Bayesian Macroeconometrics **Gerzensee Ph.D. Course, June 2019**

Exercise for Lecture 3

Consider the linear Gaussian state-space model

$$
\begin{aligned}
y_t &= \Gamma + \Psi s_t + u_t \\
s_t &= \Phi s_{t-1} + \epsilon_t
\end{aligned}
$$

where $E[u_t u_t'] = H$ and $E[\epsilon_t \epsilon_t'] = \Sigma$. Let the dimension of y_t be 2×1 and s_t be 1×1.

Fix parameters at

$$
\begin{aligned}
&\Gamma = \begin{pmatrix} 0.7 \\ 0.8 \end{pmatrix}, \quad \Psi = \begin{pmatrix} 1 \\ 0.65 \end{pmatrix}, \quad H = \begin{pmatrix} 0.47^2 & 0 \\ 0 & 0.62^2 \end{pmatrix} \\
&\Phi = 1, \quad \Sigma = 0.75^2
\end{aligned}
$$

(i) Generate 50 observations from the state-space model. You can set $s_0 = 0$ and initialize the filter based on $s_0 \sim N(0, .75^2)$.

(ii) Run the Kalman filter and store the mean and variance of $p(s_t|Y_{1:t})$ as well as the likelihood contributions $\ln p(y_t|Y_{1:t-1})$.

(iii) Plot $\mathbb{E}[s_t|Y_{1:t}]$ as well as 90% credible sets based on $p(s_t|Y_{1:t})$. Overlay the "true" states s_t.