(1)} \right] && (55)\\
\mathcal{F}_t^{(2)} &= \zeta\beta \left(\pi_t^\iota \pi_{**}^{(1-\iota)}\right)^{-\frac{1+\lambda}{\lambda}} \mathbb{E}_t \left[\left(\frac{p_t^o}{\pi_{t+1} p_{t+1}^o} \right)^{-\frac{1+\lambda}{\lambda}-1} \Xi_{t+1|t}^p \mathcal{F}_{t+1}^{(2)} \right] && (56)\\
&\quad + (p_t^o)^{-\frac{1+\lambda}{\lambda}-1} Y_t MC_t \\
\mathcal{F}_t^{(1)} &= (1+\lambda)\mathcal{F}_t^{(2)} && (57)
\end{aligned}
$$

Final Good Producing Firms' Problem: Final goods producers take factor prices and output prices as given and choose inputs $Y_t(i)$ and output Y_t to maximize profits. Free entry ensures that final good producers make zero profits and leads to

$$
\pi_t = \left[(1-\zeta)\left(\pi_t p_t^o\right)^{-\frac{1}{\lambda}} + \zeta (\pi_{t-1}^\iota)^{-\frac{1}{\lambda}} \right]^{-\lambda} \tag{58}
$$

Aggregate Resource Constraint for CM is given by

$$
Y_t = D_t^{-1}(Z_t K_t^\alpha H_t^{(1-\alpha)} - \mathcal{F}), \tag{59}
$$

where

$$
D_t = \zeta \left[\left(\frac{\pi_{t-1}}{\pi_t} \right)^\iota \left(\frac{1}{\pi_t} \right)^{(1-\iota)} \right]^{-\frac{1+\lambda}{\lambda}} D_{t-1} + (1-\zeta)\left(p_t^o\right)^{-\frac{1+\lambda}{\lambda}}. \tag{60}
$$

Market Clearing: The goods market in the CM clears:

$$
X_t + I_t + \left(1 - \frac{1}{g_t}\right) \mathcal{Y}_t = Y_t \tag{61}
$$

GDP and GDP Deflator: Prices and inflation in the DM are given by

$$
P_t^{DM} = \frac{\sigma \mathcal{M}_t P_{t-1}}{q_t}, \quad \pi_t^{DM} = \frac{P_t^{DM}}{P_{t-1}^{DM}} = \frac{\mathcal{M}_t q_{t-1}}{\mathcal{M}_{t-1} q_t} \pi_{t-1}. \tag{62}
$$

According to our (approximate) Fisher index the GDP deflator evolves according to

$$
\pi_t^{GDP} = (\pi_t)^{(1-s_*)} (\pi_t^{DM})^{s_*}. \tag{63}
$$

Real output in terms of the CM good and GDP are

$$\mathcal{Y}_t = Y_t + \frac{\sigma \mathcal{M}_t}{\pi_t}, \quad \mathcal{Y}_t^{GDP} = \mathcal{Y}_t P_t / P_t^{GDP}. \tag{64}$$

Finally, measured real money balances and (inverse) velocity in the data are given by

$$\frac{M_{t+1}}{P_t^{GDP}} = \mathcal{M}_{t+1} \frac{P_t}{P_t^{GDP}}, \quad \frac{M_{t+1}}{P_t^{GDP} Y_t^{GDP}} = \frac{\mathcal{M}_{t+1}}{(P_t^{GDP}/P_t)\mathcal{Y}_t^{GDP}} = \frac{\mathcal{M}_{t+1}}{\mathcal{Y}_t}. \tag{65}$$

Monetary Policy: The central bank supplies the quantity of money necessary to attain the nominal interest rate

$$R_t = R_{*,t}^{1-\rho_R} R_{t-1}^{\rho_R} \exp\{\sigma_R \epsilon_{R,t}\}, \quad R_{*,t} = (r_* \pi_{*,t}) \left(\frac{\pi_t^{GDP}}{\pi_{*,t}} \right)^{\psi_1} \left(\frac{\mathcal{Y}_t^{GDP}}{\gamma \mathcal{Y}_{t-1}^{GDP}} \right)^{\psi_2} \tag{66}$$

2.6 Steady States

For estimation purposes it is useful to parameterize the model in terms of $\mathcal{Y}_*$, H_*, and $\mathcal{M}_*$ and solve the steady state conditions for A, B, and Z_*. Suppose q_* and K_* are given then we can solve for the following steady states recursively:

$$\begin{aligned}
R_* &= \pi_*/\beta \\
p_*^o &= \left[\frac{1}{1-\zeta} - \frac{\zeta}{1-\zeta}\left(\frac{1}{\pi_*}\right)^{-\frac{1-\iota}{\lambda}}\right]^{-\lambda} \\
D_* &= \frac{(1-\zeta)(p_*^o)^{-\frac{1+\lambda}{\lambda}}}{1-\zeta\left(\frac{1}{\pi_*}\right)^{-\frac{(1+\lambda)(1-\iota)}{\lambda}}} \\
Y_* &= \mathcal{Y}_* - \sigma\mathcal{M}_*/\pi_* \\
\bar{Y}_* &= Y_* D_* \\
Z_* &= (\bar{Y}_* + \mathcal{F})/(K_*^\alpha H_*^{1-\alpha}) \\
R_*^k &= \frac{\alpha Z_* p_*^o}{1+\lambda}\left[\frac{1-\zeta\beta\left(\frac{1}{\pi_*}\right)^{-(1-\iota)/\lambda}}{1-\zeta\beta\left(\frac{1}{\pi_*}\right)^{-(1-\iota)(1+\lambda)/\lambda}}\right]^{-1}\left(\frac{H_*}{K_*}\right)^{1-\alpha} \\
W_* &= \frac{1-\alpha}{\alpha}\frac{K_*}{H_*}R_*^k \\
I_* &= \delta K_* \\
X_* &= Y_* - I_* - (1-1/g_*)\mathcal{Y}_* \\
A &= \frac{g(q_*, K_*, \chi_*, Z_*)W_*\pi_*}{\mathcal{M}_*} \\
U_*' &= A/W_* \\
B &= U_*' X_*^\gamma \\
\pi_*^{DM} &= \pi_*^{GDP} = \pi_*
\end{aligned} \tag{67}$$

To determine q_* and K_* we solve the following equations jointly:

$$R_* = 1 + \sigma\left[\frac{\chi_* u'(q_*)}{g_q(q_*, K_*, \chi_*, Z_*)} - 1\right] \tag{68}$$

$$1 = \beta(1 + R_*^k - \delta) - \sigma\beta\frac{\gamma(q_*, K_*, \chi_*, Z_*)}{U_*'} \tag{69}$$

In the price-taking version, we replace (67), (68) and (69) with

$$\begin{aligned}
A &= \frac{q_* c_q(q_*, K_*, \chi_*, Z_*)W_*\pi_*}{\mathcal{M}_*} \\
R_* &= 1 + \sigma\left[\frac{\chi_* u'(q_*)}{c_q(q_*, K_*, Z_*)} - 1\right] \\
1 &= \beta(1 + R_*^k - \delta) - \sigma\beta\frac{c_k(q_*, K_*, Z_*)}{U_*'}
\end{aligned}$$

We deduce from the firms' problems:

$$
\begin{aligned}
\mathcal{F}_*^{(1)} &= \left(1-\zeta\beta\pi_*\left(\frac{1}{\pi_*}\right)^{-(1-\iota)/\lambda}\right)^{-1}(p_*^o)^{-\frac{1+\lambda}{\lambda}}Y_* \\
\mathcal{F}_*^{(2)} &= \left(1-\zeta\beta\pi_*\left(\frac{1}{\pi_*}\right)^{-(1-\iota)(1+\lambda)/\lambda}\right)^{-1}(p_*^o)^{-\frac{1+\lambda}{\lambda}-1}Y_*MC_* \\
\mathcal{F}_*^{(1)} &= (1+\lambda)\mathcal{F}_*^{(2)} \\
MC_* &= \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}W_*^{1-\alpha}(R_*^k)^{\alpha}Z_*^{-1} \\
\pi_* &= \left[(1-\zeta)\left(\pi_* p_*^o\right)^{-\frac{1}{\lambda}}+\zeta\left(\pi_*^{\iota}\right)^{-\frac{1}{\lambda}}\right]^{-\lambda}
\end{aligned}
$$

which lead to the conditions for p_*^o above. The term D_* measures the steady state price dispersion. The larger π_*, that is, the faster the price of the non-adjusters is eroding in real terms, the bigger D_*. Finally, in steady state the DM share of nominal output and the DM markup are given by

$$
\begin{aligned}
s_* &= \frac{\sigma\mathcal{M}_*}{\sigma\mathcal{M}_*+Y_*\pi_*} \\
\text{markup}(dm) &= \frac{g(q_*,K_*,\chi_*,Z_*)}{q_*c_q(q_*,K_*,Z_*)}-1.
\end{aligned}
$$

2.7 Log-Linearizations

In the subsequent presentation of the log-linearized equations we adopt the convention that we abbreviate time t expectations of a $t+1$ variable simply by a time $t+1$ subscript, omitting the expectation operator.

Household's Problem: The optimality conditions for the household can be expressed as

$$\tilde{W}_t = \gamma \tilde{X}_t \tag{70}$$

$$\tilde{X}_t = \tilde{X}_{t+1} - \frac{1}{\gamma}(\tilde{R}_t - \tilde{\pi}_{t+1}) \tag{71}$$

$$\tilde{i}_t = \frac{1}{1+\beta}\tilde{i}_{t-1} + \frac{\beta}{1+\beta}\tilde{i}_{t+1} + \frac{1}{(1+\beta)S''}\tilde{\mu}_t \tag{72}$$

$$\tilde{k}_{t+1} = (1-\delta)\tilde{k}_t + \delta\tilde{i}_t \tag{73}$$

$$\begin{aligned} \tilde{\mu}_t - \gamma\tilde{X}_t &= \beta(1-\delta)\tilde{\mu}_{t+1} - \gamma\beta(1-\delta+R^k_*)\tilde{X}_{t+1} + \beta R^k_*\tilde{R}^k_{t+1} \\ &\quad +(1-\beta(1-\delta+R^k_*))\tilde{\Gamma}_{t+1} \end{aligned} \tag{74}$$

$$\tilde{\mathcal{M}}_t = \tilde{g}_t + \tilde{W}_t + \tilde{\pi}_t \tag{75}$$

$$\tilde{R}_t = \frac{R_* - 1 + \sigma}{R_*}[\tilde{\chi}_{t+1} - \tilde{g}_{q,t+1} - \eta\frac{q_*}{(q_*+\kappa)}\tilde{q}_{t+1}] \tag{76}$$

$$\tilde{\Xi}^p_{t|t-1} = -\gamma(\tilde{X}_t - \tilde{X}_{t-1}) - \tilde{\pi}_t \tag{77}$$

Equations (70) to (77) determine wages, CM consumption, investment, capital, the shadow price of installed capital, the rental rate of capital, real money balances, the stochastic discount factor used in the firms' problem, and DM consumption. For the price-taking version, we replace (74), (75) and (76) with

$$\begin{aligned} \tilde{\mu}_t - \gamma\tilde{X}_t &= \beta(1-\delta)\tilde{\mu}_{t+1} - \gamma\beta(1-\delta+R^k_*)\tilde{X}_{t+1} + \beta R^k_*\tilde{R}^k_{t+1} \\ &\quad +(1-\beta(1-\delta+R^k_*))\tilde{c}_{k,t+1} \end{aligned} \tag{78}$$

$$\tilde{\mathcal{M}}_t = \tilde{q}_t + \tilde{c}_{q,t} + \tilde{W}_t + \tilde{\pi}_t \tag{79}$$

$$\tilde{R}_t = \frac{R_* - 1 + \sigma}{R_*}[\tilde{\chi}_{t+1} - \tilde{c}_{q,t+1} - \eta\frac{q_*}{(q_*+\kappa)}\tilde{q}_{t+1}] \tag{80}$$

Decentralized Market: We now determine the law of motion for $\tilde{g}_{q,t}$, $\tilde{\Gamma}_t$, and $\tilde{g}_t$. In addition, we are introducing some auxiliary variables. We begin with (omitting t subscripts),

$$
\begin{aligned}
u &= \frac{(q+\kappa)^{1-\eta} - \kappa^{1-\eta}}{1-\eta} \\
u' &= (q+\kappa)^{-\eta} \\
u'' &= -\eta(q+\kappa)^{-\eta-1} \\
c &= \exp\{-\tilde{Z}\} q^{\psi} k^{1-\psi} \\
c_q &= \psi \exp\{-\tilde{Z}\} q^{\psi-1} k^{1-\psi} \\
c_k &= (1-\psi) \exp\{-\tilde{Z}\} q^{\psi} k^{-\psi} \\
c_{qq} &= \psi(\psi-1) \exp\{-\tilde{Z}\} q^{\psi-2} k^{1-\psi} \\
c_{kk} &= \psi(\psi-1) \exp\{-\tilde{Z}\} q^{\psi} k^{-\psi-1} \\
c_{qk} &= \psi(1-\psi) \exp\{-\tilde{Z}\} q^{\psi-1} k^{-\psi}
\end{aligned}
$$

which can be log-linearized as follows

$$
\begin{aligned}
\tilde{u}u_* &= \frac{q_*}{(q_*+\kappa)^{\eta}}\tilde{q} \\
\tilde{u}' &= -\eta\frac{q_*}{q_*+\kappa}\tilde{q} \\
\tilde{u}'' &= -(\eta+1)\frac{q_*}{q_*+\kappa}\tilde{q} \\
\tilde{c} &= -\psi\tilde{Z} + \psi\tilde{q} + (1-\psi)\tilde{k} \\
\tilde{c}_q &= -\psi\tilde{Z} + (\psi-1)\tilde{q} + (1-\psi)\tilde{k} \\
\tilde{c}_k &= -\psi\tilde{Z} + \psi\tilde{q} - \psi\tilde{k} \\
\tilde{c}_{qq} &= -\psi\tilde{Z} + (\psi-2)\tilde{q} + (1-\psi)\tilde{k} \\
\tilde{c}_{kk} &= -\psi\tilde{Z} + \psi\tilde{q} - (1+\psi)\tilde{k} \\
\tilde{c}_{qk} &= -\psi\tilde{Z} + (\psi-1)\tilde{q} + -\psi\tilde{k}
\end{aligned}
$$

Recall that

$$
\Gamma_t = \frac{c_{k,t}g_{q,t} - c_{q,t}g_{k,t}}{g_{q,t}}
$$

which implies that $\tilde{\Gamma}_t$ evolves according to

$$
\tilde{g}_{q,t} + \tilde{\Gamma}_t = \frac{c_{k*}g_{q*}}{c_{k*}g_{q*} - c_{q*}g_{k*}}[\tilde{c}_{k,t} + \tilde{g}_{q,t}] - \frac{c_{q*}g_{k*}}{c_{k*}g_{q*} - c_{q*}g_{k*}}[\tilde{c}_{q,t} + \tilde{g}_{k,t}]. \tag{81}
$$

Now consider the equation

$$g_t(\theta\chi u_t' + (1-\theta)c_{q,t}) = \theta\chi c_t u_t' + (1-\theta)\chi c_{q,t}u_t,$$

which can be written in log-linear form as

$$\begin{aligned} & \left[\theta\chi_* u_*' + (1-\theta)\, c_{q*}\right] g_* \tilde{g}_t \\ = \quad & \theta\chi_* u_*' \left(c_* - g_*\right) \tilde{u}_t' + (1-\theta)\, \chi_* c_{q*} u_* \tilde{u}_t + (1-\theta)\, c_{q*} \left(\chi_* u_* - g_*\right) \tilde{c}_{q,t} \qquad (82) \\ & + \theta\chi_* c_* u_*' \tilde{c} + \left[-\theta\chi_* g_* u_*' + \theta\chi_* c_* u_*' + (1-\theta)\, \chi_* c_{q*} u_*\right] \tilde{\chi}_t \qquad (83) \end{aligned}$$

and determines $\tilde{g}_t$. Now consider

$$g_q = \frac{\chi u' c_q[\theta\chi u' + (1-\theta)c_q] + \theta(1-\theta)(\chi u - c)\left(\chi u' c_{qq} - c_q \chi u''\right)}{[\theta\chi u' + (1-\theta)c_q]^2}$$

In log-linear form, the equation can be rewritten as

$$\begin{aligned} & g_{q*} \left[\theta\chi_* u_*' + (1-\theta)\, c_{q*}\right]^2 \tilde{g}_{q,t} \\ = \quad & -\eta g_{q*} \left[\theta\chi_* u_*' + (1-\theta)\, c_{q*}\right] \left[\theta\chi_* u_*' \left(\tilde{u}_t + \tilde{\chi}_t\right) + (1-\theta)\, c_{q*} \tilde{c}_{q,t}\right] \qquad (84) \\ & + \chi_* u_*' c_{q*} \left[\theta\chi u_*' + (1-\theta)\, c_{q*}\right] \left(\tilde{u}_t' + \tilde{\chi}_t + \tilde{c}_{q,t}\right) \\ & + \theta \left(\chi_* u_*'\right)^2 c_{q*} \left(\tilde{u}_t' + \tilde{\chi}_t\right) + \chi_* \left(1-\theta\right) u_*' c_{q*}^2 \tilde{c}_{q,t} \\ & + \theta \left(1-\theta\right) \chi_* \left(u_*' c_{qq*} - c_{q*} u_*''\right) \left[\chi_* u_* \left(\tilde{u}_t + \tilde{\chi}_t\right) - c_* \tilde{c,} t\right] \\ & + \theta \left(1-\theta\right) \chi_* \left(\chi_* u_* - c_*\right) u_*' c_{qq*} \left(\tilde{u}_t' + \tilde{\chi}_t + \tilde{c}_{qq,tt}\right) \\ & - \theta \left(1-\theta\right) \chi_* \left(\chi_* u_* - c_*\right) u_*'' c_{q*} \left(\tilde{u}_t'' + \tilde{\chi}_t + \tilde{c}_{q,t}\right). \end{aligned}$$

Moreover,

$$g_k = \frac{\theta\chi u' c_k \left[\theta\chi u' + (1-\theta)c_q\right] + \theta(1-\theta)(\chi u - c)\chi u' c_{qk}}{\left[\theta\chi u' + (1-\theta)c_q\right]^2},$$

which leads to an equation for $\tilde{g}_{k,t}$:

$$\begin{aligned} & g_{k*}[\theta\chi u'_* + (1-\theta)c_{q*}]^2\tilde{g}_{k,t} \\ = \quad & -2g_{k*}[\theta\chi_* u'_* + (1-\theta)c_{q*}]\Big(\theta\chi_* u'_*(\tilde{u}_t\tilde{\chi}_t) + (1-\theta)c_{q*}\tilde{c}_{q,t}\Big) \\ & +\theta\chi_* u'_* c_{k*}\left[\theta\chi_* u'_* + (1-\theta)\,c_{q*}\right]\left(\tilde{u}'_t + \tilde{\chi}_t + \tilde{c}_{k,t}\right) \\ & + \left(\theta\chi_* u'_*\right)^2 c_{k*}\left(\tilde{u}'_t + \tilde{\chi}_t\right) + \chi_*\theta\,(1-\theta)\,u'_* c_{k*} c_{q*}\tilde{c}_{q,t} \\ & +\theta\,(1-\theta)\,\chi_*\left(\chi_* u_* - c_*\right)u'_* c_{qk*}\left(\tilde{u}'_t + \tilde{\chi}_t + \tilde{c}_{qk,t}\right) \\ & +\theta\,(1-\theta)\,\chi_* u'_* c_{qk*}\left[\chi_* u_*\left(\tilde{u}_t + \tilde{\chi}_t\right) - c_*\tilde{c}_t\right]. \end{aligned} \tag{85}$$

To summarize, Equations (81) to (85) determine $\tilde{\Gamma}_t$, $\tilde{g}_t$, $\tilde{g}_{q,t}$, and $\tilde{g}_{k,t}$. The first three variables appear in the characterization of the households' problem above.

Firms' Problems: Marginal costs evolve according to

$$M\tilde{C}_t = (1-\alpha)\tilde{w}_t + \alpha\tilde{R}^k_t - \tilde{Z}_t.. \tag{86}$$

Conditional on capital and factor prices, the labor demand is determined according to

$$\tilde{H}_t = \tilde{K}_t + \tilde{R}^k_t - \tilde{W}_t. \tag{87}$$

Since $\mathcal{F}^{(1)}_t$ and $\mathcal{F}^{(2)}_t$ are proportional, $\tilde{\mathcal{F}}^{(1)}_t = \tilde{\mathcal{F}}^{(2)}_t = \tilde{\mathcal{F}}_t$. The remaining optimality conditions can be written as follows.

$$\begin{aligned} \tilde{\mathcal{F}}_t \quad &= \quad (1-\mathcal{A})\left[-\frac{1+\lambda}{\lambda}\tilde{p}^o_t + \tilde{\mathcal{Y}}_t\right] \\ & \quad +\mathcal{A}\left[-\frac{\iota}{\lambda}\tilde{\pi}_t - \frac{1+\lambda}{\lambda}\tilde{p}^o_t + \frac{1+\lambda}{\lambda}\tilde{\pi}_{t+1} + \frac{1+\lambda}{\lambda}\tilde{p}^o_{t+1} + \tilde{\mathcal{F}}_{t+1} + \tilde{\Xi}^p_{t+1|t}\right] \\ \mathcal{A} \quad &= \quad \zeta\beta\left(\frac{1}{\pi_*}\right)^{-(1-\iota)/\lambda} \end{aligned} \tag{88}$$

and

$$\begin{aligned} \tilde{\mathcal{F}}_t \quad &= \quad (1-\mathcal{A})\left[-\left(\frac{1+\lambda}{\lambda}+1\right)\tilde{p}^o_t + \tilde{\mathcal{Y}}_t + M\tilde{C}_t\right] \\ & \quad +\mathcal{A}\Big[-\frac{\iota(1+\lambda)}{\lambda}\tilde{\pi}_t - \left(\frac{1+\lambda}{\lambda}+1\right)\tilde{p}^o_t + \left(\frac{1+\lambda}{\lambda}+1\right)\tilde{\pi}_{t+1} \\ & \quad + \left(\frac{1+\lambda}{\lambda}+1\right)\tilde{p}^o_{t+1} + \tilde{\mathcal{F}}_{t+1} + \tilde{\Xi}^p_{t+1|t}\Big] \\ \mathcal{A} \quad &= \quad \zeta\beta\left(\frac{1}{\pi_*}\right)^{-(1-\iota)(1+\lambda)/\lambda}. \end{aligned} \tag{89}$$

The relationship between the optimal price charged by the adjusting firms and the inflation rate is given by

$$\tilde{p}^o_t = (\mathcal{A}-1)\tilde{\pi}_t - \mathcal{A}\iota\zeta\left(\frac{1}{\pi_*}\right)^{-(1-\iota)/\lambda}\tilde{\pi}_{t-1} \tag{90}$$
$$\mathcal{A} = \frac{(p^o_*)^{1/\lambda}}{1-\zeta}$$

Equations (88) to (90) determine $\tilde{\pi}_t$, $\tilde{F}_t$, and $\tilde{\pi}^o_t$.

Resource Constraint, Market Clearing Conditions in the CM: Aggregate output across intermediate good firms evolves according to

$$\tilde{\dot{Y}}_t = \tilde{Y}_t + \tilde{D}_t = (1+\mathcal{F}/\dot{Y}_*)[\tilde{Z}_t + \alpha\tilde{K}_t + (1-\alpha)\tilde{H}_t]. \tag{91}$$

and the steady state price dispersion follows

$$\tilde{D}_t = \zeta\left(\frac{1}{\pi_*}\right)^{-\frac{(1+\lambda)(1-\iota)}{\lambda}}\left[\tilde{D}_{t-1} + \frac{(1+\lambda)}{\lambda}\tilde{\pi}_t - \frac{\iota(1+\lambda)}{\lambda}\tilde{\pi}_{t-1}\right] - \frac{p^o_*(1+\lambda)(1-\zeta)}{\lambda D_*}\tilde{p}^o_t \tag{92}$$

The goods market clearing condition is of the form

$$\tilde{Y}_t = \frac{X_*}{Y_*}\tilde{X}_t + \frac{I_*}{Y_*}\tilde{I}_t + \left(1-\frac{1}{g_*}\right)\frac{\mathcal{Y}_*}{Y_*}\mathcal{Y}_t + \frac{\mathcal{Y}_*}{Y_* g_*}\tilde{g}_t \tag{93}$$

and determines investment.

Aggregate Output and Prices, Measured Real Money Balances In log-linear terms, inflation in the DM evolves according to

$$\tilde{\pi}^{DM}_t = \tilde{M}_t - \tilde{M}_{t-1} - (\tilde{q}_t - \tilde{q}_{t-1}) + \tilde{\pi}_{t-1}. \tag{94}$$

Since all inflation rates share the same steady state, changes in the GDP deflator are given by

$$\tilde{\pi}^{GDP}_t = (1-s_*)\tilde{\pi}_t + s_*\tilde{\pi}^{DM}_t. \tag{95}$$

Real output in terms of the CM final good evolves according to

$$\tilde{\mathcal{Y}}_t = (1-s_*)\tilde{Y}_t + s_*(\tilde{M}_t - \tilde{\pi}_t). \tag{96}$$

As we showed in the main text, real GDP can be expressed as

$$\tilde{\mathcal{Y}}^{GDP}_t = (1-s_*)\tilde{Y}_t + s_*\tilde{q}_t + s_*(\tilde{\mathcal{M}}_0 - \tilde{\pi}_0 - \tilde{q}_0). \tag{97}$$

Finally, inverse velocity evolves according to

$$\widetilde{\mathcal{M}_{t+1}/\mathcal{Y}_t} = \tilde{\mathcal{M}}_{t+1} - \tilde{\mathcal{Y}}_t. \tag{98}$$

Monetary Policy: The monetary policy rule can be written as

$$\tilde{R}_t = \rho_R \tilde{R}_{t-1} + (1 - \rho_R)[\psi_1(\tilde{\pi}_t^{GDP} - \tilde{\pi}_t^*) + \psi_2(\tilde{\mathcal{Y}}_t^{GDP} - \tilde{\mathcal{Y}}_{t-1}^{GDP})] + \epsilon_{R,t}. \tag{99}$$

3 The MIU Model

The subsequent exposition is based on a slightly more general utility function:

$$U(x) = B\frac{x^{1-\gamma}}{1-\gamma}.$$

3.1 Equilibrium Conditions

Household's Problem: Given exogenous states, policy and prices,

$$U'(x_t) = \frac{A}{W_t} \tag{100}$$

$$1 = \beta E_t\left[\frac{U'(x_{t+1})}{U'(x_t)}\frac{R_t}{\pi_{t+1}}\right] \tag{101}$$

$$\begin{aligned} 1 = {} & \mu_t\left[1 - S\left(\frac{i_t}{i_{t-1}}\right) + \frac{i_t}{i_{t-1}}S'\left(\frac{i_t}{i_{t-1}}\right)\right] \\ & +\beta E_t\left\{\mu_{t+1}\frac{U'(x_{t+1})}{U'(x_t)}\left(\frac{i_{t+1}}{i_t}\right)^2 S'\left(\frac{i_{t+1}}{i_t}\right)\right\} \end{aligned} \tag{102}$$

$$k_{t+1} = (1-\delta)k_t + \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right] \tag{103}$$

$$\mu_t = \beta E_t\left\{\frac{U'(x_{t+1}}{U'(x_t)}\left[R^k_{t+1} + (1-\delta)\mu_{t+1}\right]\right\} \tag{104}$$

$$\frac{U'(x_t)}{P_t} = \beta E_t\left[\frac{U'(x_{t+1})}{P_{t+1}} + \frac{\chi_{t+1}}{P_{t+1}}\left(\frac{A}{Z_*^{1/1-\alpha}}\right)^{1-\nu_m}\left(\frac{M_{t+1}}{P_{t+1}}\right)^{-\nu_m}\right] \tag{105}$$

$$\Xi^p_{t+1|t} = \frac{U'(x_{t+1})}{U'(x_t)\pi_{t+1}} \tag{106}$$

As in the search-based model, we define $\mathcal{M}_{t+1} = M_{t+1}/P_t$.

Intermediate Goods Producing Firms' Problem: Intermediate goods firms choose their capital labor ratio as a function of the factor prices to minimize costs:

$$K_t = \frac{\alpha}{1-\alpha}\frac{W_t}{R^k_t}H_t. \tag{107}$$

Firms that are allowed to change prices are choosing a relative price $p^o_t(i)$ (relative to the aggregate price level) to maximize expected profits subject to the demand curve for their differentiated product, taking the aggregate price level P_t as well as the prices charged by other firms as given, which leads to

$$\begin{aligned} MC_t &= \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}W_t^{1-\alpha}(R_t^k)^\alpha Z_t^{-1} && (108)\\ \mathcal{F}_t^{(1)} &= (p_t^o)^{-\frac{1+\lambda}{\lambda}}Y_t + \zeta\beta\,(\pi_t^\iota)^{-1/\lambda}\,\mathbb{E}_t\left[\left(\frac{p_t^o}{\pi_{t+1}p_{t+1}^o}\right)^{-\frac{1+\lambda}{\lambda}}\Xi^p_{t+1|t}\mathcal{F}^{(1)}_{t+1}\right] && (109)\\ \mathcal{F}_t^{(2)} &= (p_t^o)^{-\frac{1+\lambda}{\lambda}-1}Y_tMC_t + \zeta\beta\,(\pi_t^\iota)^{-\frac{1+\lambda}{\lambda}}\,\mathbb{E}_t\left[\left(\frac{p_t^o}{\pi_{t+1}p_{t+1}^o}\right)^{-\frac{1+\lambda}{\lambda}-1}\Xi^p_{t+1|t}\mathcal{F}^{(2)}_{t+1}\right] && (110)\\ \mathcal{F}_t^{(1)} &= (1+\lambda)\mathcal{F}_t^{(2)} && (111) \end{aligned}$$

Final Good Producing Firms' Problem: Final goods producers take factor prices and output prices as given and choose inputs $Y_t(i)$ and output Y_t to maximize profits. Free entry ensures that final good producers make zero profits and leads to

$$\pi_t = \left[(1-\zeta)\,(\pi_t p_t^o)^{-\frac{1}{\lambda}} + \zeta(\pi_{t-1}^\iota \pi_{**}^{1-\iota})^{-\frac{1}{\lambda}}\right]^{-\lambda} \quad (112)$$

Aggregate Resource Constraint: is given by

$$Y_t = D_t^{-1}(Z_t K_t^\alpha H_t^{(1-\alpha)} - \mathcal{F}), \quad (113)$$

where

$$D_t = \zeta\left[\left(\frac{\pi_{t-1}}{\pi_t}\right)^\iota\left(\frac{1}{\pi_t}\right)^{(1-\iota)}\right]^{-\frac{1+\lambda}{\lambda}} D_{t-1} + (1-\zeta)\,(p_t^o)^{-\frac{1+\lambda}{\lambda}}\,. \quad (114)$$

The gross domestic product of this economy is given by $\mathcal{Y}_t = Y_t$.

Market Clearing: The goods market in the CM clears:

$$X_t + I_t + \left(1 - \frac{1}{g_t}\right)Y_t = Y_t \quad (115)$$

Monetary Policy: The central bank supplies the quantity of money necessary to attain the nominal interest rate

$$R_t = R_{*,t}^{1-\rho_R}R_{t-1}^{\rho_R}\exp\{\sigma_R\epsilon_{R,t}\}, \quad R_{*,t} = (r_*\pi_{*,t})\left(\frac{\pi_t}{\pi_{*,t}}\right)^{\psi_1}\left(\frac{Y_t}{\gamma Y_{t-1}}\right)^{\psi_2} \quad (116)$$

3.2 Steady States

For estimation purposes it is useful to parameterize the model in terms of $\mathcal{Y}_* = Y_*$, H_*, and $\mathcal{M}_*$ and solve the steady state conditions for A, B, and Z_*.

$$
\begin{aligned}
R_* &= \pi_*/\beta \\
p_*^o &= \left[\frac{1}{1-\zeta} - \frac{\zeta}{1-\zeta}\left(\frac{1}{\pi_*}\right)^{-\frac{1-\iota}{\lambda}}\right]^{-\lambda} \\
R_*^k &= \frac{1}{\beta} + \delta - 1 \\
D_* &= \frac{(1-\zeta)(p_*^o)^{-\frac{1+\lambda}{\lambda}}}{1-\zeta\left(\frac{1}{\pi_*}\right)^{-\frac{(1+\lambda)(1-\iota)}{\lambda}}} \\
\bar{Y}_* &= Y_* D_* \\
Z_* &= (\bar{Y}_* + \mathcal{F})/(K_*^\alpha H_*^{1-\alpha}) \\
K_* &= \frac{\alpha(\bar{Y}_* + \mathcal{F})p_*^o}{(1+\lambda)R_*^k}\left[\frac{1-\zeta\beta\left(\frac{1}{\pi_*}\right)^{-(1-\iota)/\lambda}}{1-\zeta\beta\left(\frac{1}{\pi_*}\right)^{-(1-\iota)(1+\lambda)/\lambda}}\right]^{-1} \\
W_* &= \frac{1-\alpha}{\alpha}\frac{K_*}{H_*}R_*^k \\
I_* &= \delta K_* \\
X_* &= Y_* - I_* - (1-1/g_*)Y_* \\
A &= \frac{1}{\mathcal{M}_*}\left[\frac{\chi_* \pi_*^{\nu_m} W_*}{(R_*-1)Z_*^{(1-\nu_m)/(1-\alpha)}}\right]^{1/\nu_m} \\
U'_* &= A/W_* \\
B &= U'_* X_*^\gamma
\end{aligned}
$$

3.3 Log-Linearizations

We will frequently use equation-specific constants, such as $\mathcal{A}$ and $\mathcal{B}$. Variables dated $t+1$ refer to time t conditional expectations.

Household's Problem: The optimality conditions for the household can be expressed as

$$\begin{aligned}
\tilde{W}_t &= \frac{1}{\gamma}\tilde{X}_t & (117)\\
-\gamma\tilde{X}_t &= -\gamma\tilde{X}_{t+1} + (\tilde{R}_t - \tilde{\pi}_{t+1}) & (118)\\
\tilde{i}_t &= \frac{1}{1+\beta}\tilde{i}_{t-1} + \frac{\beta}{1+\beta}\tilde{i}_{t+1} + \frac{1}{(1+\beta)S''}\tilde{\mu}_t & (119)\\
\tilde{k}_{t+1} &= (1-\delta)\tilde{k}_t + \delta\tilde{i}_t & (120)\\
\tilde{\mu}_t - \gamma\tilde{X}_t &= \beta(1-\delta)\tilde{\mu}_{t+1} - \gamma\tilde{X}_{t+1} + \beta R_*^k\tilde{R}_{t+1}^k & (121)\\
\nu_m\tilde{\mathcal{M}}_{t+1} &= \gamma\tilde{X}_t + \nu_m\tilde{\chi}_{t+1} - (1-\nu_m)\tilde{\pi}_{t+1} - \frac{1}{R_*-1}\tilde{R}_t & (122)\\
\tilde{\Xi}^p_{t|t-1} &= -\gamma(\tilde{X}_t - \tilde{X}_{t-1}) - \tilde{\pi}_t. & (123)
\end{aligned}$$

Equations (117) to (123) determine wages, consumption, investment, capital, the shadow value of installed capital, the rental rate of capital, real money balances, and the stochastic discount factor.

Firms' Problems: Marginal costs evolve according to

$$\tilde{MC}_t = (1-\alpha)\tilde{w}_t + \alpha\tilde{R}_t^k - \tilde{Z}_t. \quad (124)$$

Conditional on capital, the labor demand is determined according to

$$\tilde{H}_t = \tilde{K}_t + \tilde{R}_t^k - \tilde{W}_t \quad (125)$$

Since $\mathcal{F}_t^{(1)}$ and $\mathcal{F}_t^{(2)}$ are proportional, $\tilde{\mathcal{F}}_t^{(1)} = \tilde{\mathcal{F}}_t^{(2)} = \tilde{\mathcal{F}}_t$. The remaining optimality conditions can be written as follows.

$$\begin{aligned}
\tilde{\mathcal{F}}_t &= (1-\mathcal{A})\left[-\frac{1+\lambda}{\lambda}\tilde{p}_t^o + \tilde{\mathcal{Y}}_t\right] & (126)\\
&\quad +\mathcal{A}\left[-\frac{\iota}{\lambda}\tilde{\pi}_t - \frac{1+\lambda}{\lambda}\tilde{p}_t^o + \frac{1+\lambda}{\lambda}\tilde{\pi}_{t+1} + \frac{1+\lambda}{\lambda}\tilde{p}_{t+1}^o + \tilde{\mathcal{F}}_{t+1} + \tilde{\Xi}^p_{t+1|t}\right]\\
\mathcal{A}_1 &= \zeta\beta\left(\frac{1}{\pi_*}\right)^{-(1-\iota)/\lambda}
\end{aligned}$$

and

$$\tilde{\mathcal{F}}_t = (1-\mathcal{A})\left[-\left(\frac{1+\lambda}{\lambda}+1\right)\tilde{p}^o_t + \tilde{\mathcal{Y}}_t + \tilde{MC}_t\right] \quad (127)$$
$$+\mathcal{A}\Bigg[-\frac{\iota(1+\lambda)}{\lambda}\tilde{\pi}_t - \left(\frac{1+\lambda}{\lambda}+1\right)\tilde{p}^o_t + \left(\frac{1+\lambda}{\lambda}+1\right)\tilde{\pi}_{t+1}$$
$$+\left(\frac{1+\lambda}{\lambda}+1\right)\tilde{p}^o_{t+1} + \tilde{\mathcal{F}}_{t+1} + \tilde{\Xi}^p_{t+1|t}\Bigg]$$
$$\mathcal{A}_2 = \zeta\beta\left(\frac{1}{\pi_*}\right)^{-(1-\iota)(1+\lambda)/\lambda}.$$

The relationship between the optimal price charged by the adjusting firms and the inflation rate is given by

$$\tilde{p}^o_t = (\mathcal{A}-1)\tilde{\pi}_t - \mathcal{A}\iota\zeta\left(\frac{1}{\pi_*}\right)^{-(1-\iota)/\lambda}\tilde{\pi}_{t-1} \quad (128)$$
$$\mathcal{A}_p = \frac{(p^o_*)^{1/\lambda}}{1-\zeta}$$

Equations (126) to (128) determine $\tilde{\pi}_t$, $\tilde{\mathcal{F}}_t$, and $\tilde{p}^o_t$.

Resource Constraint, Market Clearing Conditions: Aggregate output across evolves according to

$$\tilde{\bar{Y}}_t = \tilde{Y}_t + \tilde{D}_t = (1+\mathcal{F}/\bar{Y}_*)[\tilde{Z}_t + \alpha\tilde{K}_t + (1-\alpha)\tilde{H}_t]. \quad (129)$$

and the steady state price dispersion follows

$$\tilde{D}_t = \zeta\left(\frac{1}{\pi_*}\right)^{-\frac{(1+\lambda)(1-\iota)}{\lambda}}\left[\tilde{D}_{t-1} + \frac{(1+\lambda)}{\lambda}\tilde{\pi}_t - \frac{\iota(1+\lambda)}{\lambda}\tilde{\pi}_{t-1}\right] - \frac{p^o_*(1+\lambda)(1-\zeta)}{\lambda D_*}\tilde{p}^o_t \quad (130)$$

The goods market clearing condition is of the form

$$\tilde{Y}_t = \frac{X_*}{X_*+I_*}\tilde{X}_t + \frac{I_*}{X_*+I_*}\tilde{I}_t + \tilde{g}_t. \quad (131)$$

Monetary Policy: The monetary policy rule can be written as

$$\tilde{R}_t = \rho_R\tilde{R}_{t-1} + (1-\rho_R)[\psi_1(\tilde{\pi}_t - \tilde{\pi}^*_t) + \psi_2(\tilde{Y}_t - \tilde{Y}_{t-1})] + \epsilon_{R,t}. \quad (132)$$

4 Construction of Target Inflation Series

We apply a bandpass filter to the GDP deflator inflation rate. Since the agents generate forecasts of future target inflation rates with a random walk model we will use a one-sided bandpass filter that removes cycles of a duration of less than 64 quarters. Our filter is based on the approach by Geweke (1978) and Pierce (1980). We construct a time-domain moving average representation of the ideal one-sided filter (truncated at 500 lags) and then replace missing lagged observations by optimal backcasts obtained from an estimated AR(4) model. The resulting filtered inflation series is plotted in the top left panel of Figure 1 in the published paper. The panel also shows 1-year and 10-year-ahead inflation expectations obtained from the Survey of Professional Forecasters, maintained by the Federal Reserve Bank of Philadelphia.

To combine the three series we use a small state-space model with measurement equations

$$\widetilde{\pi}_t^{BP} = \widetilde{\pi}_{*,t} + 0.025\epsilon_{1,t}, \quad \widetilde{\pi}_t^{1y} = \widetilde{\pi}_{*,t} + \eta_{2,t}, \quad \widetilde{\pi}_t^{10y} = \widetilde{\pi}_{*,t} + \eta_{3,t},$$

and state transitions

$$\widetilde{\pi}_{*,t} = \widetilde{\pi}_{*,t-1} + \sigma_\pi \epsilon_{\pi,t}, \quad \eta_{2,t} = \rho_2 \eta_{2,t-1} + \sigma_2 \epsilon_{2,t}, \quad \eta_{3,t} = \rho_3 \eta_{3,t-1} + \sigma_3 \epsilon_{3,t},$$

where the $\epsilon_{i,t}$'s are *iid* standard normal random variables and $\widetilde{\pi}_t^{BP}$, $\widetilde{\pi}_t^{1y}$, and $\widetilde{\pi}_t^{10y}$ are bandpass filtered inflation, 1-year-ahead forecasts, and 10-year-ahead forecasts, respectively. We fixed the innovation standard deviation for $\widetilde{\pi}_t^{BP}$ to implicitly control the weight on the bandpass filtered series and estimated the remaining parameters. If one regresses the filtered series $\widetilde{\pi}_{*,t}$ on the three observed measures, the coefficients are 0.57 ($\widetilde{\pi}_t^{BP}$), 0.22 ($\widetilde{\pi}_t^{1y}$), and 0.23 ($\widetilde{\pi}_t^{10y}$). Moreover, the dynamics of $\widetilde{\pi}_{*,t}$ are well approximated by the random walk that the DSGE model agents use to forecast the target inflation rate.

5 Supplemental Tables and Figures

Table 1: compares unrestricted and restricted ($\sigma = 0.06$) parameter estimates for the SBM(B) model.

Table 2: compares unrestricted and restricted ($\sigma = 0.06$) parameter estimates for the SBM(PT) model.

Table 2: compares unrestricted and restricted ($\nu = 5.17$) parameter estimates for the MIU.

Table 4: compares posterior means of DSGE model implied steady states.

Table 5: compares variance decompositions from MIU and SBM(B).

Table 6: conditional on the posterior mean parameter estimates, we simulate a sample of 10,000 observations and report inflation standard deviations and first-order autocorrelations. While the autocorrelation of CM inflation is around 0.9, the autocorrelation of DM inflation is slightly negative. As a consequence, the autocorrelation of GDP deflator inflation is between 0.35 to 0.5, which is smaller than in the estimated MIU model.

Table 7: we construct a posterior predictive distribution for the correlation between interest rates and inverse velocity conditional on the target inflation shock. It is only if we fix σ and ν in the DSGE models to values that imply large interest rate elasticities of money demand that the DSGE model implied posterior predictive distribution matches that implied by the VAR.

Figure 1: The top panel depicts the welfare gain of reducing the target inflation rate below 2.5%. In the bottom panel we report the posterior expected probability that the regret of choosing a particular target inflation rate is more than 0.01%. For the estimated value of ν welfare is maximized at 0% inflation, which is the prediction of a cashless DSGE model. If ν is choosen to match the long-run interest rate elasticity, the optimal target inflation rate is around -1%.

Table 1: Posterior Distributions: Unrestricted versus Restricted SBM(B)

	SBM(B) σ estim.		SBM(B) $\sigma = 0.06$	
Name	Mean	90% Intv	Mean	90% Intv
		Household		
θ	0.95	[0.95, 0.96]	0.96	[0.95, 0.97]
$\tilde{\sigma}$	0.63	[0.56, 0.70]	0.13	[0.13, 0.13]
		Firms		
α	0.32	[0.31, 0.34]	0.29	[0.28, 0.30]
λ	0.14	[0.12, 0.16]	0.16	[0.15, 0.18]
ζ	0.83	[0.79, 0.87]	0.79	[0.75, 0.83]
ι	0.72	[0.54, 0.91]	0.14	[0.00, 0.28]
S''	4.89	[2.50, 7.36]	5.40	[3.05, 8.02]
		Central Bank		
ψ_2	0.86	[0.64, 1.06]	0.87	[0.71, 1.03]
ρ_R	0.61	[0.56, 0.66]	0.65	[0.61, 0.70]
σ_R	0.36	[0.31, 0.41]	0.33	[0.28, 0.37]
$\sigma_{R,2}$	0.85	[0.63, 1.07]	0.78	[0.58, 0.98]
$\tilde{\pi}^*_{0,A}$	0.05	[-3.21, 3.26]	-0.68	[-3.57, 2.75]
σ_π	0.05	[0.04, 0.05]	0.05	[0.04, 0.05]
		Shocks		
ρ_g	0.84	[0.81, 0.88]	0.87	[0.83, 0.90]
σ_g	1.01	[0.90, 1.11]	1.09	[0.96, 1.21]
ρ_χ	0.97	[0.97, 0.98]	0.91	[0.88, 0.95]
σ_χ	1.80	[1.63, 1.97]	4.08	[3.67, 4.51]
ρ_z	0.83	[0.76, 0.90]	0.77	[0.70, 0.84]
σ_z	1.04	[0.90, 1.17]	1.89	[1.40, 2.40]

Table 2: Posterior Distributions: Unrestricted versus Restricted SBM(PT)

	SBM(PT) σ estim.		SBM(PT) $\sigma = 0.06$	
Name	Mean	90% Intv	Mean	90% Intv
		Household		
θ	0.00	[0.00, 0.00]	0.00	[0.00, 0.00]
$\tilde{\sigma}$	0.59	[0.52, 0.66]	0.13	[0.13, 0.13]
		Firms		
α	0.27	[0.26, 0.28]	0.28	[0.27, 0.29]
λ	0.19	[0.18, 0.21]	0.17	[0.16, 0.19]
ζ	0.84	[0.80, 0.88]	0.80	[0.75, 0.86]
ι	0.57	[0.31, 0.82]	0.20	[0.00, 0.41]
S''	5.08	[2.42, 7.71]	5.48	[2.71, 8.11]
		Central Bank		
ψ_2	0.83	[0.64, 1.02]	0.88	[0.69, 1.06]
ρ_R	0.60	[0.55, 0.65]	0.65	[0.61, 0.70]
σ_R	0.37	[0.31, 0.42]	0.33	[0.29, 0.38]
$\sigma_{R,2}$	0.85	[0.62, 1.08]	0.80	[0.58, 1.01]
$\tilde{\pi}^*_{0,A}$	0.02	[-3.22, 3.28]	0.01	[-3.40, 3.33]
σ_π	0.05	[0.04, 0.05]	0.05	[0.04, 0.05]
		Shocks		
ρ_g	0.87	[0.83, 0.90]	0.87	[0.83, 0.90]
σ_g	1.06	[0.94, 1.16]	1.09	[0.96, 1.21]
ρ_χ	0.96	[0.95, 0.97]	0.91	[0.88, 0.94]
σ_χ	1.88	[1.70, 2.05]	4.11	[3.67, 4.53]
ρ_z	0.83	[0.77, 0.89]	0.75	[0.67, 0.83]
σ_z	1.06	[0.91, 1.21]	2.13	[1.38, 2.88]

Table 3: Posterior Distributions: Unrestricted versus Restricted MIU

	MIU ν estim.		MIU $\nu = 5.17$	
Name	Mean	90% Intv	Mean	90% Intv
		Households		
ν	31.754	[24.764, 38.079]	5.167	[5.167, 5.167]
		Firms		
α	0.282	[0.271, 0.293]	0.282	[0.271, 0.292]
λ	0.165	[0.151, 0.179]	0.165	[0.151, 0.178]
ζ	0.756	[0.728, 0.784]	0.750	[0.719, 0.785]
ι	0.036	[0.000, 0.073]	0.039	[0.000, 0.079]
S''	5.285	[2.640, 7.963]	4.988	[2.460, 7.468]
		Central Bank		
ψ_2	1.027	[0.846, 1.224]	1.024	[0.836, 1.209]
ρ_R	0.669	[0.622, 0.719]	0.658	[0.606, 0.710]
σ_R	0.338	[0.284, 0.389]	0.346	[0.290, 0.403]
$\sigma_{R,2}$	0.810	[0.572, 1.020]	0.830	[0.591, 1.052]
$\tilde{\pi}^*_{0,A}$	-0.058	[-3.439, 3.126]	0.033	[-3.262, 3.461]
σ_π	0.049	[0.044, 0.053]	0.049	[0.044, 0.053]
		Shocks		
ρ_g	0.896	[0.865, 0.931]	0.884	[0.847, 0.923]
σ_g	1.140	[0.989, 1.299]	1.095	[0.938, 1.239]
ρ_χ	0.982	[0.974, 0.991]	0.954	[0.929, 0.979]
σ_χ	1.298	[1.170, 1.415]	3.279	[2.985, 3.611]
ρ_z	0.799	[0.719, 0.887]	0.823	[0.745, 0.904]
σ_Z	2.082	[1.451, 2.696]	1.927	[1.293, 2.576]

Table 4: Steady States (Posterior Means)

	SBM(B)		SBM(PT)		MIU	
	σ estim.	$\sigma = 0.06$	σ estim.	$\sigma = 0.06$	ν estim.	$\nu = 5.17$
A	16.1	14.6	24.3	20.6	18.6	40.0
B	0.44	0.52	0.65	0.73	0.70	1.48
Z_*	4.10	5.32	5.48	5.56	5.54	5.54
$I_*/\mathcal{Y}_*$	0.16	0.16	0.16	0.17	0.17	0.17
$K_*/\mathcal{Y}_*$	11.1	11.7	11.8	11.9	12.0	12.0
W_*H_*/Y_*	0.60	0.61	0.61	0.62	0.62	0.62
Overall Markup	0.14	0.16	0.16	0.16	0.17	0.17
DM Share	0.21	0.04	0.20	0.04		
DM Markup	0.17	0.12	0.000			

Notes: Aggregate output is normalized to $\mathcal{Y}_* = 1$ in all economies.

Table 5: Posterior Variance Decomposition (Business Cycle Freq)

Shock	SBM(B)		MIU	
	Mean	90% Intv	Mean	90% Intv
Output				
Gov Spending	0.51	[0.43, 0.61]	0.43	[0.35, 0.50]
Money Demand	0.05	[0.03, 0.07]	0.00	[0.00, 0.00]
Monetary Policy	0.12	[0.07, 0.17]	0.16	[0.11, 0.22]
Technology	0.32	[0.23, 0.40]	0.40	[0.33, 0.52]
Target Inflation	0.01	[0.00, 0.01]	0.01	[0.01, 0.02]
Inflation				
Gov Spending	0.18	[0.14, 0.23]	0.05	[0.03, 0.06]
Money Demand	0.01	[0.00, 0.01]	0.00	[0.00, 0.00]
Monetary Policy	0.23	[0.17, 0.28]	0.13	[0.09, 0.17]
Technology	0.50	[0.45, 0.58]	0.71	[0.67, 0.77]
Target Inflation	0.08	[0.05, 0.12]	0.11	[0.07, 0.13]
Inverse Velocity				
Gov Spending	0.44	[0.38, 0.49]	0.34	[0.28, 0.40]
Money Demand	0.52	[0.46, 0.57]	0.52	[0.47, 0.58]
Monetary Policy	0.02	[0.02, 0.03]	0.02	[0.01, 0.03]
Technology	0.02	[0.01, 0.03]	0.11	[0.08, 0.17]
Target Inflation	0.00	[0.00, 0.00]	0.01	[0.00, 0.01]
Real Money Balances				
Gov Spending	0.11	[0.07, 0.14]	0.07	[0.04, 0.10]
Money Demand	0.70	[0.65, 0.74]	0.89	[0.84, 0.92]
Monetary Policy	0.13	[0.09, 0.17]	0.03	[0.02, 0.04]
Technology	0.07	[0.05, 0.11]	0.01	[0.00, 0.02]
Target Inflation	0.00	[0.00, 0.00]	0.00	[0.00, 0.01]

Table 6: Inflation Volatility and Persistence

Model		Std Dev			AC(1)		
		$\tilde{\pi}^{GDP}$	$\tilde{\pi}^{CM}$	$\tilde{\pi}^{DM}$	$\tilde{\pi}^{GDP}$	$\tilde{\pi}^{CM}$	$\tilde{\pi}^{DM}$
SBM(B)	σ estimated	1.54	1.13	5.13	0.34	0.91	-0.11
SBM(B)	$\sigma = 0.06$	1.39	1.20	10.5	0.53	0.74	-0.17
SBM(PT)	σ estimated	1.43	1.04	5.26	0.40	0.90	-0.06
SBM(PT)	$\sigma = 0.06$	1.35	1.16	11.2	0.51	0.75	-0.18
MIU	ν estimated	1.70			0.80		
MIU	$\nu = 5.17$	1.64			0.78		

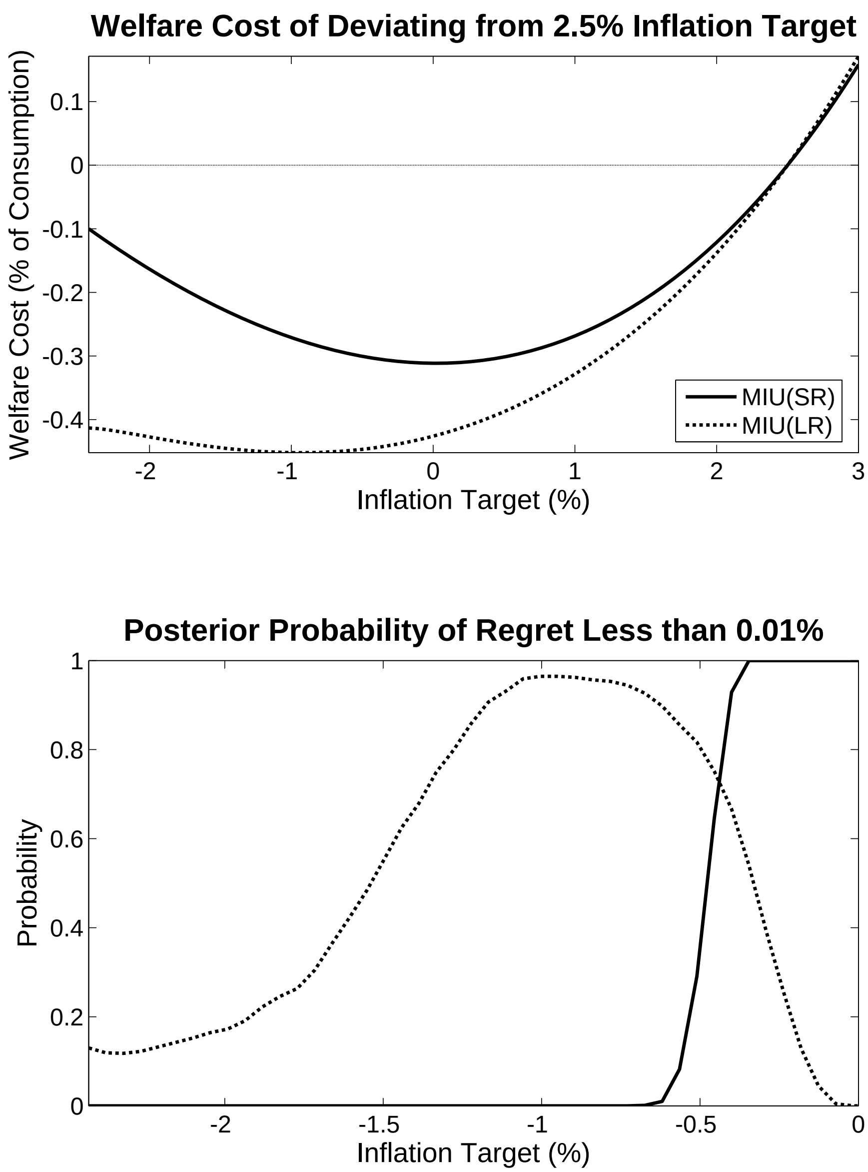
Notes: Sample moments are computed based on simulated time series of 10,000 observation, conditional on posterior mean estimate. The target inflation shock is set to zero. AC(1) is the first-order autocorrelation.

Table 7: Sample Moments Conditional on Target Inflation Shocks

		StD(Interest)		STD(Inv.Veloc.)		Corr(Interest, Inv.Veloc.)	
		Mean	90% Intv	Mean	90% Intv	Mean	90% Intv
SBM(B)	σ estimated	1.18	[0.53, 1.97]	.003	[.001, 005]	-0.44	[-0.99, 0.45]
SBM(B)	$\sigma = 0.06$	1.20	[0.59, 1.85]	0.02	[0.01, 0.04]	-0.90	[-0.99, -0.80]
SBM(PT)	σ estimated	1.19	[0.52, 2.11]	.003	[.001, 005]	-0.28	[-0.97, 0.54]
SBM(PT)	$\sigma = 0.06$	1.18	[0.51, 1.91]	0.02	[0.01, 0.04]	-0.91	[-0.99, -0.81]
MIU	ν estimated	1.21	[0.55, 1.87]	0.01	[.004, 0.02]	0.54	[0.09, 0.94]
MIU	$\nu = 5.17$	1.20	[0.55, 1.89]	0.03	[0.01, 0.04]	-0.96	[-0.99, -0.92]
VAR(4)		0.39	[0.10, 0.71]	0.01	[.002, 0.03]	-0.88	[-0.99, -0.93]

Notes: For the three models we report means and 90% credible intervals of the predictive distribution of sample moments (computed from 200 artificial observations) conditional on the target inflation shock $\epsilon_{\pi,t}$.

Figure 1: Welfare Implications of Estimated MIU Model



Notes: The top panel depicts the welfare gain of reducing the target inflation rate below 2.5%. MIU(SR) refers to the unrestricted version and MIU(LR) refers to the version in which we restrict $\nu = 5.17$. In the bottom panel we report the posterior expected probability that the regret of choosing a particular target inflation rate is more than 0.01%.

Additional References

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