Exercises for Lecture 4

Exercise 1: Gibbs Sampling

Consider a bivariate normal distribution:

$$Y \sim N(0, \Sigma), \tag{1}$$

where Y is a 2×1 vector.

(i) Derive the conditional distribution of $Y_1|Y_2$.

(ii) Derive the conditional distribution of $Y_2|Y_1$.

(iii) Write a Gibbs sampler that for $i = 1$ to N generates draws from $Y_1|Y_2^{i-1}$ and $Y_2|Y_1^i$.

(iv) Computational experiment: let

$$\Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

For various values of ρ: (i) run the Gibbs sampler 50 times with $N = 100$; draw the initialization $Y_2^0 \sim N(1, 1)$; discard the first 50 draws and approximate the means of Y_1 and Y_2 by the average of draws $i = 51, \ldots, 100$. (ii) Compute the bias, the variance, and the RMSE of the Monte Carlo approximation.
How does the quality of the Monte Carlo approximation change with ρ?

Exercise 2: DFM Estimation with MATLAB

(i) The Matlab programs accompanying this lecture implement the Bayesian analysis of a dynamic factor model (assuming that the factor is univariate). Familiarize yourself with the structure of the programs by reading the readme files.

(ii) Compare the Matlab implementation of the Gibbs sampler with the description in the lecture notes.

(iii) Run the programs using the default setting.

(iv) Compile a data set of macroeconomic time series that you think can be modeled with a one-factor DFM. Adapt the Matlab procedures and estimate the factor model based on your data set. Hint: you can obtain U.S. macroeconomic time series from the FRED database maintained by the Federal Reserve Bank of St. Louis.