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Small-Scale DSGE Model

• Textbook treatments: Woodford (2003), Gali (2008)
• Intermediate and final goods producers
• Households
• Monetary and fiscal policy
• Exogenous processes
• Equilibrium Relationships
Final Goods Producers

- Perfectly competitive firms combine a continuum of intermediate goods:

\[ Y_t = \left( \int_0^1 Y_t(j)^{1-\nu} dj \right)^{\frac{1}{1-\nu}}. \]

- Firms take input prices \( P_t(j) \) and output prices \( P_t \) as given; maximize profits

\[ \Pi_t = P_t \left( \int_0^1 Y_t(j)^{1-\nu} dj \right)^{\frac{1}{1-\nu}} - \int_0^1 P_t(j) Y_t(j) dj. \]

- Demand for intermediate good \( j \):

\[ Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-1/\nu} Y_t. \]

- Zero-profit condition implies

\[ P_t = \left( \int_0^1 P_t(j)^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}. \]
Intermediate Goods Producers

- Intermediate good $j$ is produced by a monopolist according to:
  \[ Y_t(j) = A_t N_t(j). \]

- Nominal price stickiness via quadratic price adjustment costs
  \[ AC_t(j) = \frac{\phi}{2} \left( \frac{P_t(j)}{P_{t-1}(j)} - \pi \right)^2 Y_t(j). \]

- Firm $j$ chooses its labor input $N_t(j)$ and the price $P_t(j)$ to maximize the present value of future profits:
  \[ E_t \left[ \sum_{s=0}^{\infty} \beta^s Q_{t+s} \left( \frac{P_{t+s}(j)}{P_{t+s}} Y_{t+s}(j) - W_{t+s} N_{t+s}(j) - AC_{t+s}(j) \right) \right]. \]
Households

- Household derives disutility from hours worked $H_t$ and maximizes

$$E_t \left[ \sum_{s=0}^{\infty} \beta^s \left( \frac{(C_{t+s}/A_{t+s})^{1-\tau} - 1}{1 - \tau} \right)
+ \chi_M \ln \left( \frac{M_{t+s}}{P_{t+s}} \right) - \chi_H H_{t+s} \right].$$

- Budget constraint:

$$P_t C_t + B_t + M_t + T_t = P_t W_t H_t + R_{t-1} B_{t-1} + M_{t-1} + P_t D_t + P_t SC_t.$$
• Central bank adjusts money supply to attain desired interest rate.
• Monetary policy rule:
  \[ R_t = R_t^* \left( 1 - \rho_R \right) R_{t-1}^{\rho_R} e^{\epsilon_{R,t}} \]
  \[ R_t^* = r \pi_t^* \left( \frac{\pi t}{\pi^*} \right)^{\psi_1} \left( \frac{Y_t}{Y_t^*} \right)^{\psi_2} \]
• Fiscal authority consumes fraction of aggregate output: \( G_t = \zeta_t Y_t \).
• Government budget constraint:
  \[ P_t G_t + R_{t-1} B_{t-1} + M_{t-1} = T_t + B_t + M_t. \]
Exogenous Processes

- **Technology:**
  \[ \ln A_t = \ln \gamma + \ln A_{t-1} + \ln z_t, \quad \ln z_t = \rho_z \ln z_{t-1} + \epsilon_{z,t}. \]

- **Government spending / aggregate demand:** define \( g_t = 1/(1 - \zeta_t) \); assume
  \[ \ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \epsilon_{g,t}. \]

- **Monetary policy shock** \( \epsilon_{R,t} \) is assumed to be serially uncorrelated.
Equilibrium Conditions

- Consider the symmetric equilibrium in which all intermediate goods producing firms make identical choices; omit $j$ subscript.

- Market clearing:

$$Y_t = C_t + G_t + AC_t \quad \text{and} \quad H_t = N_t.$$ 

- Complete markets:

$$Q_{t+s|t} = \left( \frac{C_{t+s}}{C_t} \right)^{-\tau} \left( \frac{A_t}{A_{t+s}} \right)^{1-\tau}.$$ 

- Consumption Euler equation and New Keynesian Phillips curve:

$$1 = \beta E_t \left[ \left( \frac{C_{t+1}/A_{t+1}}{C_t/A_t} \right)^{-\tau} \frac{A_t}{A_{t+1}} \frac{R_t}{\pi_{t+1}} \right]$$

$$1 = \phi (\pi_t - \pi) \left[ \left( 1 - \frac{1}{2\nu} \right) \pi_t + \frac{\pi}{2\nu} \right]$$

$$-\phi \beta E_t \left[ \left( \frac{C_{t+1}/A_{t+1}}{C_t/A_t} \right)^{-\tau} \frac{Y_{t+1}/A_{t+1}}{Y_t/A_t} (\pi_{t+1} - \pi) \pi_{t+1} \right]$$

$$+ \frac{1}{\nu} \left[ 1 - \left( \frac{C_t}{A_t} \right)^{\tau} \right].$$
In the absence of nominal rigidities ($\phi = 0$) aggregate output is given by

$$Y_t^* = (1 - \nu)^{1/\tau} A_t g_t,$$

which is the target level of output that appears in the monetary policy rule.
Steady State

- Set $\epsilon_{R,t}$, $\epsilon_{g,t}$, and $\epsilon_{z,t}$ to zero at all times.

- Because technology $\ln A_t$ evolves according to a random walk with drift $\ln \gamma$, consumption and output need to be detrended for a steady state to exist.

- Let

  \[ c_t = \frac{C_t}{A_t}, \quad y_t = \frac{Y_t}{A_t}, \quad y_t^* = \frac{Y_t^*}{A_t}. \]

- Steady state is given by:

  \[
  \pi = \pi^*, \quad r = \frac{\gamma}{\beta}, \quad R = r\pi^*,
  \]

  \[
  c = (1 - \nu)^{1/\tau}, \quad y = gc = y^*.
  \]
• A medium-scale DSGE model with capital and nominal wage rigidities: Smets and Wouters (2003, 2007)

• DSGE model for fiscal policy analysis: Leeper, Plante, and Traum (2010)