

# Technical Appendix to “Estimation and Evaluation of DSGE Models: Progress and Challenges”

## A DSGE Model

The subsequent exposition is based on a slightly more general utility function:

$$U(x) = B \frac{x^{1-\gamma}}{1-\gamma}.$$

### A.1 Equilibrium Conditions

**Household’s Problem:** Given exogenous states, policy and prices,

$$U'(x_t) = \frac{A}{W_t} \tag{A.1}$$

$$1 = \beta E_t \left[ \frac{U'(x_{t+1})}{U'(x_t)} \frac{R_t}{\pi_{t+1}} \right] \tag{A.2}$$

$$1 = \mu_t \left[ 1 - S \left( \frac{i_t}{i_{t-1}} \right) + \frac{i_t}{i_{t-1}} S' \left( \frac{i_t}{i_{t-1}} \right) \right] \\ + \beta E_t \left\{ \mu_{t+1} \frac{U'(x_{t+1})}{U'(x_t)} \left( \frac{i_{t+1}}{i_t} \right)^2 S' \left( \frac{i_{t+1}}{i_t} \right) \right\} \tag{A.3}$$

$$k_{t+1} = (1 - \delta)k_t + \left[ 1 - S \left( \frac{i_t}{i_{t-1}} \right) \right] \tag{A.4}$$

$$\mu_t = \beta E_t \left\{ \frac{U'(x_{t+1})}{U'(x_t)} [R_{t+1}^k + (1 - \delta)\mu_{t+1}] \right\} \tag{A.5}$$

$$\frac{U'(x_t)}{P_t} = \beta E_t \left[ \frac{U'(x_{t+1})}{P_{t+1}} + \frac{\chi_{t+1}}{P_{t+1}} \left( \frac{A}{Z_*^{1/1-\alpha}} \right)^{1-\nu_m} \left( \frac{M_{t+1}}{P_{t+1}} \right)^{-\nu_m} \right] \tag{A.6}$$

$$\Xi_{t+1|t}^p = \frac{U'(x_{t+1})}{U'(x_t)\pi_{t+1}} \tag{A.7}$$

As in the search-based model, we define  $\mathcal{M}_{t+1} = M_{t+1}/P_t$ .

**Intermediate Goods Producing Firms’ Problem:** Intermediate goods firms choose their capital labor ratio as a function of the factor prices to minimize costs:

$$K_t = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k} H_t. \tag{A.8}$$

Firms that are allowed to change prices are choosing a relative price  $p_t^o(i)$  (relative to the aggregate price level) to maximize expected profits subject to the demand curve for their differentiated product, taking the aggregate price level  $P_t$  as well as the prices charged by other firms as given, which leads to

$$MC_t = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}W_t^{1-\alpha}(R_t^k)^\alpha Z_t^{-1} \quad (\text{A.9})$$

$$\mathcal{F}_t^{(1)} = (p_t^o)^{-\frac{1+\lambda}{\lambda}}Y_t + \zeta\beta(\pi_t^\iota)^{-1/\lambda}\mathbb{E}_t\left[\left(\frac{p_t^o}{\pi_{t+1}p_{t+1}^o}\right)^{-\frac{1+\lambda}{\lambda}}\Xi_{t+1|t}^p\mathcal{F}_{t+1}^{(1)}\right] \quad (\text{A.10})$$

$$\mathcal{F}_t^{(2)} = (p_t^o)^{-\frac{1+\lambda}{\lambda}-1}Y_tMC_t + \zeta\beta(\pi_t^\iota)^{-\frac{1+\lambda}{\lambda}}\mathbb{E}_t\left[\left(\frac{p_t^o}{\pi_{t+1}p_{t+1}^o}\right)^{-\frac{1+\lambda}{\lambda}-1}\Xi_{t+1|t}^p\mathcal{F}_{t+1}^{(2)}\right] \quad (\text{A.11})$$

$$\mathcal{F}_t^{(1)} = (1+\lambda)\mathcal{F}_t^{(2)} \quad (\text{A.12})$$

**Final Good Producing Firms' Problem:** Final goods producers take factor prices and output prices as given and choose inputs  $Y_t(i)$  and output  $Y_t$  to maximize profits. Free entry ensures that final good producers make zero profits and leads to

$$\pi_t = \left[(1-\zeta)(\pi_t p_t^o)^{-\frac{1}{\lambda}} + \zeta(\pi_{t-1}^\iota \pi_{**}^{1-\iota})^{-\frac{1}{\lambda}}\right]^{-\lambda} \quad (\text{A.13})$$

**Aggregate Resource Constraint:** is given by

$$Y_t = D_t^{-1}(Z_t K_t^\alpha H_t^{1-\alpha}) - \mathcal{F}, \quad (\text{A.14})$$

where

$$D_t = \zeta \left[ \left( \frac{\pi_{t-1}}{\pi_t} \right)^\iota \left( \frac{1}{\pi_t} \right)^{(1-\iota)} \right]^{-\frac{1+\lambda}{\lambda}} D_{t-1} + (1-\zeta)(p_t^o)^{-\frac{1+\lambda}{\lambda}}. \quad (\text{A.15})$$

The gross domestic product of this economy is given by  $\mathcal{Y}_t = Y_t$ .

**Market Clearing:** The goods market in the CM clears:

$$X_t + I_t + \left(1 - \frac{1}{g_t}\right)Y_t = Y_t \quad (\text{A.16})$$

**Monetary Policy:** The central bank supplies the quantity of money necessary to attain the nominal interest rate

$$R_t = R_{*,t}^{1-\rho_R} R_{t-1}^{\rho_R} \exp\{\sigma_R \epsilon_{R,t}\}, \quad R_{*,t} = (r_* \pi_{*,t}) \left( \frac{\pi_t}{\pi_{*,t}} \right)^{\psi_1} \left( \frac{Y_t}{\gamma Y_{t-1}} \right)^{\psi_2} \quad (\text{A.17})$$

## A.2 Steady States

For estimation purposes it is useful to parameterize the model in terms of  $\mathcal{Y}_* = Y_*$ ,  $H_*$ , and  $\mathcal{M}_*$  and solve the steady state conditions for  $A$ ,  $B$ , and  $Z_*$ .

$$\begin{aligned}
R_* &= \pi_*/\beta \\
p_*^o &= \left[ \frac{1}{1-\zeta} - \frac{\zeta}{1-\zeta} \left( \frac{1}{\pi_*} \right)^{-\frac{1-\iota}{\lambda}} \right]^{-\lambda} \\
R_*^k &= \frac{1}{\beta} + \delta - 1 \\
D_* &= \frac{(1-\zeta)(p_*^o)^{-\frac{1+\lambda}{\lambda}}}{1-\zeta \left( \frac{1}{\pi_*} \right)^{-\frac{(1+\lambda)(1-\iota)}{\lambda}}} \\
\bar{Y}_* &= Y_* D_* \\
Z_* &= (\bar{Y}_* + \mathcal{F}) / (K_*^\alpha H_*^{1-\alpha}) \\
K_* &= \frac{\alpha(\bar{Y}_* + \mathcal{F})p_*^o}{(1+\lambda)R_*^k} \left[ \frac{1 - \zeta\beta \left( \frac{1}{\pi_*} \right)^{-(1-\iota)/\lambda}}{1 - \zeta\beta \left( \frac{1}{\pi_*} \right)^{-(1-\iota)(1+\lambda)/\lambda}} \right]^{-1} \\
W_* &= \frac{1-\alpha}{\alpha} \frac{K_*}{H_*} R_*^k \\
I_* &= \delta K_* \\
X_* &= Y_* - I_* - (1 - 1/g_*)Y_* \\
A &= \frac{1}{\mathcal{M}_*} \left[ \frac{\chi_* \pi_*^{\nu_m} W_*}{(R_* - 1) Z_*^{(1-\nu_m)/(1-\alpha)}} \right]^{1/\nu_m} \\
U'_* &= A/W_* \\
B &= U'_* X_*^\gamma
\end{aligned}$$

## A.3 Log-Linearizations

We will frequently use equation-specific constants, such as  $\mathcal{A}$  and  $\mathcal{B}$ . Variables dated  $t + 1$  refer to time  $t$  conditional expectations.

**Household's Problem:** The optimality conditions for the household can be expressed as

$$\tilde{W}_t = \frac{1}{\gamma} \tilde{X}_t \quad (\text{A.18})$$

$$-\gamma \tilde{X}_t = -\gamma \tilde{X}_{t+1} + (\tilde{R}_t - \tilde{\pi}_{t+1}) \quad (\text{A.19})$$

$$\tilde{i}_t = \frac{1}{1+\beta} \tilde{i}_{t-1} + \frac{\beta}{1+\beta} \tilde{i}_{t+1} + \frac{1}{(1+\beta)S''} \tilde{\mu}_t \quad (\text{A.20})$$

$$\tilde{k}_{t+1} = (1-\delta)\tilde{k}_t + \delta\tilde{i}_t \quad (\text{A.21})$$

$$\tilde{\mu}_t - \gamma \tilde{X}_t = \beta(1-\delta)\tilde{\mu}_{t+1} - \gamma \tilde{X}_{t+1} + \beta R_*^k \tilde{R}_{t+1}^k \quad (\text{A.22})$$

$$\nu_m \tilde{\mathcal{M}}_{t+1} = \gamma \tilde{X}_t + \nu_m \tilde{\chi}_{t+1} - (1-\nu_m)\tilde{\pi}_{t+1} - \frac{1}{R_* - 1} \tilde{R}_t \quad (\text{A.23})$$

$$\tilde{\Xi}_{t|t-1}^p = -\gamma(\tilde{X}_t - \tilde{X}_{t-1}) - \tilde{\pi}_t. \quad (\text{A.24})$$

Equations (A.18) to (A.24) determine wages, consumption, investment, capital, the shadow value of installed capital, the rental rate of capital, real money balances, and the stochastic discount factor.

**Firms' Problems:** Marginal costs evolve according to

$$\tilde{M}C_t = (1-\alpha)\tilde{w}_t + \alpha \tilde{R}_t^k - \tilde{Z}_t. \quad (\text{A.25})$$

Conditional on capital, the labor demand is determined according to

$$\tilde{H}_t = \tilde{K}_t + \tilde{R}_t^k - \tilde{W}_t \quad (\text{A.26})$$

Since  $\tilde{\mathcal{F}}_t^{(1)}$  and  $\tilde{\mathcal{F}}_t^{(2)}$  are proportional,  $\tilde{\mathcal{F}}_t^{(1)} = \tilde{\mathcal{F}}_t^{(2)} = \tilde{\mathcal{F}}_t$ . The remaining optimality conditions can be written as follows.

$$\begin{aligned} \tilde{\mathcal{F}}_t &= (1-\mathcal{A}) \left[ -\frac{1+\lambda}{\lambda} \tilde{p}_t^o + \tilde{\mathcal{Y}}_t \right] \\ &+ \mathcal{A} \left[ -\frac{\iota}{\lambda} \tilde{\pi}_t - \frac{1+\lambda}{\lambda} \tilde{p}_t^o + \frac{1+\lambda}{\lambda} \tilde{\pi}_{t+1} + \frac{1+\lambda}{\lambda} \tilde{p}_{t+1}^o + \tilde{\mathcal{F}}_{t+1} + \tilde{\Xi}_{t+1|t}^p \right] \\ \mathcal{A}_1 &= \zeta \beta \left( \frac{1}{\pi_*} \right)^{-(1-\iota)/\lambda} \end{aligned} \quad (\text{A.27})$$

and

$$\begin{aligned}
\tilde{\mathcal{F}}_t &= (1 - \mathcal{A}) \left[ - \left( \frac{1 + \lambda}{\lambda} + 1 \right) \tilde{p}_t^o + \tilde{\mathcal{Y}}_t + \tilde{M}C_t \right] \\
&+ \mathcal{A} \left[ - \frac{\iota(1 + \lambda)}{\lambda} \tilde{\pi}_t - \left( \frac{1 + \lambda}{\lambda} + 1 \right) \tilde{p}_t^o + \left( \frac{1 + \lambda}{\lambda} + 1 \right) \tilde{\pi}_{t+1} \right. \\
&\left. + \left( \frac{1 + \lambda}{\lambda} + 1 \right) \tilde{p}_{t+1}^o + \tilde{\mathcal{F}}_{t+1} + \tilde{\Xi}_{t+1|t}^p \right] \\
\mathcal{A}_2 &= \zeta \beta \left( \frac{1}{\pi_*} \right)^{-(1-\iota)(1+\lambda)/\lambda}.
\end{aligned} \tag{A.28}$$

The relationship between the optimal price charged by the adjusting firms and the inflation rate is given by

$$\begin{aligned}
\tilde{p}_t^o &= (\mathcal{A} - 1) \tilde{\pi}_t - \mathcal{A} \zeta \left( \frac{1}{\pi_*} \right)^{-(1-\iota)/\lambda} \tilde{\pi}_{t-1} \\
\mathcal{A}_p &= \frac{(p_*^o)^{1/\lambda}}{1 - \zeta}
\end{aligned} \tag{A.29}$$

Equations (A.27) to (A.29) determine  $\tilde{\pi}_t$ ,  $\tilde{\mathcal{F}}_t$ , and  $\tilde{p}_t^o$ .

**Resource Constraint, Market Clearing Conditions:** Aggregate output across evolves according to

$$\tilde{Y}_t = \tilde{Y}_t + \tilde{D}_t = (1 + \mathcal{F}/\bar{Y}_*) [\tilde{Z}_t + \alpha \tilde{K}_t + (1 - \alpha) \tilde{H}_t]. \tag{A.30}$$

and the steady state price dispersion follows

$$\tilde{D}_t = \zeta \left( \frac{1}{\pi_*} \right)^{-\frac{(1+\lambda)(1-\iota)}{\lambda}} \left[ \tilde{D}_{t-1} + \frac{(1 + \lambda)}{\lambda} \tilde{\pi}_t - \frac{\iota(1 + \lambda)}{\lambda} \tilde{\pi}_{t-1} \right] - \frac{p_*^o(1 + \lambda)(1 - \zeta)}{\lambda D_*} \tilde{p}_t^o \tag{A.31}$$

The goods market clearing condition is of the form

$$\tilde{Y}_t = \frac{X_*}{X_* + I_*} \tilde{X}_t + \frac{I_*}{X_* + I_*} \tilde{I}_t + \tilde{g}_t. \tag{A.32}$$

**Monetary Policy:** The monetary policy rule can be written as

$$\tilde{R}_t = \rho_R \tilde{R}_{t-1} + (1 - \rho_R) [\psi_1 (\tilde{\pi}_t - \tilde{\pi}_t^*) + \psi_2 (\tilde{Y}_t - \tilde{Y}_{t-1})] + \epsilon_{R,t}. \tag{A.33}$$

## B Data

The data set is identical to the one used in Aruoba and Schorfheide (2010). The empirical analysis is based on quarterly U.S. postwar data on aggregate output, inflation, inflation expectations, interest rates, and (inverse) velocity of money. Unless otherwise noted, the data are obtained from the FRED2 database maintained by the Federal Reserve Bank of St. Louis. Per capita output is defined as real GDP (GDPC96) divided by civilian non-institutionalized population (CNP16OV). I take the natural log of this measure and extract a linear trend and link the deviations from this trend to the stationary fluctuations around the deterministic steady state, that the DSGE model produces. Inflation is defined as the log difference of the GDP deflator (GDPDEF) and our measure of nominal interest rates corresponds to the federal funds rate (FEDFUNDS). Money is incorporated as an observable by using inverse M1 velocity. I use the sweep-adjusted M1S series provided by Cynamon, Dutkowsky and Jones (2006). The M1S series is divided by quarterly nominal output to obtain inverse velocity and we relate the natural logarithm of the resulting series to the log deviations from  $100 * \ln(\mathcal{M}^*/\mathcal{Y}^*)$ . The estimation sample ranges from 1965:I to 2005:I and I use the likelihood functions conditional on data from 1964:I to 1964:IV to estimate the DSGE model and the VARs.

In order to obtain a measure of the inflation target, three series are combined: GDP deflator filtered through a one-sided band-pass filter as well as 1-year and 10-year-ahead inflation expectations obtained from the Survey of Professional Forecasters, maintained by the Federal Reserve Bank of Philadelphia. Since the agents generate forecasts of future target inflation rates with a random walk model, a one-sided bandpass filter that removes cycles of a duration of less than 64 quarters is used. A time-domain moving average representation of the ideal one-sided filter (truncated at 500 lags) is constructed and then missing lagged observations are replaced by optimal backcasts obtained from an estimated AR(4) model.

To combine the three series a small state-space model with measurement equations

$$\tilde{\pi}_t^{BP} = \tilde{\pi}_{*,t} + 0.025\epsilon_{1,t}, \quad \tilde{\pi}_t^{1y} = \tilde{\pi}_{*,t} + \eta_{2,t}, \quad \tilde{\pi}_t^{10y} = \tilde{\pi}_{*,t} + \eta_{3,t},$$

and state transitions

$$\tilde{\pi}_{*,t} = \tilde{\pi}_{*,t-1} + \sigma_{\pi} \epsilon_{\pi,t}, \quad \eta_{2,t} = \rho_2 \eta_{2,t-1} + \sigma_2 \epsilon_{2,t}, \quad \eta_{3,t} = \rho_3 \eta_{3,t-1} + \sigma_3 \epsilon_{3,t}$$

is used. The  $\epsilon_{i,t}$ 's are *iid* standard normal random variables and  $\tilde{\pi}_t^{BP}$ ,  $\tilde{\pi}_t^{1y}$ , and  $\tilde{\pi}_t^{10y}$  are bandpass filtered inflation, 1-year-ahead forecasts, and 10-year-ahead forecasts, respectively. The innovation standard deviation for  $\tilde{\pi}_t^{BP}$  is fixed to implicitly control the weight on the bandpass filtered series and estimated the remaining parameters. If one regresses the filtered series  $\tilde{\pi}_{*,t}$  on the three observed measures, the coefficients are 0.57 ( $\tilde{\pi}_t^{BP}$ ), 0.22 ( $\tilde{\pi}_t^{1y}$ ), and 0.23 ( $\tilde{\pi}_t^{10y}$ ). Moreover, the dynamics of  $\tilde{\pi}_{*,t}$  are well approximated by the random walk that the DSGE model agents use to forecast the target inflation rate.

## C Empirical Analysis

### C.1 DSGE Model Estimation

The methods used to estimate the DSGE model are described in detail in An and Schorfheide (2007a). The following DSGE model parameters are fixed during the estimation:  $\delta = 0.014$ ,  $\gamma = 1$ ,  $\chi_* = 1$ ,  $g_* = 1.2$ ,  $\ln(\mathcal{M}_*/Y_*) = -0.38$ ,  $\ln(H_*/Y_*) = -3.35$ ,  $\ln Y_* = 1$ ,  $\psi_1 = 1.7$ , and the log-linearization point  $\pi_{*,A} = 4$ . Moreover, we set  $\beta = 1/(1 + r_A/400)$ , where  $r_A = 2.5$ . Marginal prior distributions for the remaining parameters are summarized in columns 2 to 4 of Table A-1. The joint prior is obtained by the product of the marginal densities, multiplied by the function  $f(\theta)$  defined in Equation (23) of Section 3.3 of the chapter. Posterior means and 90% credible intervals are provided in columns 5 and 6 of Table A-1.

### C.2 VAR Estimation

The VAR used as a reference model in Section 2.3 is identical to the one used in Aruoba and Schorfheide (2010). Output, inflation, interest rates, and inverse velocity are collected in the  $4 \times 1$  vector  $y_{1,t}$  and the target inflation rate in the scalar  $y_{2,t}$ . Moreover, let  $y_t = [y'_{1,t}, y_{2,t}]'$ . Assume that  $y_t$  follows a Gaussian vector autoregressive law of motion subject

to the restrictions that the target inflation rate evolves according to a random walk process and that the innovations to the target inflation rate are orthogonal to the remaining shocks. These restrictions are consistent with the assumptions that underlie the DSGE model and identify the propagation of unanticipated changes in the target inflation. The VAR takes the form

$$y_{1,t} = \Phi_0 + \Phi_1 y_{t-1} + \dots + \Phi_p y_{t-p} + \Psi \Delta y_{2,t} + u_{1,t} \quad (\text{A.34})$$

$$y_{2,t} = y_{2,t-1} + \sigma_{\pi_*} \epsilon_{\pi_*,t}, \quad (\text{A.35})$$

where  $u_{1,t} \sim \mathcal{N}(0, \Sigma_{11})$  and is independent of  $\epsilon_{\pi_*,t}$ . The VAR composed of (A.34) and (A.35) with  $p = 4$  is estimated using the version of the ‘‘Minnesota’’ prior described in Del Negro and Schorfheide (2010). The hyperparameters are  $\lambda_1 = 0.1$ ,  $\lambda_2 = 3.1$ ,  $\lambda_3 = 5$ ,  $\lambda_4 = 1$ ,  $\lambda_5 = 1$ . Our prior assumes that the elements of  $\Psi$  are independently distributed according to  $\mathcal{N}(0, \lambda_4^{-2})$ .

### C.3 DSGE-VAR Analysis

The DSGE-VAR framework described in Del Negro and Schorfheide (2010) is modified to account for the fact that one of the observables, namely the target-inflation rate, is non-stationary. Moreover, the DSGE model prior for the VAR coefficients is augmented by a standard Minnesota prior with the hyperparameter settings described above. Consider the VAR of the form

$$y_t = B_1 y_{t-1} + \dots + B_p y_{t-p} + B_c + \Sigma_{tr} \Omega \epsilon_t, \quad (\text{A.36})$$

where  $\Sigma_{tr}$  is the unique lower triangular Cholesky factor of the one-step-ahead forecast error covariance matrix  $\Sigma$ ,  $\Omega$  is an orthogonal matrix, and  $\epsilon_t \sim N(0, I)$ . Let  $B = [B_1, \dots, B_p, B_c]'$ ,  $x'_t = [y'_{t-1}, \dots, y'_{t-p}, 1]$  and write the VAR in matrix form as  $Y = XB + U$ . The prior distribution of the VAR parameters given the DSGE model parameters  $\theta$ ,  $(B, \Sigma) | \theta$ , is represented by dummy observations  $Y_*(\theta)$  and  $X_*(\theta)$ . The resulting prior takes the Matric-normal inverted-Wishart (MNIW) form

$$B, \Sigma | \theta \sim MNIW \left( B^*(\theta), X_*(\theta)' X_*(\theta), S^*(\theta), \lambda_D T + T_M - k \right), \quad (\text{A.37})$$



where  $\lambda_D$  is a hyperparameter,  $T$  is the size of the actual sample,  $T_M$  is the number of dummy observations for the Minnesota prior, and

$$\begin{aligned} B^*(\theta) &= [X_*(\theta)'X_*(\theta)]^{-1}X_*(\theta)'Y_*(\theta), \\ S^*(\theta) &= Y_*(\theta)'Y_*(\theta) - Y_*(\theta)'X_*(\theta)[X_*(\theta)'X_*(\theta)]^{-1}X_*(\theta)'Y_*(\theta). \end{aligned}$$

In the remainder of this subsection, I describe the construction of the moment matrices  $X_*(\theta)'X_*(\theta)$ ,  $X_*(\theta)'Y_*(\theta)$ , and  $Y_*(\theta)'Y_*(\theta)$ .

In order to combine the DSGE model prior and the Minnesota prior, the moment matrices are expressed as follows:

$$X_*(\theta)'X_*(\theta) = (\lambda_D T)\Gamma_{XX}^D(\theta) + X_*^{M'}X_*^M, \dots$$

The first part is derived from the DSGE model and the second part correspond to the dummy observations that are used to specify the Minnesota prior. I will subsequently focus on the first part. If the vector  $y_t$  is stationary, then  $\Gamma_{XX}^D(\theta)$  is the population covariance matrix of  $x_t$ . An extension to the case of non-stationary  $y_t$ 's can be obtained as follows. Recall that the DSGE model has a state-space representation of the form

$$y_t = \Psi_0 + \Psi_s s_t, \quad s_t = \Phi_1 s_{t-1} + \Phi_\epsilon \epsilon_t.$$

Assume that the state vector  $s_t$  in period  $t = -\tau$  was equal to zero,  $s_{-\tau} = 0$ , and that  $\epsilon_t \sim iidN(0, \Sigma_\epsilon)$ . By iterating the state-transition equation forward, one can obtain the distribution of  $s_0$  and hence  $y_0$ . Iterating the state-transition forward for another  $p$  periods, yields the joint distribution of  $y_0, \dots, y_p$ . The matrices  $\Gamma_{XX}^D$ ,  $\Gamma_{XY}^D$ , and  $\Gamma_{YY}^D$  are now constructed from the appropriate elements of the joint covariance matrix of  $y_0, \dots, y_p$ . If some of the elements of  $s_t$  are non-stationary and others are stationary, the stationary ones can be initialized in period  $-\tau$  through their ergodic distribution, and the non-stationary ones with a pointmass at zero. In our application,  $s_t$  contains one non-stationary element, namely that target inflation rate, and we set  $\tau = 40$ .

Table A-1: PRIOR AND POSTERIOR DISTRIBUTIONS

Name	Density	Prior		Posterior	
		Para (1)	Para (2)	Mean	90% Intv
Households					
$\nu$	Gamma	20.0	5.00	31.7	[24.8, 38.2]
Firms					
$\alpha$	Beta	0.30	.025	0.28	[0.27, 0.29]
$\lambda$	Normal	0.15	0.01	0.16	[0.15, 0.18]
$\zeta$	Beta	0.60	0.15	0.75	[0.72, 0.79]
$\iota$	Beta	0.50	0.25	0.03	[0.00, 0.07]
$S''$	Gamma	5.00	2.50	5.37	[2.68, 8.11]
Central Bank					
$\psi_2$	Gamma	1.00	0.50	1.02	[0.83, 1.21]
$\rho_R$	Beta	0.50	0.20	0.67	[0.63, 0.72]
$\sigma_R$	InvGamma	0.50	4.00	0.33	[0.28, 0.39]
$\sigma_{R,2}$	InvGamma	1.00	4.00	0.80	[0.59, 1.01]
$\tilde{\pi}_{0,A}^*$	Normal	0.00	2.00	-0.11	[-3.27, 3.26]
$\sigma_\pi$	InvGamma	0.05	4.00	0.05	[ 0.04, 0.05]
Shocks					
$\rho_g$	Beta	0.80	0.10	0.90	[0.86, 0.93]
$\sigma_g$	InvGamma	1.00	4.00	1.15	[0.99, 1.30]
$\rho_\chi$	Beta	0.80	0.10	0.98	[0.97, 0.99]
$\sigma_\chi$	InvGamma	1.00	4.00	1.30	[1.18, 1.42]
$\rho_z$	Beta	0.90	0.05	0.80	[0.70, 0.89]
$\sigma_z$	InvGamma	2.00	4.00	2.08	[1.32, 2.81]

*Notes:* Para (1) and Para (2) list the means and the standard deviations for Beta, Gamma, and Normal distributions;  $s$  and  $\nu$  for the Inverse Gamma distribution, where  $p_{IG}(\sigma|\nu, s) \propto \sigma^{-\nu-1} e^{-\nu s^2/2\sigma^2}$ .