Experimental model of topological defects in Minkowski space–time based on disordered ferrofluid: magnetic monopoles, cosmic strings and the space–time cloak

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In the presence of an external magnetic field, cobalt nanoparticle-based ferrofluid forms a self-assembled hyperbolic metamaterial. The wave equation, which describes propagation of extraordinary light inside the ferrofluid, exhibits $2 + 1$ dimensional Lorentz symmetry. The role of time in the corresponding effective three-dimensional Minkowski space–time is played by the spatial coordinate directed along the periodic nanoparticle chains aligned by the magnetic field. Here, we present a microscopic study of point, linear, planar and volume defects of the nanoparticle chain structure and demonstrate that they may exhibit strong similarities with such Minkowski space–time defects as magnetic monopoles, cosmic strings and the recently proposed space–time cloaks. Experimental observations of such defects are described.

Metamaterials are artificial materials built from conventional microscopic materials in order to engineer their properties in a desired way. Such artificial metamaterials offer novel ways for controlling...
electromagnetic, acoustic, elastic and thermal properties of matter. However, considerable
difficulties still exist in fabrication of three-dimensional metamaterials. Therefore, they are
typically confined to small sizes in two dimensions. Recent demonstration that cobalt
nanoparticle-based ferrofluid subjected to an external magnetic field exhibits hyperbolic
metamaterial properties \cite{1,2} opens up many new directions in transformation optics and
metamaterial research, as such a metamaterial is created by three-dimensional self-assembly, and
its dimensions are not limited by nanofabrication issues. On the other hand, three-dimensional
self-assembly of macroscopic metamaterial samples unavoidably leads to formation of various
defects in the bulk of the metamaterial, which may affect its optical properties. While very
important from the practical standpoint, this issue is also very interesting for purely scientific
reasons. It appears that the wave equation which describes propagation of extraordinary light
inside the hyperbolic metamaterial exhibits $2 + 1$ dimensional Lorentz symmetry, with the
role of time in the corresponding effective three-dimensional Minkowski space–time played
by the spatial coordinate directed along the optical axis of the metamaterial \cite{3,4}. Therefore,
point, linear and planar defects of the self-assembled nanoparticle chain structure may exhibit
strong similarities with such Minkowski space–time defects as cosmic strings \cite{5} and magnetic
monopoles \cite{6}. On the other hand, volume defects of the aligned nanoparticle chain structure
may correspond to the recently proposed space–time cloak geometry \cite{7}.

Modern developments in gravitation research strongly indicate that classic general relativity
is an effective macroscopic field theory, which needs to be replaced with a more fundamental
theory based on yet unknown microscopic degrees of freedom. However, our ability to obtain
experimental insights into the future fundamental theory is strongly limited by low energy
scales available to terrestrial particle physics and astronomical observations. The emergent
analogue space–time programme offers a promising way around this difficulty. Looking at
such systems as superfluid helium and cold atomic Bose–Einstein condensates, physicists
learn from nature and discover how macroscopic field theories arise from known well-studied
atomic degrees of freedom. A ferrofluid in an external magnetic field provides us with
another easily accessible experimental system, which lets us trace the emergence of effective
Minkowski space–time from well-understood microscopic degrees of freedom. Here, we will
describe direct experimental observations of microscopic defects of the effective Minkowski
space–time and discuss their influence on the hyperbolic metamaterial properties of the
ferrofluid. As pointed out recently by Mielczarek \cite{8}, the properties of self-assembled magnetic
nanoparticle-based hyperbolic metamaterials exhibit strong similarities with the properties of
some quantum gravity models, such as loop quantum cosmology. Moreover, the physical
vacuum appears to exhibit hyperbolic metamaterial properties when subjected to a very
strong magnetic field \cite{9,10}. Therefore, our experimental observations have many important
fundamental implications, which reach far beyond transformation optics and electromagnetic
metamaterial theory.

Our experimental technique uses three-dimensional self-assembly of cobalt nanoparticles in
the presence of an external magnetic field. Magnetic nanoparticles in a ferrofluid are known
to form nanocolumns aligned along the magnetic field \cite{11}. Moreover, depending on the
magnitude of magnetic field, nanoparticle concentration and solvent used, phase separation
into nanoparticle-rich and nanoparticle-poor phases may occur in the ferrofluid \cite{1}. This phase
separation occurs on a $0.1–1\mu m$ scale. For our experiments, we have chosen cobalt magnetic
fluid 27–0001 from Strem Chemicals composed of $10\text{nm}$ cobalt nanoparticles in kerosene coated
with sodium dioctylsulfosuccinate and a monolayer of LP4 fatty acid condensation polymer. The
average volume fraction of cobalt nanoparticles in this ferrofluid is $8.2\%$. Cobalt has metallic
properties (the real part of its refractive index $n$ is smaller than its imaginary part $k$: $n < k$, so
that $\varepsilon' = n^2 - k^2 < 0$) in the infrared range, as evident from figure 1a plotted using data for the
optical properties of cobalt reported in \cite{12}. Therefore, self-assembled cobalt nanoparticle chains
are suitable for fabrication of hyperbolic metamaterials. When cobalt nanoparticles are completely
aligned by the external magnetic field, so that the wire array geometry schematically shown in
figure 2a is formed, the diagonal components of the ferrofluid permittivity may be calculated
Figure 1. (a) Real and imaginary parts of $\varepsilon$ for cobalt according to [9]. (b) Corresponding wavelength dependencies of the real parts of $\varepsilon_z$ and $\varepsilon_{xy}$ at $\alpha = 8.2\%$. While $\varepsilon_{xy}$ stays positive and almost constant, $\varepsilon_z$ changes sign to negative around $\lambda = 1\mu m$. (c) Schematic view of our experimental set-up. (Online version in colour.)

Using Maxwell–Garnett approximation as

$$\varepsilon_z = \alpha \varepsilon_m + (1 - \alpha) \varepsilon_d$$

(0.1)

and

$$\varepsilon_{xy} = \frac{2\alpha \varepsilon_m \varepsilon_d + (1 - \alpha) \varepsilon_d (\varepsilon_d + \varepsilon_m)}{(1 - \alpha) (\varepsilon_d + \varepsilon_m) + 2 \alpha \varepsilon_d},$$

(0.2)

where $\alpha$ is the average volume fraction of the cobalt nanoparticle phase (assumed to be small), and $\varepsilon_m$ and $\varepsilon_d$ are the dielectric permittivities of cobalt and kerosene, respectively [13]. Wavelength dependencies of $\varepsilon_z$ and $\varepsilon_{xy}$ calculated using equations (0.1) and (0.2) at $\alpha = 8.2\%$ are plotted in figure 1b. While $\varepsilon_{xy}$ stays positive and almost constant, $\varepsilon_z$ changes sign to negative around $\lambda = 1\mu m$. Therefore, at $\lambda > 1\mu m$, the ferrofluid subjected to external magnetic field becomes a hyperbolic metamaterial, which has been demonstrated in an experiment [1].
Figure 2. (a) Geometry of the metal nanowire-based hyperbolic metamaterials. (b–f) Experimentally measured microscopic transmission images of cobalt nanoparticle-based ferrofluid obtained using an infrared microscope and a 1.55 µm laser as an illumination source. Frames (b) and (c) show microscopic images of the ferrofluid before and after application of external magnetic field. The self-assembled nanowires in (c) are oriented along the direction of the magnetic field, so that a hyperbolic metamaterial is formed. Frame (d) shows a microscopic image of the ferrofluid after the magnetic field is turned off and on multiple times leading to the appearance of defects in the nanowire array structure. While on average the filaments remain aligned along the direction of the magnetic field (the same direction as in (c)), multiple defects appear in the filament structure. Frames (e) and (f) show quasi-three-dimensional representations of images in frames (c) and (d), respectively. While image (e) reveals a well-ordered periodic wire array structure, image (f) shows multiple defects, which appear due to external magnetic field cycling. (Online version in colour.)
The hyperbolic character of the magnetized ferrofluid has been confirmed by polarization-dependent transmission and reflection measurements at single laser lines and broadband Fourier transform infrared spectroscopy performed in the 500–22 000 nm wavelength range [1]. Note that in the near-infrared frequency range of interest, the magnetic permeability of the ferrofluid may be assumed to be \( \mu = 1 \), and all the off-diagonal terms of the dielectric permittivity tensor of the ferrofluid may be assumed to be zero due to the relatively small induced optical activity of kerosene and weak external magnetic field used. A wave equation describing the propagation of extraordinary photons inside the ferrofluid may be written in the form of a three-dimensional Klein–Gordon equation describing a massive field \( \phi_{\omega} = E_z \) in three-dimensional Minkowski space–time:

\[
- \frac{1}{\varepsilon_{xy}} \frac{\partial^2 \phi_{\omega}}{\partial z^2} + \frac{1}{(-\varepsilon_z)} \left( \frac{\partial^2 \phi_{\omega}}{\partial x^2} + \frac{\partial^2 \phi_{\omega}}{\partial y^2} \right) = \frac{\omega_0^2}{c^2} \phi_{\omega} = \frac{m^* c^2}{\hbar^2} \phi_{\omega}
\]  

(0.3)

in which the spatial coordinate \( z = \tau \) behaves as a ‘timelike’ variable, and the effective mass is \( m^* = \hbar \omega_0/c^2 \). For example, it is easy to check that equation (0.3) remains invariant under the effective Lorentz coordinate transformation

\[
\begin{align*}
  z' &= \frac{1}{\sqrt{1 - (\varepsilon_{xy}/\varepsilon_z)\beta}} \left( z - \beta x \right) \\
  x' &= \frac{1}{\sqrt{1 - (\varepsilon_{xy}/\varepsilon_z)\beta}} \left( x - \beta \frac{\varepsilon_{xy}}{\varepsilon_z} z \right)
\end{align*}
\]

(0.4)

where \( \beta \) is the effective Lorentz boost. The opposite signs of \( \varepsilon_z \) and \( \varepsilon_{xy} \) ensure that spatial coordinate \( z \) plays the role of time in the Lorentz-like symmetry described by equation (0.4). Instead of the ‘optical path length’ \( d_{\text{opt}} = n d \), which is typically introduced in conventional optics, we may introduce an ‘optical interval’ \( ds_{\text{opt}}^2 \) as

\[
ds_{\text{opt}}^2 = -\varepsilon_{xy} dz^2 + (-\varepsilon_z)(dx^2 + dy^2),
\]

(0.5)

which remains invariant under the effective Lorentz transformations. Similar to our own 3 + 1 dimensional Minkowski space–time, opposite signs of \( \varepsilon_z \) and \( \varepsilon_{xy} \) in a hyperbolic metamaterial lead to Lorentz symmetry of its effective ‘optical space–time’. Thus, equation (0.3) describes world lines of massive particles which propagate in effective 2 + 1 dimensional Minkowski space–time [3,4]. When the ferrofluid develops phase separation into cobalt-rich and cobalt-poor phases, its microscopic structure and local field intensity in the hyperbolic frequency range may be observed directly using an infrared microscope, so that microscopic properties of the effective Minkowski space–time defects may be studied in the experiment. Our experiments were performed using a narrow 10 \( \mu \)m pathlength optical cuvette placed under the infrared microscope. The cuvette was filled with the ferrofluid and illuminated from below with a linear polarized 1.55 \( \mu \)m laser. The transmission images of the infrared microscope were studied as a function of magnitude and direction of the external magnetic field. The schematic view of our experimental set-up is shown in figure 1c.

Optical microscope transmission images of the ferrofluid illuminated with the 1.55 \( \mu \)m laser before and after application of an external magnetic field are shown in figure 2b,c. While the imaginary part of \( \varepsilon_z \) is rather large at \( \lambda = 1.55 \mu \)m, this was the longest wavelength available to us to perform optical microscopy of the samples. However, the level of metamaterial losses at 1.55 \( \mu \)m was adequate to measure transmission of a 10 \( \mu \)m pathlength optical cuvette filled with the ferrofluid. The periodic pattern of self-assembled stripes visible in figure 2c appears due to phase separation. The stripes are oriented along the direction of the magnetic field. Quasi-three-dimensional representation of this image is shown in figure 2e. Both images demonstrate an almost defect-free well-ordered periodic alignment of cobalt-rich filaments. However, by turning the external magnetic field off and on again multiple times, it was possible to create defects in the periodic alignment of these filaments. An example of a microscopic image of such accumulated...
defects within the volume of a polarized ferrofluid is shown in figure 2d. While on average the filaments remain aligned along the direction of magnetic field (the same direction as in figure 2c), multiple defects appear in the filament structure. The complicated topology of the defects is clearly revealed in the quasi-three-dimensional representation of this image shown in figure 2f. It should be pointed out that despite these defects, the measured ferrofluid polarization properties remain the same, indicating that the ferrofluid remains a hyperbolic metamaterial on a large scale. Therefore, these defects play the role of microscopic defects in effective Minkowski space–time. Let us analyse typical examples of these defects in more detail.

Let us start with some examples of simple point defects (‘events’ in the effective Minkowski space–time) shown in figure 3. Such point defects may arise either due to impurities present in

Figure 3. (a–c) Examples of the possible point defects in the periodic wire array structure formed by self-assembly of cobalt nanoparticles in an external magnetic field. The red arrows represent nanoparticle magnetic moments. The green circle in (a) represents an impurity. The defect shown in (a) may be treated as a magnetic monopole–antimonopole pair, which resides in an effective three-dimensional Minkowski metamaterial space–time. The defect shown in (b) represents a simple braid structure, while the defect shown in (c) represents a y-split. Multiple examples of such point defects may be found in figure 2f. Magnified experimentally observed images of such point defects from figure 2f are shown in (d), (e) and (f), respectively. The defect areas are marked by circles. (Online version in colour.)
the ferrofluid (as shown in figure 3a) or due to twisting and/or splitting of the ferrofluid filaments (figure 3b,c). Experimental examples of such defects are indeed easy to find in the microscopic images, as shown in figure 3d–f. It appears that these defects exhibit some similarity with the proposed point defects of Minkowski space–time, such as magnetic monopole solutions [6]. As cobalt nanoparticles possess magnetic dipole moments, which are oriented by the external magnetic field, bending the ferrofluid filaments leads to bending of the magnetic field flux lines, as shown in figure 3a–c. As a result, effective ‘virtual’ magnetic charges appear in these locations. Unlike the unperturbed regions of the effective metamaterial three-dimensional Minkowski space–time, ∇xyB is non-zero in these locations, where ∇xy = (∂/∂x, ∂/∂y) is the two-dimensional divergence operator, and z coordinate is identified as a timelike variable. Thus, scattering of extraordinary photons by such point defects may be treated as a model of photon scattering by monopole–antimonopole pairs. It is somewhat similar to photon scattering by electron–positron pairs. Such scattering may be studied in polarization-dependent IR microscopy experiments, as illustrated in figure 4. However, we must emphasize that a true magnetic monopole would appear as a line (a world line) in 2 + 1 dimensional space–time, and the point defects which we observe correspond to virtual monopole–antimonopole pairs.

Measured polarization dependencies of the ferrofluid transmission as a function of magnetic field shown in figure 4a are consistent with the hyperbolic metamaterial character of ferrofluid anisotropy in the infrared wavelength range at a sufficiently large magnetic field. Zero degree polarization corresponds to an E field of the electromagnetic wave perpendicular to the direction of the external magnetic field (the case of an ordinary wave). Such polarization is suitable for imaging the microscopic structure of the ferrofluid. On the other hand, microscopic measurements conducted at 90° polarization allow us to assess the effect of microscopic defects on extraordinary wave propagation. As discussed above (see equation (0.3)), propagation of extraordinary photons through the volume of hyperbolic metamaterial may be described in terms of particle world line evolution in an effective 2 + 1 dimensional Minkowski space–time. Thus, studying extraordinary light scattering by the point defects of the ferrofluid may be treated as a model study for point defects in a Minkowski space–time. An example of such a study is presented in figure 4b,c. The image in figure 4b measured at zero degree polarization shows the same point defect (y-split) as in figure 3f. The microscopic image of the same region measured at 90° polarization clearly demonstrates enhanced extraordinary photon scattering by the ferrofluid filament split. Increased scattering by the effective space–time defects is indeed expected, as in the absence of such defects the ferrofluid should behave as ‘empty space–time’.

Let us now proceed with examples of linear and planar defects observed in the bulk of the polarized ferrofluid. It appears that linear defects may behave as particle world lines (see figure 5a as an example), while planar defects (figure 5b) may resemble cosmic strings, which are hypothesized to exist in Minkowski space–time [5]. While transformation optics-based models of cosmic strings have been proposed [14], their experimental realization was not possible due to difficulties associated with three-dimensional nanofabrication issues. On the other hand, our experimental observations indicate that linear and planar defects in effective Minkowski space–time appear naturally in the disordered ferrofluid. The cosmic string represents a topological defect of space–time that may be described in geometrical terms by delta function-valued torsion and curvature components. It has been noted by several authors [5,15,16] that the space–time geometries of cosmic strings in 3 + 1 dimensions exhibit close relations to the distortions of solids, which may likewise be regarded as topological defect lines carrying torsion and curvature. Similar analogies hold in 2 + 1 dimensional gravity [17], and therefore applicable to the aligned ferrofluid. One of the possible scenarios for such a string analogue to appear in the aligned ferrofluid is that represented in figure 5b. It shows schematically a quasi-three-dimensional view of a 1 + 1 dimensional sheet in space–time, which may form a toy model of a ‘cosmic string’. This structure is based on shifting a twisted linear defect shown in figure 5a along y-coordinate. Such an analogue of a cosmic string may arise due to twisting of the self-assembled cobalt nanoparticle filaments. Observations of the disordered ferrofluid using polarization-dependent IR microscopy presented in figure 5c–f indeed reveal multiple linear and planar
Figure 4. (a) Experimentally measured transmission of the cobalt-based ferrofluid at $\lambda = 1.55 \, \mu m$ as a function of external magnetic field and polarization angle. Zero degree polarization corresponds to $E$ field of the electromagnetic wave perpendicular to the direction of external magnetic field. Frames (b) and (c) show microscopic images of a point defect in the ferrofluid measured at 0° and 90° polarization, respectively. Image (c) reveals increased scattering by the point defect. (Online version in colour.)

defects. Comparative examination of these images indicates that twisted filaments observed in zero degree polarization images (figure 5e and its zoom in figure 5c) give rise to scattering features observed at 90° polarization (figure 5f). Due to the small (10 µm) pathlength of the cuvette, the ‘side view’ images presented in figure 5c,e,f do not allow clear differentiation between linear and planar defects of the periodic filament structure. On the other hand, a microscopic image of the ferrofluid sample taken along the axes of the nanowires shown in figure 5d (which was obtained by directing the external magnetic field perpendicular to the ferrofluid-filled cuvette) enables such differentiation. A planar defect of the ordered filament structure (a grain boundary) is clearly seen in figure 5d. It is highlighted by the green dashed line, while the broken rows of filaments are indicated by red dashed lines. Thus, planar defects of the ferrofluid filament structure, which correspond to analogues of cosmic strings in an effective 2 + 1 dimensional Minkowski space–time, have been indeed detected in our experiments. Note that the space–time analogy does help in understanding of our experimental results. Evaluation of figure 5e,f clearly demonstrates that compared to ordinary photons, the defects of ferrofluid lattice exhibit a much stronger effect on extraordinary photon propagation. Unlike ordinary photons, extraordinary photons perceive the
Figure 5. (a) Schematic geometry of a linear defect in the periodic wire array structure formed by twisting of self-assembled cobalt nanoparticle filaments. Such a linear defect may represent a world line of a particle at rest. (b) Quasi-three-dimensional view of a $1 + 1$ dimensional sheet in space–time, which forms one of the possible toy models of a ‘cosmic string’. This structure is based on shifting a twisted linear defect shown in (a) along $y$-coordinate. Only two layers of the three-dimensional structure are shown for clarity. (c) Magnified image of twisted filaments observed in the disordered ferrofluid. (d) A microscopic image of the ferrofluid sample taken along the axes of the nanowires, which was obtained by directing the external magnetic field perpendicular to the cuvette. A planar defect of the ordered filament structure (a grain boundary) is highlighted by the green dashed line. (e,f) Microscopic images of the ferrofluid exhibiting multiple linear and planar defects measured at $0^\circ$ and $90^\circ$ polarization, respectively. Image (f) reveals increased photon scattering by these defects. (Online version in colour.)

Ferrofluid as an effective Minkowski space–time. Increased scattering by the defects is indeed expected, since in the absence of such defects from the point of view of extraordinary photons, the ferrofluid should behave as ‘empty space–time’.
Finally, let us examine some volume defects observed in the bulk of the polarized ferrofluid. While several analogues of such defects have been explored in the experiments with plasmonic hyperbolic metamaterials [18,19], reduced dimensionality of such metamaterials severely limits the range of possible configurations. On the other hand, the true three-dimensional nature of newly developed self-assembled ferrofluid-based hyperbolic metamaterials lets us study such interesting new possibilities as the recently proposed space–time cloak geometry [7]. The space–
time cloak is obtained via \((x, t) \rightarrow (\bar{x}', \bar{t}')\) space–time transformation, which creates a void near the space–time origin, as illustrated in figure 6a. The transformation is a composition of a Lorentz boost \((x, t) \rightarrow (\bar{x}, \bar{t})\) with velocity \(v = c/n\) (described by equation (0.4)), followed by applying a ‘curtain map’

\[
\bar{x}' = \left[ \frac{(\delta + |c\bar{t}|)}{(\delta + n\sigma)} (\bar{x} - \text{sgn}(\bar{x})\sigma) + \text{sgn}(\bar{x})\sigma \right], \quad \bar{t}' \rightarrow \bar{t},
\]  

(0.6)

followed by an inverse Lorentz transformation \((\bar{x}', \bar{t}) \rightarrow (x', t')\) [7]. In the resulting space–time metric, any space–time event which occurs inside the space–time cloak (the black area in figure 6a) is concealed from a distant observer. Note that the blue arrows in figure 6a,b follow the local \(\bar{x}' = \text{const.}\) direction. In the ferrofluid, this direction corresponds to the direction of cobalt nanoparticle filaments. Therefore, macroscopic bending of filaments, which is similar to the bending of blue arrows in figure 6a,b, may lead to the appearance of a space–time cloak geometry in the effective 2 + 1 dimensional metamaterial Minkowski space–time. Experimental images of disordered ferrofluid indeed provide multiple examples of such volume defects. An example of such a defect is indicated by the dashed oval in figure 6c. In order to relate it to equation (0.6), a magnified image of the space–time cloak area indicating its orientation with respect to \((\bar{x}', \bar{t}')\) and \((x', t')\) coordinate planes is shown in figure 6d. The cloaked area is marked by the green dashed line. The optical field intensity in the defect interior appears to be reduced, which is consistent with the space–time cloaking effect proposed in [7].
In conclusion, we have presented a systematic microscopic study of point, linear, planar and volume defects in the self-assembled wire array hyperbolic metamaterials based on cobalt nanoparticle filaments aligned by an external magnetic field. As extraordinary light propagation in such metamaterials may be described in terms of an effective Minkowski space–time metric, microscopic defects of the metamaterial exhibit a strong similarity to such Minkwoski space–time defects as magnetic monopoles, cosmic strings and the space–time cloak. Our experimental observations have many important fundamental implications which reach far beyond transformation optics and electromagnetic metamaterial theory. As demonstrated in [20,21], nonlinear optics of the polarized ferrofluid may be described in terms of analogue gravity. Moreover, as pointed out recently by Mielczarek [8], the properties of self-assembled magnetic nanoparticle-based hyperbolic metamaterials exhibit strong similarities to the properties of some quantum gravity models, such as loop quantum cosmology. Thus, microscopic studies of polarized ferrofluid provide a unique playground to study a microscopic analogue model of gravity in action.

Authors’ contributions. I.S. conceived of the study, designed the study, coordinated the study and drafted the manuscript; V.S. carried out the experiments, participated in data analysis and helped draft the manuscript; A.S. carried out the experiments, participated in data analysis and helped draft the manuscript; all authors gave final approval for publication.

Competing interests. We declare we have no competing interests.

Funding. This work is supported by NSF grant no. DMR-1104676.

References


