Exercise for Lecture 3

Consider the linear Gaussian state-space model

\[ y_t = \Gamma + \Psi s_t + u_t \]
\[ s_t = \Phi s_{t-1} + \epsilon_t \]

where \( E[u_t u_t'] = H \) and \( E[\epsilon_t \epsilon_t'] = \Sigma \). Let the dimension of \( y_t \) be \( 2 \times 1 \) and \( s_t \) be \( 1 \times 1 \).

Fix parameters at

\[ \Gamma = \begin{pmatrix} 0.7 \\ 0.8 \end{pmatrix}, \quad \Psi = \begin{pmatrix} 1 \\ 0.65 \end{pmatrix}, \quad H = \begin{pmatrix} 0.47^2 & 0 \\ 0 & 0.62^2 \end{pmatrix} \]
\[ \Phi = 1, \quad \Sigma = 0.75^2 \]

(i) Generate 50 observations from the state-space model. You can set \( s_0 = 0 \) and initialize the filter based on \( s_0 \sim N(0, 75^2) \).

(ii) Run the Kalman filter and store the mean and variance of \( p(s_t|Y_{1:t}) \) as well as the likelihood contributions \( \ln p(y_t|Y_{1:t-1}) \).

(iii) Plot \( \mathbb{E}[s_t|Y_{1:t}] \) as well as 90% credible sets based on \( p(s_t|Y_{1:t}) \). Overlay the “true” states \( s_t \).