Maxwell fish-eye and Eaton lenses emulated by microdroplets

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Despite strong experimental and theoretical evidence supporting superresolution imaging based on microlenses, the imaging mechanisms involved are not well understood. Based on the transformation optics approach, we demonstrate that a microlens may act as a two-dimensional fish-eye or an inverted Eaton lens. An asymmetric inverted Eaton lens may exhibit considerable image magnification, which has been confirmed experimentally. © 2010 Optical Society of America

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Current interest in electromagnetic metamaterials has been motivated by recent work on superlenses, cloaking, and transformation optics [1–3]. This interest has been followed by considerable efforts aimed at the introduction of metamaterial structures that could be realized experimentally. Unfortunately, it appears difficult to develop metamaterials with low-loss, broadband performance. The difficulties are especially severe in the visible frequency range, where good magnetic performance is limited. On the other hand, very recently we have demonstrated that many transformation optics and metamaterial-based devices requiring anisotropic dielectric permittivity and magnetic permeability could be emulated by specially designed tapered waveguides [4]. This approach leads to low-loss, broadband performance in the visible frequency range, which is difficult to achieve by other means. We have applied this technique to broadband electromagnetic cloaking in the visible range [4]. In this Letter, we apply the same technique to experimental realization of Maxwell fish-eye and inverted Eaton micro-lenses, which have been suggested to act as super imaging devices, even in the absence of negative refraction [5]. Realization of these micro-lenses using electromagnetic metamaterials would require sophisticated nanofabrication techniques. In contrast, our approach leads to a much simpler design, which involves two-dimensional (2D) imaging using a small liquid droplet.

Despite strong experimental and theoretical evidence supporting super-resolution imaging based on microlenses and micro-droplets, the imaging mechanisms involved are not well understood. Imaging by surface plasmon polaritons [6] has been proposed as the main super-resolution mechanism in imaging experiments using glycerin micro-droplets on gold film surfaces [7]. Resolution of the order of λ/8 has been observed in these experiments. On the other hand, magnification of near-field image components has been suggested in recent experiments with self-assembled plano-spherical nanolenses [8,9], which demonstrated resolution of the order of λ/4. Our analysis in terms of the effective metamaterial parameters indicates that the shape of microlenses and micro-droplets provides natural realization of the effective refractive index distribution in fish-eye and inverted Eaton micro-lenses [4]. The starting point of our analyses is the dispersion law of guided modes in a tapered waveguide. In the case of a metal-coated dielectric waveguide, it can be written in a simple analytical form:

\[
\frac{\omega^2 n_d^2}{c^2} = k_x^2 + k_y^2 + \frac{\pi^2 l^2}{d(r)^2},
\]

where \(n_d\) is the refractive index of the dielectric, \(d(r)\) is the waveguide thickness, and \(l\) is the transverse mode number. We assume that the thickness \(d\) of the waveguide in the \(z\) direction changes adiabatically with radius \(r\). A photon launched into the \(l\)th mode of the waveguide stays in this mode as long as \(d\) changes adiabatically [10]. If we wish to emulate refractive index distribution \(n(r)\) of either 2D fish-eye or 2D inverted Eaton lenses,

\[
\frac{\omega^2 n^2(r)}{c^2} = k_x^2 + k_y^2,
\]

then we need to produce the following profile of the micro-droplet:

**Fig. 1.** (Color online) Typical profiles of a micro-droplet that emulates either the fish-eye lens \((R = 7 \mu m)\) or the inverted Eaton lens \((R = 5 \mu m)\) for the following set of parameters: \(l = 1\), \(\lambda = 1.5 \mu m\), \(n_d = 1.5\), and \(n_1 = 0.65\).

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This is easy to do for some particular mode $l$ of the waveguide. Typical microdroplet/microlens profiles that emulate a fish-eye lens described by the equation

$$n = 2n_1 \left(1 + \frac{r^2}{R^2}\right)^{-1},$$

(4)

(where $2n_1$ is the refractive index at the center of the lens, and $R$ is the scale) or the inverted Eaton lens [11] described by

$$n = \begin{cases} 
1 & \text{for } r < R, \\
\sqrt{\frac{2R}{r}} - 1 & \text{for } r > R.
\end{cases}$$

(5)

are shown in Fig. 1. Real glycerin microdroplets have shapes that are somewhere in between these cases. Since the refractive index distribution in the fish-eye lens is obtained via the stereographic projection of a sphere onto a plane [5], points near the droplet edge correspond to points located near the equator of the sphere. Therefore, these points are imaged into points located near the opposite droplet edge, as shown in Fig. 2(a).

The inverted Eaton lens has similar imaging properties, as shown in Fig. 2(c). Each droplet depicted in Fig. 2 was simulated using scattered-field finite-element formulation. The continuity of the tangential field components was enforced at the host–droplet interface. The host with the droplet was surrounded by a perfectly matched (absorbing) layer to suppress reflection from the exterior boundaries of the simulation domain.

We have tested this imaging mechanism using glycerin microdroplets formed on the surface of gold film, which were illuminated near the edge using the tapered fiber tips of a near-field scanning optical microscope (NSOM), as shown in Fig. 3. As expected from the numerical simulations, an image of the NSOM tip was easy to observe at the opposite edge of the microdroplet. As has been demonstrated in [12–15], perfect imaging using a Maxwell fish-eye or an Eaton lens requires a drain. In our case, the droplet boundary may perhaps act as such a drain. However, more detailed experimental study of this issue is needed.

While the fish-eye lens design is difficult to modify to achieve image magnification, modification of the Eaton lens is straightforward. As shown in Fig. 4, two halves

Moving source close to the 'equator' as a proof of magnification

Magnification with Eaton lens

Fig. 4. (Color online) Numerical simulations of image magnification ($M = 2$) using the inverted Eaton lens. Since the sides of the lens play no role in imaging, the overall shape of the imaging device can be altered to achieve the shape of a "deformed droplet."

Fig. 5. (Color online) Experimental testing of image magnification of the "deformed droplet." The NSOM probe tip was moved along the droplet edge. The bottom row presents results of our numerical simulations in the cases of one and two point sources. The shape of the "deformed droplet" used in numerical simulations closely resembles the shape of the actual droplet.
of the Eaton lens having different values of parameter $R$ can be brought together to achieve image magnification. The image magnification in this case is $M = R_1/R_2$. Our numerical simulations in the case of $M = 2$ are presented. Because the sides of the lens play no role in imaging, the overall shape of the imaging device can be altered to achieve the shape of a “deformed droplet.” Using the experimental technique described below, we have created glycerin droplets with shapes that are very close to the shape of the “deformed droplet” used in the numerical simulations. Image magnification of the “deformed droplet” has been tested by moving the NSOM probe tip along the droplet edge, as shown in Fig. 5. It appears to be close to the $M = 2$ value predicted by the simulations. We should also note that some issues related to image magnification were considered in [16].

In our imaging experiments, the “deformed droplets” were formed in desired locations by bringing a small probe [Fig. 6(a)] wetted in glycerin into close proximity to a sample. The probe was prepared from a tapered optical fiber and has an epoxy microdroplet near its apex. Bringing the probe to a surface region covered with glycerin led to a glycerin microdroplet formation under the probe [Fig. 6(b)]. The shape of the glycerin droplet was determined by the shape of the seed droplet of epoxy. Our droplet deposition procedure allowed us to form droplet shapes that were reasonably close to the shape of a magnifying Eaton lens, as shown in Figs. 5 and 7. In addition, the liquid droplet boundary may be expected to be rather smooth because of the surface tension, which is essential for the proper performance of the droplet boundary as a 2D fish-eye or Eaton lens.

Image magnification of the 2D magnifying Eaton lens has been measured as demonstrated in Fig. 7. The positions of the NSOM tip and its image in the second frame are shown by the red dots (marked by arrows) in the first frame. The ratio of the arrow lengths, which connect the NSOM tip and image locations in the two frames shown, is close to the theoretically predicted value $M = 2$.

In conclusion, we have demonstrated that small dielectric microlenses behave as 2D imaging devices, which can be approximated by 2D fish-eye or inverted Eaton lenses. Deformed microlenses/microdroplets were observed to exhibit image magnification, which is consistent with numerical predictions.

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